ALLAN J. GOTTLIEB

Back to Kresge Without Road-Runner

Last week I went to a VLSI conference in Kresge Auditorium at M.I.T. where, as an undergraduate many years ago, I used to watch movies on weekends. I remember those days well. In particular, I have many fond memories of active audience participation during the Road Runner cartoons that often accompanied the featured movie. I was invited to the conference to speak about results obtained by our ultracomputer research project here at NYU, but it was *very* hard for me to stand on the Kresge stage and look out at the audience; the Road Runner memories were so strong. Indeed, whenever I projected a new overhead transparency, I felt as though a sharp "beep beep" was in order.

FLASH! Phil Hogin just called to say that he and a consultant from Cornell University have decided on a winner of the $100 JAN 2 sweepstakes. Mr. Hogin reports that nearly all the responders found the parametric solution that he had also discovered but that no one obtained a closed-form solution. That is, errors were found in all the closed-form attempts. Apparently, the consultant had a hard time finding the error in one of these but eventually succeeded. The winning entry supplied a converging series whose error can be made $O(r^k)$ for arbitrary k by including enough terms of the series. The solution will appear next issue. The envelope, please. The winner is . . . (fumble, fumble, rip) . . . David Meyer!

Problems

JUL 1. We begin with a bridge problem from Winslow Hartford. The hand was played by 80-year-old Elmer Schwartz of Cleveland and reported by the *Cleveland Plain Dealer* for November 18, 1984: The contract is six hearts by South, East having bid spades; and the ♠7 is led.

	♠ A J 9 8	
	♥ Q 9	
	♦ Q 5 4 2	
	♣ 4 3 2	
♠ 7 6 5		♠ K Q 10 4 3
♥ 3 2		♥ 8 7 6
♦ J 10 9 6		♦ A 3
♣ J 9 6 5		♣ Q 10 8
	♠ 2	
	♥ A K J 10 5 4	
	♦ K 8 7	
	♣ A K 7	

JUL 2. Phelps Meaker needs help with his sloping lampshade. A six-sided lampshade assembled from parchment trapezoids is larger at the bottom that at the top. The two parallel sides and the height of the trapezoids are in the ratio 2:3:4. How much does a side slope from the vertical?

JUL 3. Jerry Grossman would like you to find a pattern or better yet a closed form for f(n) defined by:

$f(1) = 1$

$f(n) = f[n - f(n - 1)] + 1$ for $n > 1$

JUL 4. Smith D. Turner (∫dt) asks about the spy letter that came in out of the rain:

A spy received a rain-soaked letter. The stamp was gone, and all the postmark was obliterated except two contiguous letters, "ON." The contents bore no letterhead, but on decoding the letter the spy concluded it must have been mailed in Bonn, London, or Washington. What is the probability it was mailed in London?

JUL 5. Allen Wiegner wants us to analyze his M&M consumption algorithm:

I love M&M's, but have an odd way of eating them. I remove one at a time at random from the bag and place it on the table. Whenever there are two of the same color on the table, I eat those two. After I have removed 100 M&M's from the bag, how many might I expect to find left on the table? (Assume there are five colors altogether, distributed equally.) After 101? If brown ones are twice as common as the other colors, how do the expected values change after 100 and after 101?

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012.

ɔeed Department

Ↄ 1. Back in the dark ages (1968, in ɛt) I was boasting to a friend about an ɜctronic calculator (then an unheard- novelty) on which I had computed 9^{100} in 15 seconds. "Point nine nine to e one-hundredth?" my friend said. 'hat's easy. It's .37" How did he do it?

Ↄ 2. Ron Raines recalls an advantage the "old style" watches:
How can you tell which direction is ɔrth using only an analog watch and e position of the sun? How would the ıswer differ if you were in the South- n Hemisphere? Ignore daylight sav- gs time.

ɔlutions

Ⅵ 1. Construct a deal such that North-South can ıke seven no-trump against the worst defense th North-South having the minimum number of ʒh-card points. By worst defense we mean that ː declarer can specify which (legal) cards the de- ıders are to play at each trick.
Our first solution is from Robert Bart. North- uth can make seven no-trump with zero points d no card higher than a nine.

