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Playing 20 Hands With No Winners, and the Feeding of Cats and Explorers

Since this is the first issue of a new academic year, I once more review the ground rules under which this department is conducted.

In each issue I present five regular problems (the first of which is chess, bridge, or computer-related) and two "speed" problems. Readers are invited to submit solutions to the regular problems, and three issues later one submitted solution is printed for each problem; I also list other readers whose solutions were successful. For example, solutions to the problems you see below will appear in the February/March issue. Since I must submit that column sometime in November (today is July 19), you should send your solutions to me during the next few weeks. Late solutions, as well as comments on published solutions, are acknowledged in the section "Better Late Than Never" in subsequent issues.

For "speed" problems the procedure is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same column in which the problem is published. For example, the solutions to this issue's "speed" problems are given below. Only rarely are comments on "speed" problems published or acknowledged.

There is also an annual problem, published in the first issue of each new year; and sometimes I go back into history to republish problems that remained unsolved after their first appearance.

Problems

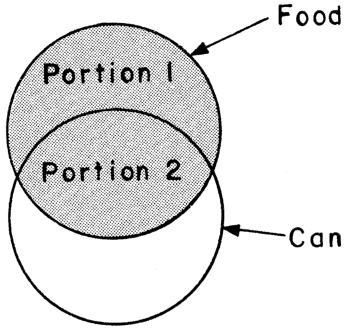
OCT 1. We begin with a bridge problem from Lawrence C. Kells, who writes that a friend directed a duplicate bridge tournament in which every hand was played 20 times. One of these hands produced a strange result. At four of the tables the final contract was one club, played once from each of the foursides. At four of the other tables it was played at one diamond once from each side. At the remaining tables it was played once from each side at one heart, one spade, and one no-trump. Every one of these contracts was set. Analysis of the hand proved that none of the declarers made a mistake in play. Unfortunately, Mr. Kells failed to see what the deal was. Can you reconstruct it? (That is, a deal where any contract anybody bids can be set no matter how hard he tries to make it.)

OCT 2. Smith D. Turner (∫dt) has a question about the following nim-like game, called the game of thirty-one, that was popular with New York City magicians about 40 years ago:
Twenty-four cards, the 1-6 of each suit, are put face-up on the table. Two players pick up alternately, each keeping track of his pip-total. One wins by hitting *thirty-one,* or forcing one's opponent to exceed 31. Does the first or second player have a sure win, and how does he play to insure it?

OCT 3. Albert Mullin asks us an educational problem:
A very wise dean wishes to improve the quality of research and education at her university. She believes some of the "fault" is with the department heads, but, being wise, recognizes some of the "fault" may be with herself. For starters, she decides to relocate her office so as to *minimize* the average distance to her department heads. Where should she locate her office? (For simplicity, assume all offices are located in a plane and that the dean cannot co-reside with any department head).

OCT 4. Yogesh Gupta posted the following problem on an electronic bulletin board that I read:
An exploring team wants to reach a destination that is six days away. Each explorer can carry enough provisions to sustain one person for four days (and the distance an explorer travels in a day is independent of the amount of provisions he or she is carrying). What is the smallest team that permits at least one explorer to reach the destination and permits all the explorers to return home safely?

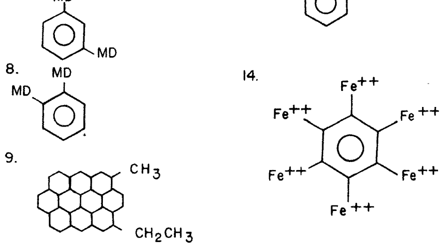
OCT 5. John Glenn has two cats, which are fed a single can of cat food. The food comes out of the can as a single intact cylinder, and the can is then used to cut the cylinder into two shares of food. Where should the can be positioned to generate equal shares?



Speed Department

SD 1. Phelps Meaker wants to know at what time between two and three o'clock the hour and minute hands are in a symmetrical position.

SD 2. We end with a series of "chemistry" questions from *The Tech,* M.I.T.'s student newspaper:
Give the names of the following compounds:



Solutions

M/J 1. How does South make five clubs after an opening lead of the ♦Q, and what lead sets this contract?

