Seeking the Constants in the Physical Constants

Since this is the first issue of a new academic year, I once more review the ground rules under which this department is conducted.

In each issue I present five regular problems (the first of which is chess, bridge, or computer-related) and two "speed" problems. Readers are invited to submit solutions to the regular problems, and three issues later one submitted solution is printed for each problem; I also list other readers whose solutions were successful. For example, solutions to the problems you see below will appear in the February/March issue. Since I must submit that column sometime in November, you should send your solutions to me during the next few weeks. Late solutions, as well as comments on published solutions, are acknowledged in the section "Better Late Than Never" in subsequent issues.

For "speed" problems the procedure is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same column in which the problem is published. For example, the solutions to the issue's "speed" problems are given below. Only rarely are comments on "speed" problems published or acknowledged.

There is also an annual problem, published in the first issue of each new year; and sometimes I go back into history to republish problems that remained unsolved after their first appearances.

Problems

OCT 1. Lawrence Kells reports that a friend of his once held

\[
\begin{array}{cccc}
1 & 3 & 2 & 1 \\
4 & 3 & 2 & 2 \\
5 & 4 & 3 & 2 \\
\end{array}
\]

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 551 MERCER ST., NEW YORK, N.Y. 10012.

OCT 2. Hy Tran wants you to show that for \( k, r, \) and positive integers the expression

\[
\sum_{k=1}^{r} \frac{r!}{k!(r-k)!}
\]

is always even—i.e., an integral multiple of 2.

OCT 3. The following problem first appeared in Computers and People in 1985. A "numble" is an arithmetical problem in which digits have been replaced by capital letters; there are two messages, one which can be read right away and a second one in the digit cipher. The problem is to solve for the digits. Each capital letter in the arithmetical problem stands for just one digit 0 to 9. A digit may be represented by more than one letter. The second message, expressed in numerical digits, is to be translated (using the same key) into letters so that it may be read; but the spelling may use puns or deliberate (but evident) misspellings, or may be otherwise irregular, to discourage cryptanalytic methods of deciphering.

\[
\begin{array}{c}
T \times W \ O \ L \ F \\
K \ T \ L \ O \\
E \ W \ T \ F \\
S \ W \ E \ W \\
H \ H \ W \ L \\
\hline
W \ L \ I \ W \ O \ O
\end{array}
\]

OCT 4. Here is a ladder problem from Joseph Molitoris and George Butwin. It looks familiar so I would not be surprised to hear that a similar if not identical problem appeared in Puzzle Corner ten or fifteen years ago:

Given an alley of width \( W \). Two ladders of length 40 ft. and 30 ft. are laid against opposite walls. They intersect 10 ft. above the ground. Find \( W \) and \( L \), the width and one of the lengths of intersection. (See diagram next page.)
Also solved by the proposer Jim Landau, who reports that Allen Beadle, mentioned in the original statement of MJ 1, is also the (heretofore) anonymous slide rule expert who "computed," 9999! mentally. Moreover, it was Beadle who told Landau the Zorn's Lemma riddle.

MJ 2. What is the pressure due to gravity at the center of a spherical heavenly body of uniform density, mass M, and radius R?

The following solution is from Dennis White:

Consider a small right cylinder with its axis placed radially within the spherical body:

\[ p(r) = \frac{GM^2}{R^3} \]

The gravitational force on it is provided by the central sphere "underneath" with radius \( r \) (outer layers have a net zero attraction on it), and this sphere's mass can be considered to be concentrated at 0. Taking the cylinder to be a point mass, this force is \( F_G = -\frac{GM^2}{4\pi r^2} \Delta p(\Delta \rho) = -\frac{4\pi GM^2 \Delta \rho \Delta r}{3} \) where \( \rho \) is the body's density. For the cylinder to be at equilibrium, this must be matched by the buoyant force provided by the pressure differential \( dp = p(r + dr) - p(r) \). (The pressure is a function of \( r \) alone by symmetry.) This force is \( F_B = \Delta p \Delta r = \Delta p(r + dr) - \Delta p(r) \).

Setting \( F_B = F_G \) we have

\[ \Delta p = \frac{4\pi GM^2 \Delta \rho \Delta r}{3} \]

Solved, with the boundary condition \( p(0) = 0 \), this gives

\[ p(r) = \frac{2\pi G M^2 R^3}{3} \]

or substituting

\[ p = \frac{M^2(3k+1)}{4\pi R^3} \]

and so \( p(0) = \frac{M^2}{3\pi} \), the pressure at the center, is

\[ p(0) = \frac{(3k+1)G^2M^2}{3\pi R^3} \]

Thus also solved by Matthew Fountain, Winslow Hartford, and the proposer, Bruce Calder.

MJ 3. Find nine single-digit numbers with sum 45 and product 362880. One solution is (1,2,3,4,5,6,7,8,9).

The following solution is from Bob Marshall, who says "thanks for publishing this puzzle that I could solve!"