♠ 5 4 3 2
♥ 8 7 6 5 4 3 2
♦ —
♣ 3 2

West	East
A K Q J 10 8 6	♠ —
A K Q	♥ J 10 9
Q J 10 9	♦ A K
—	♣ A K Q J 10 8 6 4

♠ 9 7
♥ —
♦ 9 8 7 6 5 4 3 2
♣ 9 7 5

est drops the ♠8 and ♠6 under South's ♠9 and 7 as East pitches the diamond honors. South then shes the ♣9, ♣7, and ♣5, East under-playing th the ♣8, ♣6 and ♣4 while West throws his amond honors. South takes the rest of the tricks .th good diamonds.
Also solved by Allen Wiegner, Allen Zaklad, Mat- ew Fountain, Steve Feldman, Winslow Hartford, ıd the proposer, Howard Sard.

M 2. Find the age of Mrs. Grooby, Farmer Dunk's other-in-law. The given facts: there are 20 shillings the pound sterling, an acre is 4840 square yards, ıd a rood is a quarter of an acre. Do *not* assume e puzzle was invented this year. Also, these hints ːlp: one number in the puzzle is the area of Dog's ead in roods, but it relates to something in the ızzle quite different from that area. One of the ımbers across is the same as one of the numbers ɔwn. All numbers are integers, and no number ·gins with an '0'. So here are the clues:

cross
Area of Dog's Mead in square yards.
Age of Farmer Dunk's daughter, Martha.
The difference between the length and breadth Dog's Mead in yards.
Number of roods in Dog's Mead times number ne down.
The year when Little Piggly came into occupation ⁄ Dunk family.
ı. Farmer Dunk's age.
. The year Farmer Dunk's youngest child, Mary, as born.
ı. Perimeter of Dog's Mead in yards.
ı. The cube of Farmer Dunk's walking speed in iles per hour.
ı. Number fifteen across minus number nine ɔwn.

Down
1. The value of Dog's Mead in shillings per acre.
2. The square of Mrs. Grooby's age.
3. The age of Mary.
4. The value of Dog's Mead in pounds sterling.
6. The age of Farmer Dunk's first-born, Edward, who will be twice as old as Mary next year.
7. The square, in yards, of the breadth of Dog's Mead.
8. The number of minutes Farmer Dunk needs to walk one and one third times around Dog's Mead.
9. See number ten down.
10. Ten across times nine down.
12. One more than the sum of the digits in the second column down.
13. Length of tenure, in years, of Little Piggly by the Dunk family.

The following solution is from Jim Rutledge:

[1] 3	8	[2] 7	[3] 2	0	■	[4] 1
5	■	[5] 3	2	■	[6] 4	4
5	■	9	■	[7] 3	5	2
■	[8] 1	6	[9] 1	0	■	■
[10] 7	2	■	[11] 1	9	[12] 1	[13] 3
9	■	■	■	[14] 7	9	2
[15] 2	7	■	[16] 1	6	■	5

Assuming that this story puzzle takes place before or during the 20th century, 11A must begin with a 1 and 9D must perforce be 11 due to its use as a multiplier for 10A in which the leading digit is both a member of the multiplicand and the product. Since the last digit of 15A minus 9D is also the last digit of a squared term, 15A must be 27 (64 would produce a final 3—not the product of a square). 16A is 16 and, again assuming a pre-21st-century milieu, 8D must be 12. This fact yields the perimeter of Dog's Mead:
$(4/3)P = 3 \text{ mph} \times (1/5) \text{ hr}$
$P = (9/20) \text{ mi} = 792 \text{ yds.}$
Consequently, 14A is 792, and the length plus the breadth of Dog's Mead is 396 yards. According to 6A, 10 yds $\leq L - B \leq$ 99 yds and the range of choices for L and B are 203/193 to 247/149. Of these, only the squares of the breadths 174 and 176, viz. 30276 and 30976, end in "76." The two possible areas are 222 × 174 = 38628 sq. yds. and 220 × 176 = 38720 sq. yds. Given that all numbers are integers and 1 acre = 4840 sq. yds., the actual area must be 38720 sq. yds., precisely 8 acres. Consequently, 1A is 38720, 6A is 44 and 7D is 30976. Also, the number of roods, 32, times 9D equals 7A, 352. This determines the age of Farmer Dunk's first-born Edward to be 45. His youngest daughter Mary then must be 22, 3D, and his daughter Martha must be either 22, 32, or 42. These figures yield a possible range for Mrs. Grooby's age of 85 to 86, where $85^2 = 7225$ and $86^2 = 7396$, and eliminate 42 as a choice for Martha. By simple addition, 12D is 19. Considering 11A, 8A, and 13D together, Mary could have been born in 1913 or 1914, Little Piggly could have arrived in 1610 or 1510, and 13D could have been 325 or 426, respectively. Since Martha's age could be 22 (as the second of twins) or 32, the hint concerning the area of Dog's Mead in roods was well-given. Therefore, Martha is 32, 5A; Mrs. Grooby is 86 and the square of her age is 7396, 2D; 8A is 1610, 11A is 1913, and 13D is 325. The value of Dog's Mead may be determined by simple calculation. Its worth in shillings per acre is between 300 and 399. At 20 shillings to the pound sterling, this translates