	♠ A 8 4 ♥ A K 10 ♦ A 7 6 5 4 ♣ K J	
♠ K ♥ Q J 9 4 3 2 ♦ Q 9 8 3 2 ♣ Q		♠ Q J 10 3 2 ♥ 8 6 5 ♦ J ♣ 10 7 5 4
	♠ 9 7 6 5 ♥ 7 ♦ K 10 ♣ A 9 8 6 3 2	

Benjamin Feinswog suggests winning the opening lead with the ♦K and then cashing the ♣K and ♣J, discarding a spade. Now the ♥10 is ruffed and the ♣A cashed. At this point the ♦10 is led and, whether East ruffs or not, the ♠A provides an entry to the ♦A permitting a second spade discard. Thus South loses only one spade and one trump. Mr. Feinswog notes that a lead of the ♦2 (among others) will set the contract.

Also solved by Richard Hess (who believes that *any* lead will set the contract), Matthew Fountain, and the proposer, Doug Van Patter.

M/J 2. A large bed of flowers and greenery is laid out in the form of a regular polygon of N sides. A walk composed of N trapezoidal concrete slabs surrounds the flower bed. A circumscribing circle passing through the outer corners of the walk and an inscribing circle tangent to the inner flats of the trapezoids have circumferences in the ratio of almost exactly 191:165. The total area within the outer periphery of the walk and the area of the walk itself are in the ratio of 4:1. Find N.

The following solution is from Richard Heldenfels:

The general equations of regular polygons and a trigonometric relationship were all that was required to obtain a direct solution. The ratio of circumscribing and inscribing circles provides the following:

$a_o\tan(180/N)/a_i\sin(180/N) = 191/165$,

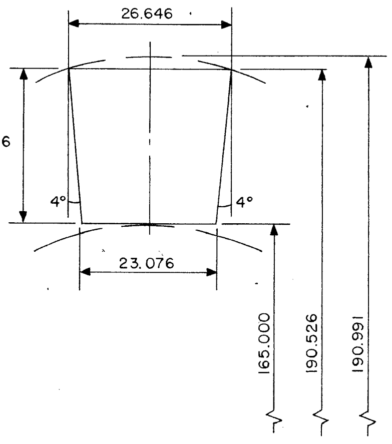
where the a's are the length of the outer and inner sides and N is the number of sides. The ratio of areas provides the following:

$A_1/A_0 = (a_1/a_0)^2 = 3/4$.

These equations can be combined and simplified to yield:

$\cos(180/N) = (165/191)(2/\sqrt{3})$.

The solution is: 180/N = 4.0391°. Since the ratio of circles was not exact, N = 45. The ratio of 191:165 is greater than that ratio for N = 45 polygon by the factor of 1.000048. A drawing of one trapezoidal concrete slab follows:



Also solved by Charles Freeman, Dennis White, Frank Carbin, George Parks, Harry Zaremba, John Prussing, John Woolston, Matthew Fountain, Michael Jung, Naomi Markovitz, Richard Hess, Steve Feldman, and the proposer, Phelps Meaker.

M/J 3. Given a number N between 500 and 1000, rapidly construct a series of numbers using eight of the nine digits from 1 through 9 once, the other digit twice, and a few zeroes so the series totals N.

Naomi Markovitz must have spent a great deal of time in tricky parlors:

H	T	U	H	T	U	H	T	U	H	T	U
	1			1			1			1	
	2			2			2			2	
	3		3			3			3		
	4			4			4			4	
	5			5			5			5	
	6			6			6			6	
	7			7			7			7	
	8			8			8			8	
	9			9				9			9
											3

