ALLAN J. GOTTLIEB

Can You Solve This "Maximdidge?"

For the information of new readers, I review the ground rules under which this department is conducted.

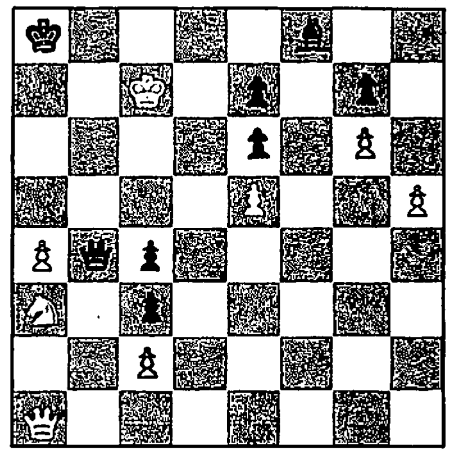
In each issue I present five regular problems (the first of which is chess, bridge, or computer-related) and two "speed" problems. Readers are invited to submit solutions to the regular problems, and three issues later one submitted solution is printed for each problem; I also list other readers whose solutions were successful. For example, solutions to the problems you see below will appear in the February/March issue. Since I must submit that column sometime in November, you should send your solutions to me during the next few weeks. Late solutions, as well as comments on published solutions, are acknowledged in the section "Better Late Than Never" in subsequent issues.

For "speed" problems the procedure is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same column in which the problem is published. For example, the solutions to this issue's "speed" problem are given below. Only rarely are comments on "speed" problems published or acknowledged.

There is also an annual problem, published in the first issue of each new year; and sometimes I go back into history to republish problems that remained unsolved after their first appearance.

Solutions

OCT 1. We begin with a chess problem from *The Tech*, M.I.T.'s student newspaper. White is to move and mate in 10.

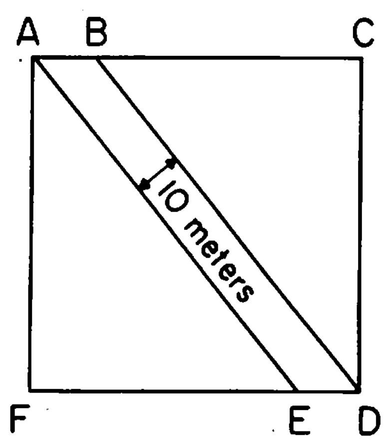


SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012.

OCT 2. Nob. Yoshigahara wants to know the smallest positive integer A such that the first 10 digits in $\sqrt{A}$ consist of 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0, each occurring exactly once.

OCT 3. the following problem is from Jim Landau:

A road 10 meters wide cuts not-quite-diagonally across a 1-kilometer square as shown. What is the area of triangle AEF?



OCT 4. Our next problem, a "maximdidge" from Neil Macdonald, first appeared in *Computers and People*:

A maxim (common saying, proverb, some good advice, etc.) using 14 or fewer different letters is enciphered (using a simple substitution cipher) into the

10 decimal digits or equivalent signs, plus a few more signs. To compress any extra letters into the set of signs, the encipherer may use puns, minor misspellings, equivalents (like CD or KS for X), etc. But the spaces between words are kept. The problem is to decipher the following:

Ωθμ ωΩμτ θΩΔ δμφΔ Δζμ ζΩℓτμ Δζφ Δ ΔℓΩΔτ.

OCT 5. We end this section with an infamous solid geometry problem submitted by George Byrd:

A regular pyramid and a regular tetrahedron both have sides of the same length. Place one face of the tetrahedron on one triangular face of the pyramid so that the three vertices of both faces coincide. How many faces does the resulting solid figure have?

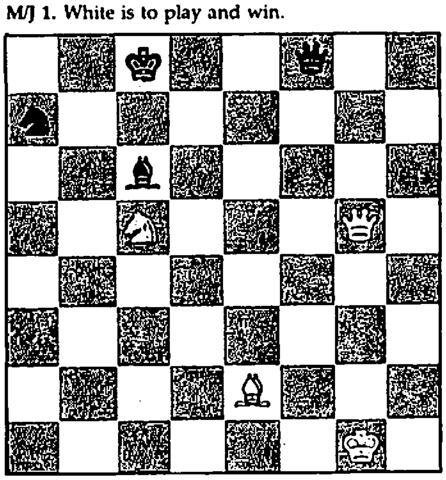
Speed Department

SD 1. The short sides of three equal 45-45-90 triangles are joined to form a three-sided pyramid whose base is an equilateral triangle. What is the slant angle between the sides and the base of the pyramid? (It's easier for a regular tetrahedron.)