into between 15 and 20 pounds per acre. Knowing that there are 8 acres, the total value of the land is between 120 and 160 pounds sterling. Therefore, 4D must be 142 pounds. This equals 2840 shillings and dividing by 8 yields 355 shillings per acre, 1D. Finally, Farmer Dunk's age remains in question since 62 and 72 are both viable possibilities. The hint, again well-given, that one of the numbers across is the same as one of the numbers down leads to Farmer Dunk being 72 years old, 10A, so that 10D is 792, the same as 14A. And thereby is the puzzle of Dog's Mead unraveled.

Also solved by Dennis White, Allen Wiegner, Arthur Connick, Bill Hauke, Brian Mannion, Donald Berkey, Jim Landau, John Carlin, Matthew Fountain, Naomi Markovitz, Norman Spencer, Robert Bart, Runyon Colie, Steve Feldman, Victor Newton, and Winslow Hartford.

F/M 3. N = (2K + 1) people own a bank. What is the minimum number of different locks that must be put on a safe so that when keys to these locks are distributed to the different people, every majority contains a complete set of keys, but no minority does?

Mike Harris found all the keys needed to solve this problem:

The group of N = 2k + 1 owners can be divided into a minority of k members and a majority of k + 1 members in M different ways, where

$$M = \frac{(2K+1)!}{(K+1)!\,K!} = \binom{N}{K}$$

To assure that each minority cannot open all the locks, and that each majority can open all the locks, do the following procedure in sequence for each of the M splits of the group: Attach a new lock to the safe and issue a key to each of the k + 1 majority of that split. Thus, the total number of locks is M, the total number of keys is (k + 1) M and each owner is issued

$n = [(k + 1)M]/(2k + 1) = (2k)!/(k!)^2$.

As an example, suppose Ann, Bob, and Carol own the bank. Three locks are attached to the safe and each owner has two keys. Labeling the locks 0, 1, and 2, key assignments are as shown:

Ann	0,1
Bob	1,2
Carol	2,0

If Dick and Ellen buy into the bank, M = 10 locks (numbered 0 through 9) must be used and n = 6 keys are issued to each owner as shown:

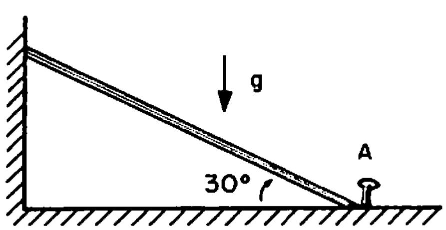
Ann	0,1,2,3,4,5
Bob	0,1,2,6,7,8
Carol	0,3,4,6,7,9
Dick	1,3,5,6,8,9
Ellen	2,4,5,7,8,9

If Frank and Ginger buy in, M = 35 locks must be used and n = 20 keys are issued to each owner.

Also solved by Allen Wiegner, Matthew Fountain, Naomi Markovitz, Robert Bart, Winslow Hartford, and the proposer, Joe Verducci.

F/M 4. A rigid, uniform stick one meter long is held leaning against a smooth vertical wall at a 30° angle to a smooth horizontal surface. When the stick is released, how fast will its center pass point A?

