PUZZLE CORNER

ALLAN J. GOTTLIEB

A Nontrivial Bet: Poker Face vs. High Roller

This introduction will be short, containing just three short contributions from you, the readers. Jonathan Aronson was looking through an old *Technology Review* and read the solution given for finding the radius of a sphere circumscribing a regular tetrahedron. He found an alternate solution using the trick of inscribing the tetrahedron in a cube and points out that other properties of the tetrahedron can be derived quickly using this trick.

Greg Huber adapted an argument of his based on the four 4s problem to show that if our yearly problem is expanded to permit logs and square roots, then any integer can be expressed using the digits 1,9,8,8. Robert Bart notes that the yearly problem is in for hard times, since until 2134 all years will have repeated digits or a zero. Although Bart suggests loosening the rules, I prefer to sit tight and wait it out until 2134.

Problems

M/J 1. We begin with a Bridge problem from Doug Van Patter:

North
♠ 4 3
♥ K 7 6
♦ A Q J 5 2
♣ K Q 5

South
♠ K 6
♥ J 10 8 5 3 2
♦ K 7
♣ A 6 4

Bidding

N	E	S	W
1D	1S	2H	Pass
3H	Pass	4H	Pass
Pass	Pass		

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012.

M/J 2. Our next problem is from Matthew Fountain, usually a sure bet: Two gamblers, High Roller and Poker Face, love to gamble but not with each other, as High Roller always wins at dice and Poker Face always wins at cards. A mutual friend suggests a fair and nontrivial bet that they could make. Each could privately write three amounts adding to $1,000 on a slip of paper. Then they could compare their amounts, the largest against largest, the smallest against the smallest, and the median against the median. The one with the larger amount in two of these three comparisons would win the bet and take the $1,000. Can you help Poker Face decide what to write on his slip?

M/J 3. Inspired by 1987 F/M 5, John Weissberg has a roller coaster problem: Imagine a roller coaster with a ruler, initially pointing straight up, balanced on it. Even ignoring air drag, the ruler's attitude is unstable, and it will eventually fall over. Is there a path, e.g. a series of hills and loops, for the coaster such that the ruler's attitude remains stable and it will not fall over?

M/J 4. Sidney Shapiro believes (suspects?) that he has an interesting problem: Four suspects, each of different height, are in a house surrounded by a posse. The actual criminal is known to be the tallest. The sheriff is constrained to make only a single arrest. The suspects can be arrested only as they leave the building. They do so one at a time. Which suspect—the first to leave, the second, etc.—should the sheriff arrest?

M/J 5. Richard Hess has a point, a line, a compass, and a straight edge. He wants to construct the perpendicular to the line through the point using the compass only once and the straight edge as many times as necessary.

Speed Department

SD 1. Here is one from Jim Landau. The mathematical connection is admittedly very tenuous but we both like it:

George Washington turned down an offer to become king of the Colonies and became our first president instead. However, without compromising his democratic principles, he could still have chosen to style himself President George I. In that case, can you identify the following Presidents:

1. Franklin I
2. Hiram I
3. James V
4. Thomas II
5. Leslie I

SD 2. Thomas Harriman notes that in a game of backgammon your opponent may offer to double the stake once (unless you double him back): if you refuse the offer, you immediately lose the original stake. How low can your probability of winning be and still have it advantageous to accept?

Solutions

JAN 1. You are in a four-spade contract and take the opening lead of ♥2 in dummy. Assuming rubber bridge with neither side vulnerable, how do you play the trumps? What if match points are being used?

Dummy
♠ 5
♥ A K 8
♦ Q 9 3
♣ A K 8 6 5 3

Declarer
♠ A K 10 9 8 7 2
♥ 6
♦ J 10 8 6
♣ Q

Our first solution is from Richard Hess, who notes that at trick two South can either take the trump finesse or go for the drop. The only distributions of opponents' trumps that give different outcomes for the two options are shown in the following table.

East	West	Drop	Finesse	Probability
x	QJxx	Make 9	Make 10	.08478
Q	Jxxx	Make 10	Make 9	.02826
J	Qxxx	Make 10	Make 9	.02826
QJ	xxx	Make 11	Make 10	.033915

In rubber bridge it is better to finesse, giving a higher probability of making game. With match points, however, it is better to play for the drop, having a probability of .090435 of bettering opponents who take the finesse and having a probability of only .08478 of being bettered by such opponents.