SD 2. We close with a puzzle that appeared in *IEEE Potentials*. What is the missing number in the following sequence?
10, 11, 12, 13, 14, 15, 16, 17, 20, 22, 24,— 100, 121, 10000.

Solutions

M/J 1. White is to play and win.



A poetic solution from John Bobbitt:

White moves his Bishop to the square R6,
And tells Black that he's in Check!
Black moves his King to the square N1,
And says, "Oh, what the heck.

"I cannot move King to Bishop 3
Because you'll fork my Queen.
Moving Bishop down to space N2
Is worse—you'll take it clean."

Move 2 for the White is with his Queen.
He moves it back to N3.
Black slides his King to the square R1.
It's either there or B3.

King to B3 is a loser, we know,
Because of the fork by White's Knight.
But now Black's King is really trapped,
Much to the delight of White.

White now moves his Bishop to square N7,
And says, "Check on you, my friend."
Black's Bishop takes White's, and Black then replies,
"You lost it. We're nearing the end."

"The end is in sight," White quickly replies.
"Knight to Queen 7 is right.
I now have two ways to get a checkmate
Or capture your Queen with my Knight.

"If you move your Queen along the back row
Knight to N6 gives a mate.
Or push up your Queen so it leaves the back row,
And I get you with Queen to Knight 8.

"Or leave your Queen there and move N or B;
Checkmate no longer is mine.
But Knight takes Queen is a winner for me—
it's only a matter of time."

Black looks and frowns and topples his King,
And offers his hand to White.
The moves from above summed up below,
And with that I bid you, "Good Knight."

1. B-R6 ch K-N1
If 1. . . . K-B3, then 2. N-K6 ch wins the Queen
2. Q-N3 ch K-R1
3. B-N7 ch! BxB forced
4. N-Q7! Resigns

Also solved by Jim Landau, Matthew Fountain, Richard Hess, Thomas Chang, and the proposer, Bob Kimble.

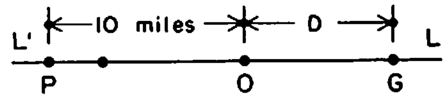
M/J 2. A BB 2 mm. in diameter sits atop a "superball" 10 cm. in diameter, the center of which is 1 meter above the ground. If both objects are released simultaneously, what is the maximum possible height the BB can attain upon rebound?

The following solution is from Matthew Fountain: I vote for 8.65 meters. I take superball to be a ball that can bounce as high as it drops. This implies that upon leaving the ground all parts of the ball must have the same speed, as otherwise the ball will be vibrating about its center of gravity, with the energy associated with this vibration subtracted from that which would otherwise be available for elevating the superball. Superball and the ground must be extremely hard, as any appreciable displacement would cause the ball or ground to vibrate. With these assumptions, it follows that superball will rebound so quickly that the BB will act as if it has fallen upon a surface rising with the speed that superball hits the ground. The BB will therefore rebound from superball as if it had approached superball with twice the speed that superball hits the ground. Consequently the BB may not rebound with more than three times this speed when measured with respect to the ground. As kinetic energy varies as the square of velocity and potential energy varies directly as the height, the BB will not be flipped more than nine times as far as the ball drops. Nine times 0.95 plus 0.10 equals 8.65. In my very crude experiments with a very old golf ball and an upside down thumbtack, I found the thumbtack sometimes to bounce to what seemed to be at least three times the height of the drop, even though the ball rebounded about half the height of the drop. My eyeball measurements were rather crude, as the drops were of the order of one inch and my measuring scale was two bricks. I interpret these results to mean that the top surface of this ball vibrates so that the BB is tossed upward. The thumbtack does not bounce at all well off the ball.

Also solved by Dennis White, Harry Zaremba, Jim Landau, Peter Kramer, Richard Hess, Thomas Chang, and the proposer, Bruce Calder.

M/J 3. From the front to the rear of an advancing army detachment was 10 miles. A rear guard messenger, dispatched to the guard house directly behind his position in the line of march, returned without loss of time and then proceeded immediately to the vanguard and again returned. He then noted that he had overtaken his guard 10 miles from the starting point and that the time spent on each errand had been the same. How far was the guard house from the starting point, and how far did the messenger travel altogether?

