ALLAN J. GOTTLIEB

How the Right Fahrenheit Becomes the Wrong Celsius

I begin with an apology. Last month several on-time solutions to A/S problems were not reported. Specifically, Walter Nissen and Judy Badner solved problem 1; Yale Zussman solved problems 1, 2, 4, and 5; Michael Gennert solved problems 1, 4, and 5; and David DeLeeuw, Chip Whiting, and Harry Lieberman solved problem 5. Sorry.

I am occasionally asked to explain how I decide if a problem should be classified as "speed." Actually, in the majority of cases it is the proposer who decides. If the problem arrives with no classification by its author, it is considered to be a regular problem unless it is clearly whimsical. When the proposer classifies the problem as borderline, I pass judgment.

Richard Hess let me know about a puzzle exhibition being held at the Folk Art Museum, 5814 Wilshire, Los Angeles. Although this issue may not appear until after the exhibition ends on 22 February, I want to mention it if for no other reason than to compliment the sponsors on their invitation, a clever T puzzle that I will use in some future issue. I was also invited to the eighth International Puzzle Party, the invitation carefully formed into a hexaflexagon.

Finally, Jim Landau noticed that two problems previously in *Puzzle Corner* were discussed in the August 1986 issue of *Byte*.

Problems

F/M 1. We begin with a chess problem that K. Heuer submitted to USENET, a loosely connected computer network used for (human) communication:
What is the shortest chess game that ends with the move "Pawn takes pawn, en passant, double check, mate"?

F/M 2. Thomas Harriman notes that the Cubs lost 13 games in a row during the summer of 1985 and asks what is the *a priori* probability that this is the major league record?

F/M 3. Frank Rubin notes that a general Fibonacci series is formed by taking $x_1 = A$, $x_2 = B$, $x_{n+2} = x_n + x_{n+1}$ for $n = 1, 2, 3, \ldots$ and that its sequent series is formed by taking $x_1 = A$ and $x_2 = B + 1$. If the tenth terms of a series and its sequent series are 127 and 161, respectively, what are the 11th terms?

F/M 4. Our next problem is from Jerry Grossman:
Weather reports on the radio typically give the temperature in degrees Fahrenheit, and the more progressive stations typically follow with an equivalent temperature on the Celsius scale. I assume that the Fahrenheit reading is obtained from a thermometer and represents the temperature to the nearest degree. I also assume that the Celsius announcement is simply obtained from the Fahrenheit by consulting a chart which gives, for each integer F, the whole degree Celsius temperature nearest to F° Fahrenheit. Under some reasonable uniformity assumptions, what is the probability that the reported Celsius temperature is wrong, i.e. not the actual temperature to the nearest whole degree Celsius?

F/M 5. Robert Cherry is worried about losing his Kupie doll while riding in a roller coaster and writes:
Given a roller coaster car that just sits on the rails and is not otherwise held down to the track, is it possible for an object in the car, say a Kupie doll sitting on your lap, to be thrown out as the car goes over the hills in the track? Assume a track with no turns to either side and that the car stays in contact with the rails at all times. Ignore air resistance.

Speed Department

SD 1. Phelps Meaker wants to know the

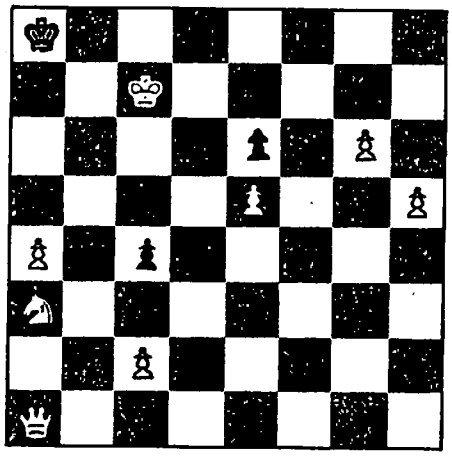
SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012.

perimeter of a five-pointed star in terms of the radius of the circumscribing circle.

SD 2. Bruce Calder has submitted what he calls a real-life speed problem:
Rebecca was laid off on March 1. She will receive her first unemployment check on March 12 for $360 and is eligible to receive $360 every two weeks for the next six months, or until she starts another job. Her budget consists of buying food, paying for entertainment and other discretionary items, and spending $510 per month for rent and other fixed expenses. She pays $135 on the 15th of every month and $375 at the end of every month. Fortunately, Rebecca has just enough money in savings to pay for all of her fixed expenses through March 31. But after April 1, Rebecca must live exclusively on the income from unemployment. How much money can she spend each week on food, entertainment and other discretionary items starting March 1 so that she never runs out of money?

