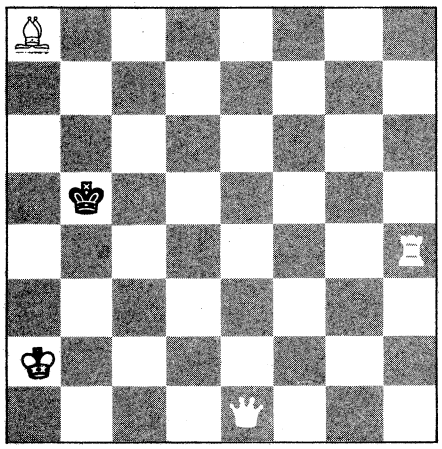
ALLAN J. GOTTLIEB

Midsummer Snowstorm

Our group is just now moving to a newly renovated floor. We had a major influence in specifying the layout, and it is satisfying to see how well it all works. I guess that if our research efforts into computer architecture turn out to fizzle we have a related field to move into.

Problems

A/S 1. Here is a chess problem from *The Tech*, M.I.T.'s student newspaper. White is to move and force a mate in two:

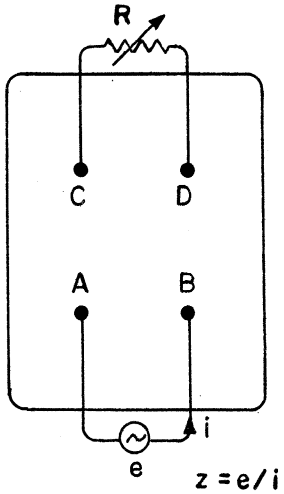


A/S 2. Another geometry problem from Phelps Meaker, who writes:

I wanted an approximation of a four-foot sphere for light-integration measurement. I made a dodecahedron of sheet metal, except that instead of flat surfaces, I substituted low five-sided pyramids. It served my purpose. How should I have designed the pyramids so that all the dihedral angles were equal?

A/S 3. Randy Barron poses a question that should appeal to all EE majors:

A four-port device contains only passive linear circuit elements. Using a fixed-frequency sine-wave generator, you can measure the complex impedance Z between two of the terminals, A and B. A variable resistor R is connected across the other two terminals, C and D. What is the locus in the complex plane traced out by Z as R varies from zero (short circuit) to infinity (open circuit)?



A/S 4. Here is a temporal cryptarithmetic problem from Nob Yoshigahara relayed to me via Richard Hess (on DICKNET interpuzzler communication network): Fill in the boxes with the digits 1,2,3...,9:

$\square\square^{min}:\square\square^{sec} \times \square = \square\square^{min}:\square\square^{sec}$

A/S 5. An out-of-season problem from Bruce Calder:

It began to snow on a certain morning, and the snow continued to fall steadily throughout the day. At noon, a snowplow started to clear a road at a constant rate in terms of the volume of snow removed per hour. The snowplow cleared two miles by 2 p.m. and one more mile by 4 p.m. At what time had the snowstorm begun?

Speed Department

SD 1. Greg Huber sent us the following problem, told to him by Douglas Hofstadter:

Simplify the product

$(x - a)(x - b) \ldots (x - z)$.

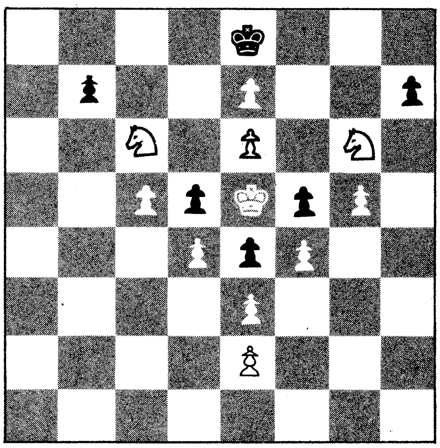
SD2. Doug Van Patter offers a bridge quickie:

North:	*East:*
♠ K 9 3	♠ 10 7 6
♥ 6 5 3	♥ A K Q 7
♦ A J 9 2	♦ Q 7 5
♣ Q 7 4	♣ J 10 9

You are East in a rubber bridge game; bidding has gone: South 1NT, North 3NT. Your partner leads the ♥10, and everyone follows to three rounds of hearts. What is your best shot at setting this game?