The following solution is from Matthew Fountain:



The stick passes the peg at 36.9 cm/sec. As neither wall nor floor move, they do not work on the stick. Its loss of potential energy during its fall is accompanied by an equal gain in kinetic energy. Letting Y be the height of the upper end of the stick, the center of gravity of the stick is at Y/2. The loss in potential energy when Y is less than its starting value of 50 cm. is (100mg)(25 − Y/2). Here m is the weight/cm of the stick and g is equal to 980 cm/sec/sec. During the fall of the stick, before the stick leaves contact with the wall, a point b cm from the lower end of the stick has a vertical velocity of (b/100)(dY/dt) and a horizontal velocity of [(100 − b)/100](dX/dt). Here X is the distance from the wall to the lower end of the stick. Kinetic energy equals (1/2)(mass)(velocity)^2. The kinetic energy during the early part of the fall is equal to

$$\int_0^{100}\{(m/2)(b/100)^2(dY/dt)^2 + [(100-b)/100]^2(dX/dt)^2 db$$

$= (100m/6)[(dX/dt)^2 + (dY/dt)^2]$.

While the stick is in contact with the wall,

$X^2 + Y^2 = 100^2$ and

$2X(dX/dt) + 2Y(dY/dt) = 0$, making

$(dY/dt)^2 = (X/Y)^2(dX/dt)^2 = [(100^2 - Y^2)^2/Y^2](dX/dt)^2$.

Substituting for $(dY/dt)^2$ and equating loss of potential energy to gain in kinetic energy,

$100mg(25 - Y/2) = (100m/6)[1 + (100^2 - Y^2](dX/dt)^2$,

making

$(dX/dt)^2 = 6g(25 - Y/2)Y^2/100^2$

$= 0.294(50Y^2 - Y^3)$.

During the fall, as long as the stick is in contact with the wall, the center of the stick—the location of the center of gravity—accelerates horizontally in reaction to the force exerted by the wall. This acceleration decreases smoothly to zero at the moment the stick leaves contact with the wall. The lower end of the stick, which has been moving horizontally with twice the speed of the center of the stick, stops accelerating at the same time. The horizontal velocity of the center of the stick is constant after the stick leaves the wall. It equals (1/2)dX/dt when $d^2X/dt^2 = 0$. Taking the derivative of

$(dX/dt)^2 = 0.294(50Y^2 - Y^3)$,

$2(dX/dt)(d^2X/dt^2) = 0.294(100Y - 3Y^2)(dY/dt)$, and

$d^2X/dt^2 = 0$ at $Y = 100/3$.

At $Y = 100/3$,

$dX/dt = \sqrt{0.294(50 - 100/3)(100/3)^2} = 73.8$.

The stick moves past the peg at 73.8/2 = 36.9 cm/sec.

Also solved by John Langhaar, Allen Wiegner, John Prussing, Winslow Hartford, and the proposer, Bruce Calder.

F/M 5. (1) Choose any four different digits.
(2) Arrange them to make the largest number.
(3) Arrange them to make the smallest number.
(4) Subtract the smaller from the larger.
(5) Take the result and go back to step 2 and see what happens.

Why does this procedure always converge to 7641? Is there a similar phenomenon for five-digit numbers, and if so why?

Winslow Hartford noticed that for five-digit numbers a fixed point does not always arise. For example, this is the situation when starting with 98632. Other numbers (eg. 86521) converge to 61974. To explain the phenomenon with four-digit numbers Matthew Fountain observes that when the procedure is followed, as illustrated by 7321 − 1237 = 6084, the outer digits of the result always sum to 10, the inner digits to 8, except when the inner digits are a pair of 9's. This restricts the results of the subtraction so that the numbers obtained are few in number. Checking all these cases shows that a number with the digits of 7641 always appears. As 7641 − 1467 = 6174, the procedure then stops.

Also solved by Reginald Bisson and the proposor, John Rudy.

Better Late Than Never

JAN 1. Peter Silverberg has responded.

JAN 2, 3, 4. John Langhaar has responded.

F/M SD1. Albert Mullin corrects an embarassing typo: Ken Thompson received the ACM *Turing* award.

Proposers' Solutions to Speed Problems

SD 1. Anyone who solves this problem will be showing his age! The trick was to be familiar with the log-log scales on a slide rule. The scale, which runs from $e^{-.01}$ to $e^{-.001}$, starts somewhere around .99. Hence, $.99 \approx e^{-.01}$ and $.99^{100} \approx e^{-1}$, which to two-digit accuracy is .37.

SD 2. If you point the hour hand of the watch at the sun, north is midway between the hour hand and 12 o'clock. In the Southern Hemisphere, north is midway between the hour hand and 6 o'clock.