Solutions

OCT 1. White is to move and mate in 10:



The following solution is from John Newman:
1. Q—R1 check K—R2
2. N—N5 check . . .
Why not try the obvious?
...Q × N
If P × Q, Black has no moves: we are talking stalemate!
3. Q—R8 check . . .
Lightning strikes! White counters Black's tainted offer with his own, and it's an offer Black can't refuse: truly an elegant move.
...K × Q
Black has to choke down this "zwishenzug."
4. P × Q K—R2
5. N—N6 check K—R3
The hapless monarch chooses the "best" move; if he goes into the corner, the end comes even more quickly.
6. K—B6 . . .
Plugging the gap. If P—N7 the Black king will wander off the reservation.
...K—R4
7. P—N7 K—N5
8. P—N8 Queen ck. K—R5
9. Q—N5 check K—R6
10. Q—N3 mate

Also solved by Richard Hess, Ronald Raines, Matthew Fountain, and Greg Spradlin.

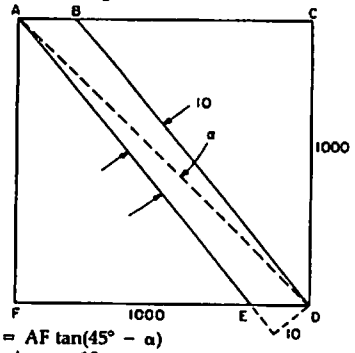
OCT 2. Find the smallest positive integer A such that the first 10 digits in $\sqrt{A}$ consist of 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0, each occurring exactly once.

This turned out to be a computer problem. Sidney Shapiro and his PC found
$\sqrt{1362} = 36.9052841$.

Also solved by Winslow Hartford, Michael Jung, Richard Hess, Matthew Fountain, Steve Silberberg, Mr. Seidel, Steve Feldman, and the proposer, Nob Yoshigahara.

OCT 3. A road 10 meters wide cuts not-quite-diagonally across a 1-kilometer square as shown. What is the area of triangle AEF?

The following solution is from Bruce Gist:



$FE = AF \tan(45° - \alpha)$
$AD \sin \alpha = 10$,
$\alpha = \sin^{-1}(10/1000\sqrt{2}) = 0.405146°$
Area AEF = (AF)(FE)/2
$= [(AF)^2 \tan(45° - \alpha)]/2$
= 492978.407 square meters.

Also solved by N. Tsang, Mr. Seidel, John Prussing, Harry Zaremba, Richard Hess, Avi Ornstein, Winslow Hartford, Chip Whiting, Matthew Fountain, Ken Rosato, Charles Piper, John Rollino, Ronald Martin, David DeLeeuw, Raymond Gallard, Stephan Goldstein, Allan MacLaren, Mark Clements, William Benjamin, Ross Rapoport, Steve Feldman, Richard Tooley, Steve Silberberg, Greg Spradlin, Sidney Shapiro, and the proposer, Jim Landau.

OCT 4. A maxim (common saying, proverb, some good advice, etc.) using 14 or fewer different letters is enciphered (using a simple substitution cipher) into the 10 decimal digits or equivalent signs, plus a few more signs. To compress any extra letters into the set of signs, the encipherer may use puns, minor misspellings, equivalents (like CD or KS for X), etc. But the spaces between words are kept. The problem is to decipher the following:

ΩθμωΩμτθΩΔδμφΔΔζμ
ζΩℓτμΔζφΔΔℓΩΔτ.

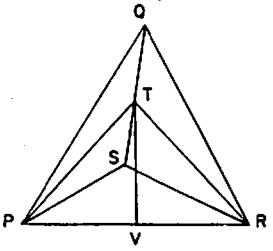
Rajesh Patel galloped to the solution:
To solve this puzzle, first I substituted each Greek letter with a number to make it easier to understand.
123 4135 216 7386 693 91053 6986 60165
This also made certain letter combinations in words easier to see. For instance, the first and third words start with the same two letters transposed. This was the most obvious step. I could only think of a few three-letter word pairs which met the criteria: one, not; ego, get; ace, cat. Of course, I thought of the correct pair last (isn't that always the case?). Next, I filled in these letters, giving me t9E, narrowing that word to toe, lie, or the. Since 9 was the beginning of the next word, "THE" made the most sense. After getting 9, the second word following it was TH8T, and the only word which I could think of was "THAT." At this point I had:
ONE 4OE5 NOT 7EAT THE HO05E THAT T0OT5
I guessed at the second word, since "DOES" was the only word I could think of. This made the sixth word HO05E and the 8th word T0OTS. The only letter that would fit into the sixth word was "R", giving me "HORSE" and "TROTS"; after this I had the answer, the last letter surely being a "B".

Also solved by John Prussing, Ray Kinsley, Audrey Clark, Avi Ornstein, Matthew Fountain and Winslow Hartford.