Being that as many zeros as desired may be used, I'll worry about whether the other digits should be in the hundreds, tens, or units column and then affix the appropriate zeros in the presentation of the requested series. I noticed in the example presented that most of the non-zero digits are in the tens column. Indeed the sum of 10 + 20 + . . . + 90 = 450, which is the same order of magnitude as any answer which may be requested. Therefore, my approach would be to start with one "copy" of each digit in the tens column and then to check, in the case of each sum desired, which digits have to be moved to the hundreds column, which to the units column, and which has to be duplicated (and the column in which the second copy will appear). If a digit, a, moves from the tens to the hundreds column, the sum will increase by 100a − 100a = 90a. If a digit, b, is moved from the tens column to the units column, the change is −10b + b, or a decrease of 9b. Take, for example, the case which is presented in the problem: N = 642. I start off with the arrangement at the left of the box above. This gives a total of 450. I have to add 642 − 450 = 192. To check which digit has to be moved to the hundreds column, I divide 190 by 90 and get 2+. I'll move 3 to the hundreds column, because if I add too much, I can subtract the extra by moving a digit to the units column. Now I have the second display in the box. I've added 3 × 90 = 270, instead of the required 192, so I'll have to subtract 78. To determine which digit to move to the units column, I divide 78 by 9 and get 8+. Again, I chose to subtract "too much" because this problem can be solved by adding the second copy of one of the digits. So I'll move the 9 into the units column. Now I have the third display in the box. I have subtracted 9 × 9 = 81 instead of the required 78. I have to add 81 − 78 = 3. I therefore add a second 3 to the units in the fourth display. So, the answer can be presented as:

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319
 20
 40
 50
 60
 70
 83
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642
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There are, however, cases in which the procedure must be slightly modified. The basic procedure will work in all cases since:

$500 \leq N \leq 1000$

we see that

$50 \leq N - 450 \leq 550$.

Dividing by 90 will always give an answer in which a single digit must be moved to the hundreds column. (Even if the division comes out even, add 1 to the quotient to determine the digit to be moved, because eventually a second copy of one digit will have to be added.) The second stage consists of dividing by 9 a number from 1 to 90. Several problems could occur:

(1) The number that has to be moved from the tens

column to the units column is no longer in the tens column, because it has already been moved to the hundreds column. Solution: If b ⩾ 3, move both 1 and b − 1 (or any other combination of 2 numbers totaling b) into the units column. If b = 2, instead of moving 2 into the hundreds column (for a gain of 2 × 90 = 180), move the 3 into the hundreds column and then 1 + 9 (or any other combination totaling 10 other than 2 + 8) into the units column:
$3 \times 90 - 1 \times 9 - 9 \times 9 = 180$
The 2 will then be available to move into the units column.
If b = 1, instead of moving 1 into the hundreds column, move 2 into the hundreds column and either 3 and 7 or 4 and 6 into the units column. The 1 will then be available to be moved into the units column.
(2) It's required to move 10 or 11 into the units column. Solution: Move in two numbers with the appropriate sum. There will always be a pair available, because the original list (1 − 9) consists of four pairs totaling 10 and four pairs totaling 11. Since only one digit has been removed from the tens column, enough choices still remain. The last stage always gives a number from 1 to 9 that can be added, so this can be accomplished by placing the appropriate single digit in the units column.

Also solved by Mathew Fountain, Richard Hess, Steve Feldman, and the proposer, Lester Steffens.

M/J 4. As we generate geometric figures to represent $y = x^n$, we have "elements" consisting of points, lines, faces, cubes, etc. as n increases. For the number of points in each figure, we have (for $n > 0$) $P = 2^n$. Derive the number of lines and faces in a five-dimensional hyper-cube.