Start by observing that all such sets must be assembled from numbers included in the prime factorization of 362880 with the optional addition of one or more 1s. (Note that the starter solution included a 1; therefore this must be permitted.) The prime factorization of 362880:

\[ 362880 = 2^9 \times 3^3 \times 5 \times 7 \]

Note immediately that each solution must include the single digit numbers 5 and 7, because these factors must be included to achieve the product and any multiple of either of these numbers is not a single digit. The case of whether or not a 1 is included in the solution will be dealt with specially because the 1 does not change the product but will change the sum. For the case of no 1s is the factors \( 2^9 \times 3^3 \) must be distributed among seven single-digit numbers. These are constructed by inspection as follows:

\[ 1, 2, 3, 4, 5, 6, 7 \]

As can be seen from the Sum column none of these sets of single-digit numbers has the required sum. For the case with a single 1 the factors \( 2^8 \times 3^3 \) must be distributed among six single-digit numbers. These are constructed by inspection as follows:

\[ 1, 2, 3, 4, 5, 6, 7, 9 \]

In this case there are two solutions—the combinations: 1, 2, 4, 4, 5, 6, 7, 8, 9 and 1, 2, 3, 4, 5, 6, 7, 8, 9. This latter solution is the "given" solution and my method would be proved non-exhaustive if it failed to find this solution. For the case with two 1s, the factors \( 2^7 \times 3^3 \) must be distributed among five single-digit numbers. These are constructed by inspection as follows:

\[ 1, 2, 3, 4, 5, 6, 7, 9 \]

Thus there are no solutions for the case of two 1s. For the case with three 1s the factors \( 2^6 \times 3^3 \) must be distributed among four single-digit numbers. However, this cannot be done as any combination of those factors distributed among four numbers yields at least one two-digit number. Therefore the search for solutions is complete. To recap, the two solutions are:

\[ 1, 2, 3, 4, 5, 6, 7, 8, 9 \]

\[ 1, 2, 3, 4, 5, 6, 7, 9 \]

Also solved by Thomas Stowe, Avi Ornstein, Mike Gennert, Michael Jung, Alan Taylor, Dick Robberti, James Landau, John Woodster, Larry Bell, Matthew Fountain, Winslow Hartford, Ken Rosato, and Steve Feldman.

MJ 4. Find the axes of the largest (in area) ellipse that can be inscribed in a triangle having sides of length 3, 4, and 5 inches.

The following solution is from the proposer, Matthew Fountain.
The major and minor axes are 2.93986 and 1.57110. When the 3.4.5 triangle is viewed from an angle that makes it appear to be equilateral, the maximum area ellipse must appear as a circle. The foreshortening takes place in the direction of the major axis of the ellipse, with no foreshortening in the direction of the minor axis, so it may be concluded that the angle of view does not affect the ratio of apparent area of ellipse to apparent area of triangle. If an ellipse does not appear as a circle when the triangle appears equilateral, it follows that if the ellipse is viewed as a circle, the triangle would then appear not an equilateral triangle, implying that the ratio of ellipse area to triangle area was smaller than the previous case. An equilateral triangle is the smallest triangle circumscribing a circle.

Similarly, a proper-size equilateral triangle can be foreshortened into a 3.4.5 triangle. The figure above shows such an equilateral triangle with sides of length 5. 5. so that its projection on a plane is the 3.4.5 triangle ABC. One vertex is the distance A above A, one vertex is the distance C above C, and the remaining vertex lies in the plane at B. It is found by writing $a^2 = \frac{5^2}{2} - \frac{3^2}{2} - \frac{4^2}{2}$, and $a - c^2 = \frac{5^2}{2} - \frac{3^2}{2}$ and solving for S. The elimination of a and c results in the equation

$3S^2 + 100S^2 + 241 = 0$,

with the solution $S = 5.09198$. The altitude of the equilateral triangle is $(5S^2)/\sqrt{2} = 4.40978$ and the diameter of the inscribed circle is two-thirds the altitude—that is, 2.93986. This is also the major axis of the ellipse in the 3.4.5 triangle. The respective areas of the equilateral triangle and the 3.4.5 triangle are $5.09198(4.40978)/2 = 11.22728$ and $(3)(4)/2 = 6$. The minor axis of the ellipse is $(2.93986)(6)/11.22728 = 1.571096$.

Also solved by Norman Wickstram, Winslow Hartford, Edward Dawson, and Dennis White.

MI 5. Through which regular polyhedron can one carve a hole such that another regular polyhedron of the same size and type can pass? For example, a cube with unit edge can pass through a suitably cut hole in another cube with unit edge. The following solution is from Matthew Fountain: The regular tetrahedron, the cube, and the regular octahedron can each be passed through a hole in a polyhedron identical to it. The dodecahedron and icosahedron cannot. One procedure to determine then this is possible is to view each polyhedron from various angles and compare silhouettes. When no silhouette can fit into another, it is possible. If a polyhedron is rotated, it is possible to obtain other silhouettes as projections of the polyhedron on one plane, in positions that assist comparisons between them. As a regular polyhedron rotates, its vertices move in the surface of a sphere, their projections on a plane moving to and from the circumference of a circle. The dodecahedron and icosahedron have so many vertices that there is no matter what axis of revolution is chosen, the vertices of either silhouettes are never going outward into the circle at the same time. This would be necessary for a silhouette to fit within another, as there is not another for any appreciable revolution of one silhouette owing it to fit into another.

