ALLAN J. GOTTLIEB

Celtics vs. Lakers: Was It *Really* Amazing?

In the last issue we announced that David Meyer was the winner of the $100 prize for the best solution to JAN. 2. As you may recall, Philip Hogin proposed the problem, offered the prize, and judged the solutions, and I had promised to publish the winning solution. I have now received the solutions from Mr. Hogin, and unfortunately, as might be expected, neither the problem nor the solution was trivial and Mr. Meyer's response is rather lengthy. So instead of publishing it, I will instead mail a photocopy of Hogin's solution to any reader who wants it. Please refer to the problem as 1985 JAN 2. In the "Better Late Than Never" section of this issue, I include an excerpt of Mr. Hogin's letter to Mr. Meyer that, in very general terms, comments on the winning solution.

Problems

A/S 1. We begin with a computer-related problem from Walter Nissen, who wants to know what is particularly interesting about the factors of the following numbers: 1271, 42477, 74989, 128929, 923521, 4424351, 4782969, 536215711, 2889101203, 98695877281, 424777960767, 1470848491213, 7532627125087, 7617609926757, 12893722612807, 17037029794091, 28917102427847, 170396299851737, 1703971665820979.

A/S 2. Nob Yoshigahara asks another problem concerning factoring, one which first appeared in the Japanese Journal *Quark*:

Find the smallest positive number whose prime factorization consists of the nine digits 1 through 9 and then find the smallest positive number whose prime factorization consists of the ten digits 0 through 9. To clarify this question, we note that 2×13 would be the answer for the three digits 1 through 3.

A/S 3. The following problem is from Phelps Meaker:

Cooking vegetables such as potatoes is carried on more rapidly as the amount of exposed surface is increased. Take a very large potato and trim it to a 2" cube. Divide it into 8 slices with 7 equally spaced cuts. Find the ratio of exposed surface area to number of cuts. Now make 7 cuts in another plane, resulting in 64 little sticks, as for French fries. Again, find the ratio of total area to total number of cuts. Again make 7 cuts, resulting in 512 tiny cubes. The ratios all have similar repeating decimals. Do you dare make another series of cuts into the fourth dimension?

A/S 4. A basketball problem from David DeLeeuw:

During the fifth game of last year's basketball championship series between the Boston Celtics and the Los Angeles Lakers, the television commentators constantly reminded the viewing audience that 10 of the last 15 winners of game number 5 of the championship series went on to become the eventual world champions. Determine whether this is really such an "amazing" statistic as the announcers would have us believe. Stated differently, given two evenly matched teams participating in a best-four-out-of-seven series, what is the probability that the team that wins the fifth game of the series will win the series?

A/S 5. We close this section with a geometry problem from Dennis White:

A "right tetrahedron" has a vertex with three right angles. If in a right tetrahedron, A,B,C denote the areas of the three faces that share the "right vertex" and D denotes the area of the face opposite the right vertex, show that $A^2 + B^2 + C^2 = D^2$ (a "Pythagorean Theorem" for areas in 3-space).

Speed Department

SD 1. Jim Landau wants to know the

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value of i^i, where i is the principal square root of -1.

SD 2. Our final problem for this issue is from *The Tech*:

Give the next number in the sequences:

a) 11, 13, 17, 25, 32, 37, 47, 58, 71, . . .

b) 7, 3, 10, 4, . . .

Solutions

APR 1. Develop a computer program that determines the day-of-the-week for any Nth day after a selected date.