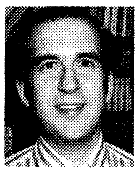
Solutions

APR 1. White is to play and mate in two:



Jerry Grossman sent us the following solution:

White moves g5-f6 (en passant), then f6-f7 (checkmate) after any reply by Black. To prove that this solution is correct, we first claim that Black's previous move was either d7-d5 or f7-f5. These are clearly the only pawn moves possible (since the White king could not have been in check when Black moved), but why couldn't the Black king have moved to its present position on the last move to escape check? It couldn't have come from d8 or f8, since there would have been no way for White to administer the double check (pawn at e7 and knight). On the other hand, if the Black king came from d7 or f7, then White would have had to have moved the pawn to e6 on the previous move but there is no now-vacant square from which this pawn could have come. Thus we have verified this first claim. Now, in order for White's pawns to be in the position they are in, they must have made at least 10 captures. Black is missing only 10 pieces, however, and hence every one must have been captured by a pawn. If Black's last move was d7-d5, then Black's queen bishop could not have ever moved and could not have been captured by a pawn. This contradiction proves that Black's last move could only have been f7-f5, and thus the so-



SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, ASSOCIATE RESEARCH PROFESSOR AT THE COURANT INSTITUTE OF MATHEMATICAL SCIENCES, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y., 10012.

lution announced above provides the only mate in two.

Also solved by Jeffrey Mattox, John Bobbitt, Matthew Fountain, and Neil Hochstedler.

APR 2. Note that each of the following is a different way of evaluating the same equation:

16601.92 + 14374.08 =
11334.4 + 19641.6 =
18521.44 + 12454.56 =
4147.36 + 26828.64 =

Write the equation in the usual form.

John Bobbitt was able to recognize the hidden quadratic and writes:

The four "sets of numbers" represent

$a^2 + 2ab + b^2 = (a + b)^2$

where a = 111.6 and b = 64.4. By forming different groupings of the four terms a^2, b^2, ab, ab, we can get the four sets. Thus,

$(a^2 + b^2) + 2ab = (a + b)^2$
$(ab + b^2) + (ab + a^2) = (a + b)^2$
$(b^2 + 2ab) + a^2 = (a + b)^2$
$b^2 + (a^2 + 2ab) = (a + b)^2$

Also solved by Harry Zaremba, Jerry Grossman, Marshall Fritz, Matthew Fountain, and Winslow Hartford.

APR 3. Find all maxima and minima of

$$\frac{\ln\left(1 + e^{-x} + \frac{x}{2}\right)}{\ln\left(1 + e^{x} - \frac{x}{2}\right)}$$

without using calculus.

The solution below is from George Bird:

Note that the expression $(1 + e^{-x})$ may be written as $(1 + 1/e^x)$. Use the second form of this expression and write it as a fraction with the common denominator e^x, thus:

$(e^x + 1)/e^x$

If we substitute this form, the expression $\ln(1 + e^{-x})$ becomes:

$\ln(1 + e^x) - \ln(e^x)$.

The function

$[\ln(1 + e^{-x}) + x/2]/[\ln(1 + e^x) - x/2]$

then becomes:

$[\ln(e^x + 1) - \ln(e^x) + x/2] / [\ln(1 + e^x) - x/2]$,

which reduces to 1/1. Therefore there are infinitely many maxima and minima, and all have value 1.

Also solved by Allen Tracht, Edwin McMillan, Harry Zaremba, Howard Stern, J. Richard Swenson, Jerry Grossman, John Bobbitt, John Prussing, Ken Haruta, Marshall Fritz, Matthew Fountain, Mike Hennessey, Naomi Markovitz, Peter Card, Ross Hoffman, Steve Feldman, Steve Silberberg, Tony Trojanowski, Winslow Hartford, and the proposer, Rick Decker.

APR 4. On each day of the year (not leap year) you are given a penny. On December 31 you are given your last penny and told that it was fresh from the U.S. Mint, but that one of the previous pennies may have been counterfeit, and therefore lighter or heavier than the standard penny. You are asked to determine the number of balancings, using a common pan balance, that would be necessary and sufficient to determine whether or not there is a counterfeit coin, and if there is, to tell whether it is heavier or lighter than the last penny that you received.