The following solution is from James Abbott:

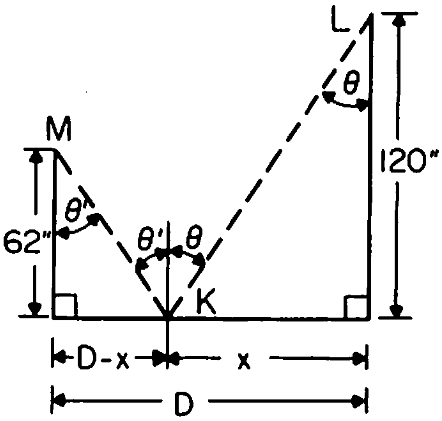


Let L' − L represent the line of march, 0 the starting point, G the guard house, and P the point at which the messenger overtakes the rear guard. Let T_0 be the instant he starts toward the guard house and T_1 the instant he reaches point P. Let Vm be the speed of the messenger along the line of march and Va the speed of the army detachment. It is assumed that these speeds remain constant for the duration of both errands. Then the time required for the first "errand" is
$T_1 - T_0 = (2D + 10)/Vm$.
To reach the vanguard his speed relative to the detachment is Vm − Va. The time he takes to reach the vanguard would be:
$\theta_n = 10/(Vm - Va)$.
On the the return leg his relative speed becomes Vm + Va and the time on the leg would be
$\theta_r = 10/(Vm + Va)$.
By definition, the sum of these two times is equal to $T_1 - T_0$, or (1)
$10/(Vm - Va) + 10/(Vm + Va) = (2D + 10)/Vm$.
A second relation is given by the fact that the detachment traveled 10 miles during the period $T_1 - T_0$:
$T_1 - T_0 = 10/Va$.
Equating the two expressions for $T_1 - T_0$ gives
$(2D + 10)/Vm = 10/Va$.
Rearranging and reducing to lowest terms yields
$Va/Vm = 5/(D + 5)$. (2)
Equations (1) and (2) give us a system of two equations with apparently three unknowns. However, since both Va and Vm are constants (by assumption) it follows that their ratio Va/Vm must also be a constant. Let this constant be R; then Va/Vm = R and Va = RVm. Substituting RVm for Va in (1) gives
$10/(Vm - RVm) + 10/(Vm + RVm) = (2D + 10)/Vm$.
Cancelling common factors (Vm in the denominators, 2 in the numerators),
$5/(1 - R) + 5/(1 + R) = D + 5$. (3)
Placing the left-hand terms over a common denominator $(1 - R^2)$ and clearing of fractions,
$5(1 + R) + 5(1 - R) = (D + 5)(1 - R^2)$ or,
$(D + 5)(1 - R^2) = 10$. (4)
Solving (4) for R^2,
$R^2 = (D - 5)/(D + 5)$.
Substituting R for Va/Vm in (2) and squaring both sides,
$R^2 = 25/(D + 5)^2$.
Equating two expressions for R^2 and simplifying,
$D^2 - 25 = 25$
$D = (50)^{1/2} = 7.701$ miles (to 3 decimal places).

To get the total distance the messenger traveled, we note that on the first "errand" he travels a distance (2D + 10) or 24.142 miles. Since his speed is assumed constant and the times for both "errands" are the same, it follows that he must travel the same distance for the second "errand." Hence the total mileage is
$2 \times 24.142 = 48.284$ miles.
Although not called for in the problem statement, it is interesting to solve for R and consider the practical implications. It turns out that R = 0.4142 and its reciprocal is 2.4142. This means that the messenger must travel almost 2.5 times the speed of the marching army. Since this speed is a little over 3 mph, the messenger must be either a Superman or else have some kind of mechanical assistance (possibly a bicycle?)

Also solved by Dennis White, Harry Zaremba, Jim Landau, Jim Rutledge, Jules Sandock, Matthew Fountain, Mary Lindenberg, Michael Strieby, Richard Hess, Steve Feldman, and Thomas Chang.

M/J 4. A supermarket shopper whose eyes are 62 inches above the ground notes that the glossy linoleum floor of an aisle reflects the fluorescent lamps installed crosswise to the aisle 10 feet above it. As the shopper moves down the aisle at a uniform rate of 42 inches per second, the reflected images move as well. When a lamp is a horizonal distance D ahead, what is the speed of its image in the floor?



M is the shopper's eyeballs.
L is the light.
K is the image of the light on the floor.