Only the proposer, Harry Zaremba, sent us a computer-oriented solution; his BASIC program appears below, followed by a nomogram that Walter Cluett attributes to the late Professor Douglas P. Adams of M.I.T. in the 1960's:

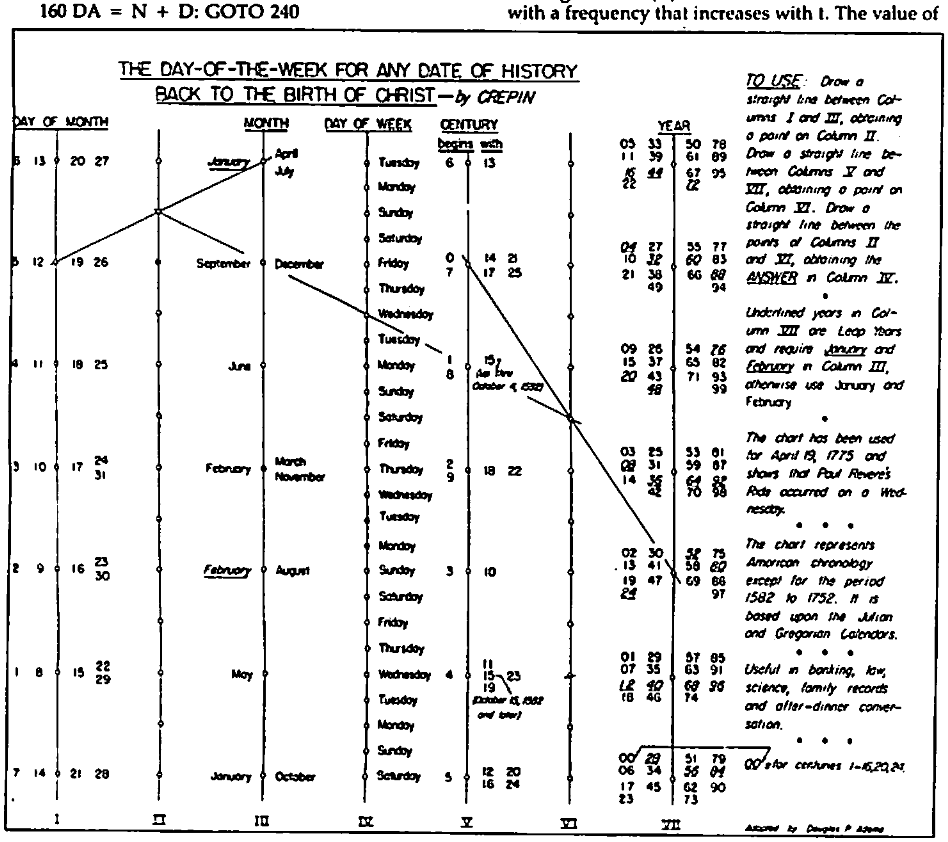
The program listed below is in BASIC language for a hand-held Sharp scientific computer, Model EL-5500II. The numeric function INT determines the integer portion of a numerical value. Input data consists of numerical values for the year, month, and day of the preselected date and for the number of days N for which the date and day-of-week is desired. Leap years are accounted for in the program.

```
10 CLEAR: PRINT = LPRINT
20 DIM MOS(12), Y(2), M(2), F(2), DWS(6)
30 FOR I = O TO 6: READ DWS(I)
40 DATA "SATURDAY," "SUNDAY," "MONDAY," "TUESDAY"
50 DATA "WEDNESDAY," "THURSDAY," "FRIDAY": NEXT I
60 FOR J = 1 TO 12: READ MOS(J)
70 DATA "JAN.," "FEB.," "MAR.," "APR."
80 DATA "MAY," "JUNE," "JULY," "AUG."
90 DATA "SEPT.," "OCT.," "NOV.," "DEC.":
NEXT J
100 INPUT "YEAR = ?"; "MONTH = ?"; M
110 INPUT "DAY = ?"; D, "NO. OF DAYS = ?";
N
120 M(1) = M: Y(1) = Y: K = 2
130 GOSUB 350
140 DF = DM - D
150 IF N > DF THEN 170
160 DA = N + D: GOTO 240
170 S = DF
180 M(1) = M(2): Y(1) = Y(2): K = 2
190 GOSUB 350
200 S = S + DM
210 T = N - S
220 IF T > 0 THEN 180
230 DA = DM + T
240 J = 1
250 IF M(1) = J THEN 270
260 J = J + 1: GOTO 250
270 PRINT MOS(J); DA; "; "; Y(1)
280 D1 = DA: K = 1
290 GOSUB 390
300 DW = F(1) - INT(F(1)/7)*7
310 J = 0
320 IF DW = J THEN 340
330 J = J + 1: GOTO 320
340 PRINT DWS(J): GOTO 100
350 D1 = 1
360 IF M(1) < 12 THEN 380
370 M(2) = 1: Y(2) = Y(1) + 1: GOTO 390
380 M(2) = M(1) + 1: Y(2) = Y(1)
390 FOR I = 1 TO K
400 A = 365* Y(I) + D1 + 31*(M(I) - 1)
410 IF M(I) > 3 THEN 440
420 B = INT(Y(I)/4) - INT(.4*M(I) + 2.3) - INT(3/
4*INT(Y(I)/100 + 1))
430 F(I) = A + B: NEXT I: GOTO 460
440 B = INT((Y(I) - 1)/4) - INT(3/4*INT((Y(I) - 1)/
100 + 1))
450 F(I) = A + B: NEXT I
460 DM = F(2) - F(1): RETURN
```