Also solved by Robert Bart and the proposer, Douglas Van Patter.

JAN 2. Compute the natural logarithms of the integers from 2 to 10, using pencil and paper *only* (no calculators, computers, or numeric tables), to four decimal places of accuracy.

We offer two different solutions. William Waite sent in a solution done with pen and paper (including all long divisions, which we have omitted). He begins with the Taylor series

$\ln[(1 + x)/(1 - x)] = 2(x + x^3/3 + x^5/5 + x^7/7 + x^9/9 \ldots)$.

Letting $x = 1/3$ we get

$\ln 2 = 2(1/3 + 1/81 + 1/1215 + 1/5309 + 1/177147 + \ldots) = 0.69315$.

Now let $x = 1/5$. This time we need only four terms to see that

$\ln(3/2) = 0.40546$

and hence

$\ln 3 = \ln(3/2) + \ln 2 = 1.09861$.

Naturally,

$\ln 4 = 2 \times \ln 2 = 1.38630$.

Now let $x = 1.9$ and keep three terms to get

$\ln (5/4) = 0.223142$

and hence

$\ln 5 = \ln(5/4) + \ln 4 = 1.60944$.

Also

$\ln 6 = \ln 2 + \ln 3 = 1.79176$.

Finally let $x = 1/6$ and keep four terms to get

$\ln(7/5) = 0.336474$

and hence

$\ln 7 = \ln(7/5) + \ln 5 = 1.94591$.

The rest is easy

$\ln 8 = \ln 2 + \ln 4 = 2.07945$

$\ln 9 = 2 \times \ln 3 = 2.19722$

$\ln 10 = \ln 2 + \ln 5 = 2.30259$.

Our second solution, from John Chandler, uses different Taylor series. Mr. Chandler writes (actually sends via electronic mail):

$$e^x = 1 + x + x^2/2 + x^3/6 + \ldots \quad (1)$$

and

$$\ln(1 + x) = x - x^2/2 + x^3/3 - x^4/4 + \ldots \quad (2)$$

Equation (1) shows a simple way of computing (which I assume we must not simply look up!): $1 + 1 + .5 + .1666667 + .0416667 + .0013889 + .0001984 + .0000248 + .0000027 = 2.7182815$, which gives us the base of natural logarithms to 5+ decimal places. Since equation (2) doesn't converge very fast for $x = 1$, let's try a short cut: squaring e gives 7.38904 (to 5 places), and this is reasonably close to 7. In fact, it's 7*1.05558. We can save a bit of multiplying by noticing that .05558 is very close to 1/18, certainly close enough for plugging into equation (2): $\ln(1.05558) = .05558 - .00155 + .00006 = .05409$, so

$\ln 7 = 2 - \ln(1.05558)$, and hence $\ln 7 = 1.94591$.

Now, $7/8 = 1 - 1/8$, so plugging that into equation (2) gives:

$\ln(7/8) = -.12500 - .00781 - .00065 - .00006 = -.13352$, so

$\ln 8 = \ln 7 - \ln 7/8$, and hence $\ln 8 = 2.07943$.

$\ln 2 = (\ln 8)/3$, and hence $\ln 2 = 0.69314$.

$\ln 4 = \ln 8 - \ln 2$, and hence $\ln 4 = 1.38629$.

Next, $49/50 = 1 - .02$, so that gives:

$\ln(49/50) = -.02000 - .00020 = -.02020$, but

$\ln 49 = 2^*\ln 7 = 3.89182$.

So $\ln 50 = 3.91202$, and

$\ln 25 = \ln 50 - \ln 2 = 3.21888$.

Hence $\ln 5 = 1.60644$.

Also, $\ln 10 = \ln 5 + \ln 2$, and hence $\ln 10 = 2.30258$.