The second figure above shows the square ABCD as the silhouette of a cube projected upon a plane parallel to one face, Rotating the cube about its center on an axis parallel to AC produces the silhouette AA'B'C'EG. As EE and GH are perpendicular to AC, the first silhouette will fit in the second when rotated slightly.

The third figure (above) shows the square ABCD as the silhouette of a regular octahedron, projected upon a plane parallel to four edges. Rotating the octahedron about its center on an axis parallel to AC produces the diamond-shaped silhouette extending between A and C. An additional rotation of the octahedron about its center on an axis parallel to BD produces a silhouette lying within the ABCD. Also solved by Winslow Hartford.
Better Late Than Never

JAN SD1. Sidney Williams has responded.

JAN 3. Dennis White notes that an unstated requirement was that the 7th term uses the digit 1.

FM 2, 3, 4. Winslow Hartford has responded.

APR 3. Joseph Muskat, Naomi Markovitz, and Winslow Hartford have responded.

APR 4. Naomi Markovitz and Winslow Hartford have responded.

APR 5. Joseph Muskat and Winslow Hartford have responded.

MJ SD1. Linda Kalver has responded.

MJ SD2. Both Turner Gilman and the proposer Jim Landau assert that the answer should be 4 meters and not 2.8284 . . . meters. Mr. Gilman writes: “The answer given would be correct if the ‘lateral rig’ was constructed logically, in which case the center of area of the triangular sail would be at the mast location. This would minimize rotational forces, but that is not the way the puzzle is presented. I assume that the lateral yard is attached to its center to a mast 2 meters high.” The key is the word center. There are some details omitted, i.e. the clearance between deck and sail and any excess height above the sail, but on the same basis that the proposer gets his 2.8284 meters I arrive at a solution of 4.0000 . . . meters!”

Proposers’ Solutions to Speed Problems

SD 1. You are just using an iterative method to find the solution to cos(x) = x.

SD 2. All the numbers 371 are prime for k equal to the consecutive numbers 1 to 7, inclusive. The smallest value of k for which the number is composite is 8: 351 = 333,333,331 = 17 × 19,607,843.

This Space Available

For your advertising message
Call: Peter Delaliotis
Technology Review
(617) 253-6290

North Coast Innovation, Inc.

Specialists in Product Manufacturing Technologies
Serving high volume manufacturers
• Automotive
• Appliances
• Consumer Products
• Pharmaceutical
• Hospital Supplies
Providing electromechanical product and process consulting services
• Analysis
• Design
• Evaluation

L. Scott Duncan ‘65
John M. Collins ‘82
7845 Delabrook Rd.
Cleveland, Ohio 44131
(216) 642-8444

Boyle Engineering Corp.

Engineers/Architects
Thomas S. Maddock ’51
1501 Quail Street
P.O. Box 7250
Newport Beach, CA
92658-7250
(714) 476-3400

Complete Professional Services:
• Water Resources
• Pollution Control
• Architects
• Highways and Bridges
• Dams and Reservoirs
• Environmental Science
• Computer Sciences/CADD
• Agricultural Services
• Management and Administration
• Transportation
• Engineering

Serving the Science Community Since 1953

James Goldstein & Partners

S. James Goldstein ’48
Elliot W. Goldstein ’77
89 Millburn Avenue
Millburn, N.J. 07041
(201) 467-8840

Blessner Associates

Consultants in Electronic Systems
In research, development and prototype products
• analog and digital signal processing
• electronic hardware
• analog-digital conversion
• active filters
• design audits and evaluation
• video systems
• simulations and modeling
• electronic reapportion
• circuit design

Barry Blessner, Ph.D.
Box 335
Rumford, R.I. 02917
(617) 484-5405

Kearsege Ventures, L.P.

A venture capital fund seeking investments in companies:
located within one hundred miles of Manchester, New Hampshire
products or services which are developed in the marketplace
rapid growth in sales and profits
Investments from $250,000 to $600,000

Kurt D. Bleicken ’62
Alan Veugleri
General Partners
66 Hanover Street
Suite 202
Manchester, N.H. 03101 (603) 625-1468

LEA Group

Consultants to industry, commerce, government and institutions.
Eugene R. Eisenberg ’43
Louis Revroot ‘50
William S. Hartery ’52
Veitns H. Ule ’76

Building Design
Environmental Engineering
Site-Civil Design
Roofing Technology
75 Kneeland Street
Boston, MA 02111
(617) 426-6300
Suite 213
9311 Mullard Drive
Laurel, MD 20726
(301) 725-3445

Artisan Industries Inc.

Problem Solving in
- ENGINEERING -
Chemical-Mechanical
Biotechnology
Microprocessor
Food-Process Development System
Instrumentation
73 Pond Street
Waltham, MA
02254-5103
(617) 893-6800
TLX 023312
FAX (617) 647-0143

James Donovan ’28
James L. Baldo ’46
Victor Takala ’43
Dr. Joseph Quatzi ’72

MIT 62 OCTOBER 1987