The subroutine from lines 350 to 460, inclusive, determines the number of days DM in a particular month, and the subroutine from lines 390 to 460 is used to determine F(I) for use in calculating the number DW that corresponds to the number of the required day-of-the-week.

APR 2. In the family of functions $f_n(x) = \int_{t=0}^{x} \sin(t^n)dt$ the infant f_1 is reasonably well behaved; it is bounded and oscillates between -1 and $+1$. How well behaved are the other family members, f_2, f_3, etc?

Matthew Fountain sent us the following solution: The infant f_1 oscillates between 0 and 2. It is $\sin(t^n)$ that oscillates between -1 and $+1$. Hereafter assuming $n>1$, $\sin(t^n)$ oscillates between -1 and $+1$ with a frequency that increases with t. The value of



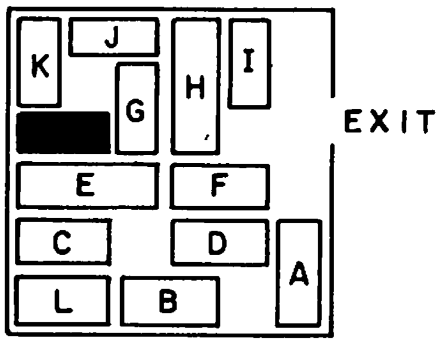
f_n is always positive, reaching its maximum at $x = \pi^{1/n}$. This maximum is 2 when n = 1 and decreases as n increases. As x increases from 0, f_n alternately increases and decreases with increasing frequency. Each successive increase or decrease is less than its preceding decrease or increase, so that at large x, f_n approaches a constant value.

APR 3. Arrange the digits 1 through 9 into two proper fractions and two positive integers so that the four numbers total 100. The result of interchanging digits in the integers is not considered a different arrangement—e.g., 1/8, 63/72, 95, and 4 is considered the same as our original arrangement.

I must apologize for an error that I (personally) introduced into this problem. Mr. Duffy did not require that there be exactly *two* positive integers, and hence the seven solution sets include:
6/7 4/28 95 3 1.
Steve Feldman found the six solutions to the problem as printed:
1/2 38/76 4 95
1/3 56/84 2 97
2/7 45/63 1 98
3/6 27/54 1 98
3/7 16/28 4 95
3/6 9/18 74 25

Also solved by Dennis White, Jim Landau, Jim Rutledge, Matthew Fountain, Michael Gennert, Ray Kinsley, and the proposer, Emmet Duffy.

APR 4. List the shortest sequence of moves necessary to free the black car in the overcrowded parking lot. Cars can move only back and forth and cannot turn.