Finally, 81/80 = 1 + 1/80, and plugging that into equation 2 gives:
$\ln 81/80 = .01250 - .00008 \approx .01242$.
But $\ln 80 = \ln 8 + \ln 10$, $\ln 80 = 4.38201$, so $\ln 81 \approx 4.39443$. So
$\ln 9 = (\ln 81)/2$, and hence $\ln 9 \approx 2.19722$.
$\ln 3 = (\ln 9)/2$, and hence $\ln 3 \approx 1.09861$.
$\ln 6 = \ln 3 + \ln 2$, and hence $\ln 6 \approx 1.79175$.

Also solved by Norman Wickstrand (who also showed that the antilogs (base 10) of 0, 0.1, 0.2, 0.3, . . ., 1.0 can all be computed easily within 1% by remembering two approximations: $\log 2 \approx .3$ and $\log\pi \approx .5$), Chip Whiting, Harry Zaremba, Richard Hess, Jonathan Aronson, Robert Bart, and the proposer, Jim Landau.

JAN 3. It is possible to obtain all the integers from 1 to 40 by adding and subtracting various combinations of only four different integers. What are these four integers?

Louis Wadel actually shows (slightly) more than twice what was required:
The answer is 1, 3, 9, 27. This can be seen easily by considering a 4-digit base 3 number, dddd, where each digit can have the value 0, +1, or −1. A given number then represents the absence, or the positive, or the negative presence of 27 ($=3^3$), 9, 3, or 1. Thus all integers from −40 to +40 can be represented, a total of 81 = 3^4 numbers.

Also solved by Raymond Gaillard, Sidney Williams, Harry Zaremba, Jim Landau, Richard Hess, Jonathan Aronson, Douglas Ell, Ken Rosato, Greg Spradlin, and Robert Bart.

JAN 4. *"The 20th Anniversary Party"* The hostess told me the youngest of her three children likes her to pose this problem, and proceeded to explain. "I normally ask guests to determine the ages of my three children, given the sum and product of their ages. Since Smith missed the problem tonight and Jones missed it at the party two years ago, I'll let you off the hook." Your response is "No need to tell me more, their ages are. . ."

Our final solution is from Robert Buegler. Copies of his program and its output are available from the editor. Mr. Buegler writes:
I've enjoyed your puzzles for a number of years but have never sent in a solution. This one intrigued me and presented an unusual challenge. I must congratulate Richard Hess for his rapid and "intuitive" response: "No need to tell me more, their ages are . . ." I could neither come to an intuitive conclusion nor see an elegant solution and decided to tackle it brute force with a Basic program on my PC. I thought there might be more than one possible solution, but after refining my approach, concluded that there was a single solution based on the following assumptions:
a. At the 18th Anniversary, the minimum age of the youngest child was 2, since that seemed to be the limit for a child to enjoy a game; and
b. The maximum age of the oldest child was 17, since this was the 18th Anniversary, and it didn't seem proper to impugn the moral integrity of the hostess.
The correct solution by Hess, at the 20th Anniversary party, was that the three children's ages were 5, 6, and 16.

To summarize briefly: In order for Jones and Smith to have incorrect solutions there had to be, at each Anniversary, two possible age combinations with the same sum and product. For each age sum from minimum to maximum at the 18th Anniversary, the program first develops an array of all age combinations with that sum. Each such array is then checked for those sums which have like products, which are stored as pairs in a second array. This process is repeated for the 20th Anniversary. The program prints the resultant pairs for each array. The program then compares the age group for each entry in the 18th Anniversary array, incremented by 2 years, with the entries in the 20th Anniversary array. For each correct match, the result is printed out, annotated as to which are the correct and incorrect solutions. The printed results demonstrate that there is only one correct solution.

Also solved by Robert Bart, Ken Rosato, Steven Feldman, Jonathan Aronson, and the proposer, Richard Hess.

Better Late Than Never

1987 OCT 2. W. Messner has responded.

OCT 4. Tracy Wichmann has responded.

N/D 1. Nathan Glasser and W. Messner have responded.

N/D 2, N/D 3. Nathan Glasser has responded.

N/D 4. Nathan Glasser and W. Messner have responded.

N/D 5. Nathan Glasser has responded.

Proposers' Solutions to Speed Problems

SD 1. Franklin Pierce, Ulysses S. Grant (born Hiram Ulysses Grant), Thomas Woodrow Wilson, Gerald Ford (born Leslie King).

SD 2. 25%.