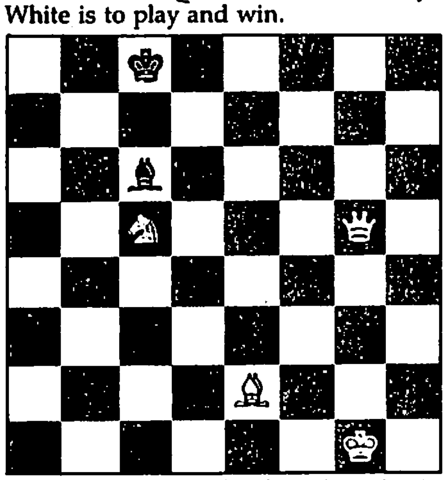
ALLAN J. GOTTLIEB

Thoughts While Shopping in a Favorite Supermarket

For the information of newcomers, here are the criteria I use to select solutions for publication. As responses to problems arrive, they are sorted by problem, with no regard to their date of arrival or postmark. When it is time for me to write the column in which solutions are to appear, I first weed out erroneous and illegible responses. For difficult problems, this may be enough; the most publishable solution becomes obvious. Usually, however, many responses remain. I next try to select a solution that supplies an appropriate amount of detail and that includes a minimal number of characters that are hard to set in type. A particularly elegant solution is, of course, preferred. I favor solutions from correspondents whose work has not previously appeared, as well as solutions that are neatly written or typed, since the latter produce fewer typesetting errors.

Problems

M/J 1. We begin with a chess problem that Bob Kimble attributes to Seletsky. White is to play and win.



M/J 2. Bruce Calder has been busy lately bouncing BBs on top of superballs: A BB 2 mm. in diameter sits atop a "Superball" 10 cm. in diameter, the center of which is 1 meter above the ground. If both objects are released simultaneously, what is the maximum possible height the BB can attain upon rebound?

M/J 3. The following problem is from John Rule:
From the front to the rear of an advancing army detachment was 10 miles. A rear guard messenger, dispatched to the guard house directly behind his position in the line of march, returned without loss of time and then proceeded immediately to the vanguard and again returned. He then noted that he had overtaken his guard 10 miles from the starting point and that the time spent on each errand had been the same. How far was the guard house from the starting point, and how far did the messenger travel altogether?

M/J 4. Phelps Meaker, whose eyes are 62 inches above the ground, notes that the floor of the aisles in his favorite supermarket is glossy linoleum and reflects the fluorescent lamps installed crosswise to the aisle 10 feet above it. As he moves down the aisle at a uniform rate of 42 inches per second, the reflected images move as well. When a lamp is a horizontal distance D ahead of him, what is the speed of its image in the floor?

M/J 5. Our last regular problem is from Nob Yoshigahara via Richard Hess:
Fill in the boxes with the digits 0, 1, 2 . . . 9.

$\square\square^{\text{min}}:\square\square^{\text{sec}} \times \square = \square^{\text{hrs}}: \square\square^{\text{min}}: \square\square^{\text{sec}}$

Speed Department

SD 1. Our first speed problem appeared in *The Tech*, M.I.T.'s student newspaper, as part of the "Freshman Quiz": Translate the following into a limerick:

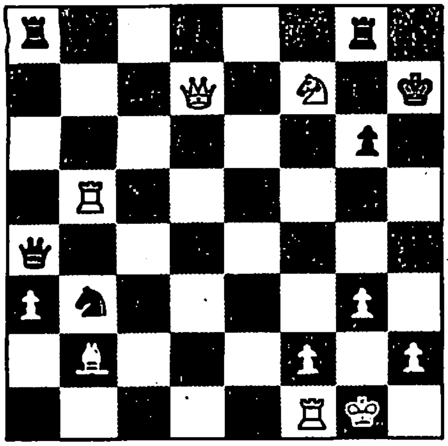
$$\int_1^{\sqrt[3]{3}} z^2 dz (\cos 3\pi/9) = \ln e^{1/3}$$

SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, THE COURANT INSTITUTE, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y. 10012.

SD 2. The 1986 M.I.T. Integration Bee included the following problem:
$\int \{[\sin(x) + \cos(x)]dx/\sqrt{1 + \sin(2x)}\}$

Solutions

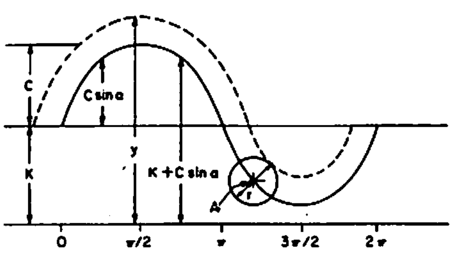
JAN 1. White to move and mate in four.