The following 23-move solution is from Darryl Ephraim:

1	A	north	3 units
2	F	east	1 unit
3	D	east	1 unit
4	B	east	2 units
5	H	south	3 units
6	I	south	1 unit
7	J	east	3 units
8	G	north	1 unit
9	H	north	3 units
10	CAR	east	1 unit
11	E	east	1 unit
12	C	east	1 unit
13	L	east	1 unit
14	K	south	4 units
15	E	west	1 unit
16	H	south	3 units
17	CAR	west	1 unit
18	G	south	1 unit
19	J	west	4 units
20	A	north	1 unit
21	I	north	1 unit
22	G	north	1 unit
23	CAR	OUT	

Also solved by Avi Ornstein, Charles Swift, Chris Hill, Dennis White, Dudley Church, Jim Landau, Jim Rutledge, Jim Walker, Jordan Wouk, K. D. Kuntz, Kevin Trammel, Kyre Cluett, Lu Ting, Mary Lindenberg, Matthew Fountain, Michael Gennert, Pierre Heftler, Ray Kinsley,Ronald Martin, Kyre Cluett, and the proposer, Nob. Yoshigahara.

APR 5. Find the smallest number that can be partitioned into four distinct positive integers such that the sum of every pair is a perfect square.

Dennis White found the solution 4205 = 2 + 359 + 482 + 3362, where the six possible sums of pairs give the squares of 19, 22, 29, 58, 61, and 62. Mr. White observed that the problem is equivalent to finding a number that can be partitioned into squares in three different ways. With the help of algebra combined with trial and error, he set up and solved a set of second-order equations involving several variables. Although not too hard to follow, the solution would be hard to typeset and hence I will instead mail copies to anyone who sends a request.

Also solved by George Thomas, George Welti, Matthew Fountain, and the proposer, P.V. Heftler.

Better Late Than Never

1985 OCT 3. John Langhaar notes that the solution given is clearly wrong if the relevant points are colinear. In that case the median and not the mean gives the correct answer. For the two-dimensional case he gives an example to show that neither the median nor the mean is correct. The problem is still open.

OCT 5. Mary Lindenberg has responded.

N/D 1. Jim Landau tightened up his (published) proof by showing that the constructed point D satisfies the necessary properties. Frank Rubin and Dennis White noticed that the "human-friendly version" given is not correct. Thomas Weiss has also responded.

N/D 3. Dennis White believes that the problem still holds if complex numbers are permitted, but not using the proof given. John Prussing found a simpler solution.

N/D 4. Thomas Weiss has responded.

JAN 1 1986. Thomas Chang has responded.

JAN 2. As mentioned in my introductory remarks above, what follows is a portion of the letter that Philip Hogin sent to David Meyer:
You will be interested to know that I received 31 responses (somewhat more than the norm—Ed.). Almost all began by pointing out, as you did, that the practical solution was a rather simple pair of parametric equations relating x and y to alpha. About half of the rest then proceeded, as you did, to attempt a nonparametric solution, concluding with some formidable approximation with the potential of a very small error. As stated in the problem, an approximate solution was acceptable, so there were many eligible solutions. Yours, however, had one distinction—it had the earliest postmark. I probably should have stated in the problem that a parametric solution was actually used to build the cam, and our purpose in presenting the problem was to see if any of you geniuses out there could come up with a closed-form solution in the form $y = f(\alpha)$—we couldn't. I am comforted to know that no one else could either, and that the problem, when addressed as presented, was found to be non-trivial.

JAN 3. Edwin Rosenberg notes that using standard number theory tables for the "index of 10" it is possible to tell whether a prime will have a maximum-length repeating decimal.

JAN 4. Thomas Chang has responded.

F/M 1. Lester Steffens, Thomas Chang, Jim Landau, and Eli Passow have responded.

F/M 2. M.J. Ralph, Thomas Chang, and Dudley Church have responded.

F/M 3, F/M 4. Thomas Chang has responded.

F/M 5. Avi Ornstein and Jim Landau have responded.

Proposers' Solutions to Speed Problems

SD 1. $e^{-\pi/2}$, a real number!

SD 2. (a) 79. Each number is formed by adding the preceeding number to the sum of its own digits. (b) 8. These are the numbers on buildings on the infinite corridor. ☐