If we find x, we can then find the speed at which K travels, as both K and M reach the vertical line under L at the same time. We note from physics that
$\sin\theta = x/\sqrt{120^2 + x^2}$
$\sin\theta' = (D - x)/\sqrt{62^2 + (D - x)^2}$
$x/\sqrt{120^2 + x^2} = (D - x)/\sqrt{62^2 + (D - x)^2}$
By simplifying for x we get
$x = (120/182)D = .65934D$
Now distance = rate × time. But the time for M to travel D is the same as the time for K to travel x. Solve for t and we get the following:
$D/R_D = x/R_K$.
Solving for R_K gives
$R_K = (x \times R_D)/D$.
Plugging in the variables knowing $R_D = 42$ in/sec we get
$R_K = (.65934\ D)(42)/D$, so $R_K = 24.69228$ in/sec.

Also solved by Dennis White, Greg Spradlin, Harry Zaremba, Jim Landau, Jim Rutledge, John Bobbitt, Jules Sandock, Matthew Fountain, Richard Hess, Thomas Chang, and the proposer, Phelps Meaker.

M/J 5. Fill in the boxes with the digits 0, 1, 2 . . . 9.

□□^{min}:□□^{sec} × □ = □^{hrs}: □□^{min}: □□^{sec}

Steve Feldman wrote a computer program to check all combinations and list those that worked. The result was the list printed below:

1) 15:69 × 3 = 0:48:27	8)67:54 × 8 = 9:03:12
2) 18:49 × 3 = 0:56:27	9)69:23 × 7 = 8:05:41
3) 18:74 × 9 = 2:53:06	10)76:98 × 4 = 5:10:32
4) 30:97 × 8 = 4:12:56	11)78:09 × 4 = 5:12:36
5) 31:97 × 8 = 4:20:56	12)79:08 × 4 = 5:16:32
6) 45:79 × 8 = 6:10:32	13)82:19 × 3 = 4:06:57
7) 50:42 × 9 = 7:36:18	14)91:73 × 4 = 6:08:52

The only solution that he feels could unequivocally be called legitimate would be number 7. Solution 2 might also be considered valid by some people. He would not call any of the others valid.

Also solved by David Simen, Donald Savage, Hy Tran, Jim Landau, John Bobbitt, Jules Sandock, Mary Lindenberg, Matthew Fountain, Steve Feldman, and the proposer, Nob Yoshigahara.

Better Late Than Never

1986 JAN 3. John Langhaar has responded, and Walter Nissen believes that the solution given does not work for p = 17. However, if one interprets the repeating part of the decimal to include the leading zero, all seems well.

JAN 4. Walter Nissen noticed that the problem was reprinted incorrectly. The solution is correct, but naturally for the original problem.

F/M 1. Charles Larson has responded.

F/M 4. Howard Zeidler has responded.

F/M 5. Stefania Anderson has responded, and the following submission is from Stanley Liu:
The following references to the recreational mathematical literature may be of some interest. It turns out that the number 6174, along with its unusual status as the unique fixed point of all four-digit integers (whose digits are not all identical) under repeated (maximum difference) reordering-subtraction operations, was first discovered and discussed by D.R. Kaprekar in 1949. Most appropriately, many subsequent writers have referred to 6174 as *Kaprekar's number* in honor of its original discoverer. I myself have been fascinated by Kaprekar's number and the associate Kaprekar operations since the late 1960s. I also read with interest the various attempts to generalize this problem to other multiple-digit numbers and to numbers in different bases. I find the discussions by Charles Trigg (in 1970 and 1971) most comprehensive. He classified all four-digit integers (not a multiple of 1111) into one of 54 representations by two "predictive indices" (i.e., the difference between the largest and smallest digits and that between the middle ones) and listed the number of Kaprekar operations required before the self-regenerative 6174 is reached. He also discussed five-digit numbers (and many others) and presented results in number bases other than ten. (One can easily carry out some of these calculations nowadays on a microcomputer, as I have done and verified his table for 6174.) I hope one day I will have more to say on this matter and add to the existing literature.

APR 2. Jim Landau notes that $^t\int_0 \sin(t^2)dt$ is often used by electrical engineers to get a wave decaying in both wavelength and amplitude.

Proposer's Solutions to Speed Problems

SD 1. Let *L* equal an edge of the base; the slant height is then *L*/2. A vertical line through the peak of the pyramid has its foot on a perpendicular bisector of the base, 2/3 of the distance from the corner of the base. The angle is
$\cos^{-1}(.866L/3)/(L/2) = \cos^{-1}(.5774)$ (54.7°).
For the tetrahedron: $\cos^{-1}(.3333)$ (70.53°).

SD 2. 31. Each number is sixteen expressed in different bases. □