Robert Bart, Stephen Callaghan, and Matthew Fountain have refuted this problem by showing that with best play mate requires five moves, not four. The key move for White is R-h5, but Black answers with Q-f4 and after PxQ replies N-d4. Now the quickest mate is RxP, BxR, N-g5, K-h8, Q-h7.

Also solved by Erwin Bramhall, Paul Tesser, Ted Clampitt, Albert Moore, Richard Hess, Steve Feldman, Ray Kingsley, Greg Spradlin, Ron Raines, and Matthew Ek.

JAN 2. Philip Hogin offered $100 for a solution to
$y = f(\alpha)$
for given values of K, C, and r in the diagram below.



For some reason this problem attracted considerable attention. Could it have been the $100 reward? To be fair, I waited until today, 21 March, the closing date for the column, to gather together all proposed solutions and send them to Mr. Hogan for his decision (and disbursement). As soon as I hear, I will announce the lucky winner.

Responses were received from Ali Nadim and Hossein Haj Hariri, Matthew Ek, Richard Garner, Tom Tiller, Bruce Calder, Daniel Morgan, David Leblang, David Meyer, Dean Peterson, Jahir Pabon, Marc LaBranche, Martin Carrera, Matthew Stenzel, Mengli Du, Robert Moeser, Matthew Fountain, Richard Hess, Bin Ly, Harry Zaremba, Albert Moore, A. Lawson, Dennis Brown, Shahriar Negahdaripour, Winslow Hartford, Norman Wickstrand, Frank Quinn, Bill Peak, Ray Kingsley, William Messner, and Charles Benesh.

JAN 3. The six-digit number 142857 has interesting properties: one times the number is 142857; two times is 285714; three times is 428571; four times is 571428; five times is 714285; and six times is 851428. All these products are rotated versions of the original. Are there other numbers having the property that multiplication by any positive integer not exceeding the number of digits in the original number produces rotated versions of the original?

The following solution is from Robert Moeser:

I recognized 142857 as an integer made from the repeating part of the division 1/7. In fact, any prime that exhibits a repeating decimal of the full theoretically possible length (p − 1 for a prime p, since all numbers except p itself and zero appear as remainders) will have the desired properties. Seventeen for example, and I checked that out. There is no way of telling in advance whether a prime will have a maximum-length repeating decimal, but those that do can be used to produce the interesting numbers. Do the division (let a computer help you, please!) and then make the repeating part into an integer. Viola! In essence, multiplying such a number by any positive integer less than the prime used to produce it is just like starting a division process that is bound to repeat itself, but has the effect of shifting the repeating digits over to the point (in the division 1/p) where the number appears as a remainder. The following remainders follow the same cycle, which of course leads eventually back to a remainder of one, and then sooner or later again to the number used as a multiplier. The net result is the same digits appearing in the quotient, only rotated!

With his solution, Matthew Fountain included the related remark that as a boy he saw a magician multiply a large number by 142857 in his head. Later Mr. Fountain realized that the magician had divided by 7 and made a correction. However it must be noted that while doing this the magician was also writing upside down and backwards and memorizing cards read to him.

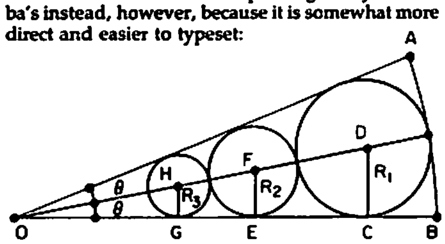
Also solved by Sidney Darlington, Eli Passow, Avi Ornstein, Edward Lynch, Winslow Hartford, Bin Ly, Richard Hess, and Naomi Markovitz.

JAN 4. Given the sector OAB of a circle of a unit radius with angle 2θ. For what θ is the ratio of the sum of the three inscribed circles' areas to the sector's area largest, and what is its value?

I received the following note from Jason Bitsky, which I feel is too good not to share:

I've been convinced for some time that, having been a corporate lawyer in New York for this long, my cognitive faculties have long since atrophied. As I'm sure you know if you've ever encountered one, every good lawyer faithfully commits himself to the ideal of rarely thinking intelligently about anything and certainly not if there's no bucks to be made. This being my normal state, I generally limit my exposure to *Technology Review* to a brief glance at a few of the more dramatic color graphics. But, as I flipped through the pages this time, problem JAN4 in "Puzzle Corner" caught my eye. It reminded me of the kind of computational exercise that entertained me often in the precatatonic phase of my life. Anyway, I decided to look brain death straight in the face and try my hand at the problem. Like an antique engine, rusty cogs began to turn, trigonometric identities flickered up out of the mental morass, and a long-moribund intellect, briefly revived, produced the following answer. I doubt very much that it is the simplest, most direct solution, but I hope it will not embarrass me.