Charles Sutton solved a generalization of M/J 4: We may consider the generation of the sequence of n-dimensional cubes, d units on a side, as follows: A cube of any number of dimensions, moved a distance d in a direction perpendicular to all its dimensions, will generate a cube of one higher dimension. If one starts with a zero-dimensional cube (a point), this procedure will generate, in succession, line segments, squares, cubes, and so on into cubes of higher dimensions (hypercubes). Note that an n-dimensional cube will have as components all cubes of lower dimensions, from zero-dimensional points up to (n − 1)-dimensional cubes. Now when an n-dimensional cube is moved a distance d in the direction of the next higher dimension to generate an (n + 1)-dimensional cube, its r-dimensional component cubes ($0 \leq r < n$) will necessarily generate (r + 1)-dimensional components of the (n + 1)-dimensional cube. It is clear that the number of points will double, since there will be just as many points in the final position of the cube as in the initial position, and hence the number of points (zero-dimensional components) in an n-dimensional cube will be 2^n. When an r-dimensional component ($r \geqslant 1$) of an n-dimensional cube moves a distance of d, there will be just as many such components in the final position as in the initial position, but there will be additional r-dimensional components generated by the motion of (r − 1)-dimensional components. Hence the number of r-dimensional components in an (n + 1)-dimensional cube can be obtained by adding the number of (r − 1)-dimensional components to twice the number of r-dimensional components in an n-dimensional cube. We can use this to complete the array at the top of the next column, giving the lower dimensional components for cubes of each number of dimensions. Filling in 1's in the diagonal (since an n-dimensional cube can be considered to have itself as a component) and powers of two in the left hand column, we can continue by adding twice the value of any entry to its left hand neighbor and writing the result below the doubled entry. The construction of this array is reminiscent of Pascal's triangle, and in fact the numbers in the rows can be seen to be the coefficients in the expansion of $(2x + 1)^n$. And the answer to Winslow Hartford's question in M/J4 is that the numbers of lines and faces in a five-dimensional hypercube are both 80.

Mr. Sutton then adds the following remarks:
I have always been intrigued by the geometry of

		Dimension of components							
		0	1	2	3	4	5	6	7
Dimension	0	1							
of cube	1	2	1						
	2	4	4	1					
	3	8	12	6	1				
	4	16	32	24	8	1			
	5	32	80	90	40	10	1		
	6	64	192	240	160	160	12	1	
	7	128	448	672	560	290	84	14	1

higher dimensional figures, and recall having read someplace that there are six regular four-dimensional solids but that only the analogues of the cube and tetrahedron exist for all higher dimensions. That set me to wondering what the triangular array I had obtained in my solution of M/J 4 would look like for the equilateral triangle, regular tetrahedron series. Actually it's fairly easy, since to step up one dimension you need only to find a point in the next higher dimension that is the same distance from the set of equidistant points in the lower dimensions. The n-dimensional analogue of the tetrahedron would have n + 1 vertices, and the number of its r-dimensional components (each having r + 1 vertices) would be the number of combinations of n + 1 things taken r + 1 at a time. Binomial coefficients! And the numbers in the horizontal row of the triangular array corresponding to the n-dimensional analogue of the tetrahedron turn out to be the coefficients of $(x + 1)^{n+1} - x^{n+1}$. As a check, for n = 3, a tetrahedron has four vertices, six edges, four faces, and one tetrahedron, coefficients of $(x + 1)^{n+1} - x^{n+1}$.

Also solved by Avi Ornstein, Charles Freeman, Dennis White, Matthew Fountain, Richard Hess, and Winslow Hartford.

M/J 5. Take the letters in the first half of alphabet in order. Place the A on the table. Place the B next to the one of the four sides of the A. Place the C next to one of the six sides at the AB (or BA) pair. Then add the D and so on. If you do this correctly, when you reach the letter M you will have created a crossword-puzzle matrix of complete common English words, no proper names, no foreign words, and no acronyms or abbreviations. Having solved the problem as posed, can you add one or more letters to the A-M set and still retain complete words?

Both Rik Anderson and Matthew Fountain found the identical solution—one that goes up to P. The key to extending past M is the word "knop." It would be hard to go further since adding Q would require U and hence R S T.

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      H
    B A C K
  F E D   N
J I G     O
  L       P
  M
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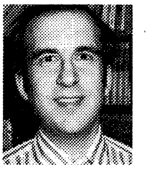
Also solved by Harry Zaremba, M. Arch, Naomi Markovitz, Richard Hess, and Steve Feldman.

Proposers' Solutions to Speed Problems

SD 1. 2:46 2/11
SD 2.
1) Polywannacracker
2) Polar bear
3) Barium Goldwater (barium gold H-to-O)
4) Hi-O, Silver
5) A dozen bananas
6) Paramedics (or paradox)
7) Metaphysics
8) Orthodox
9) Methyl ethyl chicken wire
10) Transistor
11) Cis-boom-bah
12) FORTRAN 4
13) Mercedes benzene
14) Ferrous wheel



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