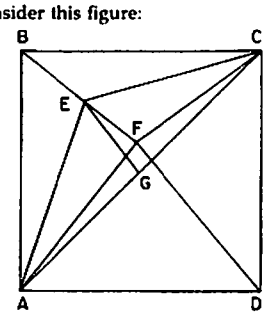
OCT 5. A regular pyramid and a regular tetrahedron both have sides of the same length. Place one face of the tetahedron on one triangular face of the pyramid so that the three vertices of both faces coincide. How many faces does the resulting solid figure have?

Matthew Fountain sent us the following analytical solution:

The number of faces resulting is 6 when two tetrahedrons are joined. When the pyramid has a square base, the number of faces resulting is 5. The addition of the tetrahedron to one face of the pyramid in the manner described results in a solid with two less faces than the combined number of faces on the tetrahedron and the pyramid. The two faces that are bonded together lie in the interior of the resulting solid and are no longer counted. However, when the tetrahedron is added to a regular pyramid having a square base, two faces of the tetrahedron each meet a face of the pyramid at an interior dihedral angle of 180°, so that two faces become extensions of the pyramid faces and do not count as separate faces. To see that the faces meet at an angle of 180°, first consider this figure:

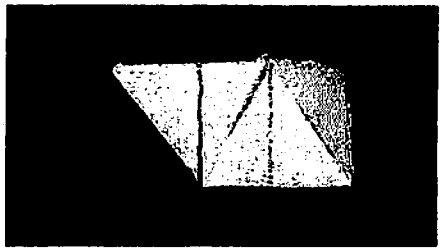


Here PQRS is a regular tetrahedron with edges of unit length. T bisects edges QS and V bisects edge PR. Angle PTR is the dihedral angle between faces PQS and QRS. PT = $\sqrt{3/4}$, and PV = 1/2.
Sin[(1/2)PTR] = PV/PT
= (1/2)/$\sqrt{3/4}$ = $\sqrt{1/3}$.
Next consider this figure:



Here ABCDF is a regular square pyramid with edges of unit length. E bisects edge BF and G bisects diagonal AC. Angle AEC is the dihedral angle between faces ABF and BCF. AE = $\sqrt{3/4}$ and AG = $\sqrt{1/2}$.
Sin[(1/2)AEC)] = AG/AE = $\sqrt{1/2}/\sqrt{3/4}$ = $\sqrt{2/3}$.
Because $\sin^2[(1/2)PTR] + \sin^2[(1/2)AEC] = (1/3) + (2/3) = 1$,
(1/2)PTR + (1/2)AEC = 90° and
PTR + AEC = 180°.

Avi Ornstein found a similar solution and then put together two appropriate D&D dice to verify the result. Turner Gilman constructed the figures and sent us the following photograph:



As I remember, this problem appeared on some standard math exam a while back. The obvious and expected answer was 7. However, one student complained that the answer was 5 and that the grading was incorrect; after a small fight the student won.

Also solved by Ken Rosato, Charles Piper, Winslow Hartford, ichard Hess, Ronald Raines, Raymond Gallar alter Cluett, Harry Zaremba, Chip Whiting, a proposer, George Byrd.

Better Lat Never

M/J 1. Tu er Gilman noticed that all references to Bishop 3 should be to Bishop 2.

M/J 4. Thomas Brendle points out that 24.69 was a typo; the correct answer is 27.69.

A/S 1. Dennis White has responded.

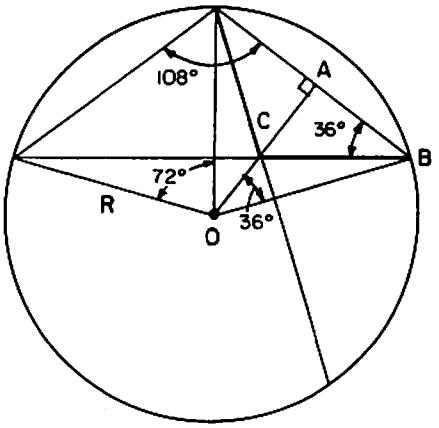
A/S 3. Michael Jung has responded.

A/S 4. Michael Jung, Dennis White, and Raisa Deber have responded.

A/S 5. Dennis White has responded.

Proposers' Solutions to Speed Problems

SD 1. In triangle OAB, AB = R sin 36°. In triangle ABC, BC = AB/cos 36°
= R (sin 36°)/(cos 36°)
= R tan 36°.
Perimeter = 10 R tan 36°



SD 2. During "steady state":
A + $360 − (2 weeks) times $X/week − $135 = B
B + $360 − (2 weeks) times $X/week − $375 = A
Therefore, $X/week = $52.50/week.