Mr. Bitsky has not embarrassed himself; indeed his solution is correct. I am printing Harry Zaremba's instead, however, because it is somewhat more direct and easier to typeset:



Since the radius OA = 1, the area of sector AOB is
$A_s = OA^2/2 \times 2\theta = \theta$.
From triangle OCD, $R_1 = OD \times \sin\theta$, or
$R_1 = (1 - R_1) \times \sin\theta$,
from which $R_1 = k$ where $k = (\sin\theta)/(\sin\theta + 1)$.
From triangle OEF, $R_2 = OF \times \sin\theta = (1 - 2R_1 - R_2)\sin\theta$, or
$R_2 = k(1 - 2K)$.
In a similar manner from triangle OGH,
$R_3 = k(1 - 2k)^2$ and for the ith circle,
$R_i = k(1 - 2k)^{(i-1)}$.
The area of the circles where $i \to \infty$ is
$A_c = \pi(R_1^2 + R_2^2 + R_3^2 + \ldots + R_i^2 + \ldots)$
$= \pi k^2 [1 + (1 - 2k)^2 + (1 - 2k)^4 + \ldots + (1 - 2k)^{2(i-1)} + \ldots]$
$= \pi k^2/[1 - (1 - 2k)^2]$
$= \pi k/[4(1 - k)]$.
Substituting for k and simplifying,
$A_c = \pi \sin\theta/4$.
The ratio of the sum of the circle areas to the area of the sector is
$R = \pi/4 \times (\sin\theta)/\theta$.
Maximum R occurs when θ approaches zero, for which $(\sin\theta)/\theta \to 1$. Thus,
$R_{max} = \pi/4 = 0.785398$.

Also solved by Dave Mohr, Michael Jung, Thomas Compton, Naomi Moskovitz, Matthew Fountain, Richard Hess, Bin Ly, Winslow Hartford, Steve Feldman, Norman Wickstrand, Harry Zaremba, Jason Bitsky, Stephen Scheinberg, Eli Passow, Norman Spencer, and the proposer, Howard Stern.

Better Late Than Never

1985 JUL 4. Phelps Meaker notes that it was John Rule and not he who proposed this problem. Moreover, P.V. Heftler and Steve Feldman noticed that we mistakenly published the solution for A/S 4 instead. Fortunately, my records proved adequate in this case, and here is the correct problem:

A manufacturer makes all possible sizes of brick-shaped blocks such that the lengths of the edges are integral multiples of the unit of length, and that the number of units in the total of twelve edges of the block is equal to two-thirds of the number of units of volume in the block. What sizes does he make?

Furthermore, here is the correct solution submitted by Avi Ornstein last June:
$4(A+B+C) = 2(A \times B \times C)/3$
$6(A+B+C) = (A \times B \times C)$
$6(A+B) = (A \times B \times C) - 6C$
$C = 6(A+B)/(A \times B - 6)$
Since the answers must all be integers, I let A and B be the two smallest sides of each brick. Acceptable solutions were found when A × B = 7, 8, 9, or 12. I found that the bricks are made in eight different sizes: (1,7,48), (1,8,27), (1,9,20), (1,12,13), (2,4,18), (2,6,8), (3,3,12), and (3,4,7.)

1985 N/D 2. Joseph Feil has responded.

1985 N/D 5. Gerald Leibowitz reports that related questions have been studied in the literature.

Y1985. The following improvements come from Robert Deutsch, Gregory Daley, Tom Harriman, and Steve Feldman.
2 = 9+8−15
11 = 9+8−5−1
28 = 1+9×(8−5)
39 = 58−19
41 = 59−18
93 = 1×98−5
95 = 1+9+85
The original solution 11 = −1 + 9 + 8 − 5 would be preferred if the initial minus is not considered an operator but part of the number minus one.

JAN SD2. Ruth Cross would prefer that the last line be given as, "Is nine times itself and no more." Charles Bostick notes that this problem appeared in a 1969 issue of *Word Ways*, a journal of recreational linguistics.

Proposer's Solutions to Speed Problems

SD 1. "Integral z-squared-dz,
From one to the cube root of three,
Times the cosine of three
times pi over nine,
equals ln-of-cube-root-of-e."

SD 2. x + C (play with the denominator). □