The following solution is from Leon Tabak:

Two balancings are necessary (clearly) and sufficient for determining which of three possible solutions is the true situation:

(1) all coins are genuine.
(2) one coin is counterfeit and it is lighter that all of the other (genuine) coins.
(3) one coin is counterfeit and it is heavier that all of the other (genuine) coins.

Divide the set of 364 coins (whose genuineness is not known) into three sets: S1, S2, and S3. Let S1 contain 122 coins. Let S2 and S3 each contain 121 coins. A fourth set, G, contains the one certified, genuine coin.

First measurement: Place S1 in the left pan of the scale. Place S2 and G on the right side of the scale. If the two balance, then all of the coins in S1 and S2 must be genuine and the counterfeit coin, if there is one, must be in S3. The possibilities that remain

if the two pans balance in this first measurement are:
(1) all coins are genuine.
(2) S3 contains an extra-light counterfeit coin.
(3) S3 contains an extra-heavy counterfeit coin.
If the right pan comes to rest at a lower level than the left pan, then two possibilities remain:
(1) S1 contains an extra-light counterfeit coin.
(2) S2 contains an extra-heavy counterfeit coin.
In either case, all of the coins in S3 must be genuine. (There is at most one counterfeit coin, and this test has shown that it must be in S1 or S2).
Second measurement: Place S1 in the left pan. Place S3 and G in the right pan. If the scale balances on the second measurement and it also balanced on the first measurement, then all coins are genuine. If the left pan is higher, but the scale balanced during the first measurement, then S3 must contain an extra-heavy counterfeit coin. If the left pan is lower, but the scale balanced during the first measurement, then S3 must contain an extra-light counterfeit coin. If the scale balances on the second measurement, but the left pan rested lower during the first measurement, then S2 must contain an extra-light counterfeit coin. If the scale balances on the second measurement, but the left pan rested higher during the first measurement, then S2 must contain an extra-heavy counterfeit coin. If the left pan is lower and the left pan was also lower during the first measurement, then S1 must contain an extra-heavy counterfeit coin. If the left pan is higher and the left pan was also higher during the first measurement, then S1 must contain an extra-light counterfeit coin. Two outcomes are inconsistent with the statement of the problem. The left pan cannot be higher in the first measurement and lower in the second, nor can it be lower in the first and higher in the second. Either outcome would imply the existence of more than one counterfeit coin.

Also solved by Rik Anderson, Dudley Church, E.P. Schacht, Frederic Jelen, Harry Zaremba, John Prussing, John Spalding, Kenneth Olshansky, Matthew Fountain, Phelps Meaker, Walter Cluett, and the proposer, Alan Faller. Mr. Faller also showed that with six balancings, one can also determine which coin was counterfeit (except for leap years when seven are required). Copies of Mr. Faller's solution are available upon request.

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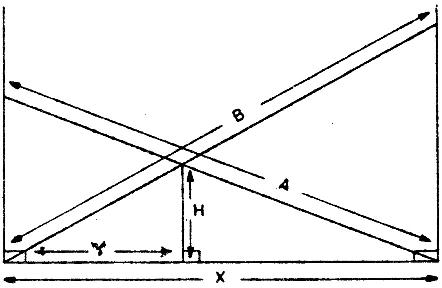
APR 5. Find X for two configurations—when $A = 15$, $B = 10$, and $H = 8$; and when $A = 16 + 2\sqrt{2}$, $B = 16 - 2\sqrt{2}$, and $H = 2$.



Howard Stern's solution is one of the few in which an exact solution was found for the second configuration:
With the above lengths labelled, similar triangles give the following relationships:
$(\sqrt{B^2 - X^2})/X = H/Y$
$(\sqrt{A^2 - X^2})/X = H/(X - Y)$.
Eliminating Y yields:
$$1/(\sqrt{B^2 - X^2}) + 1/(\sqrt{A^2 - X^2}) = 1/H \qquad (1)$$
The left side of (1), viewed as a function of X, has a minimum of $(1/A) + (1/B)$ when $X = 0$ and increases with X. For the first set of parameters given ($A = 15$, $B = 10$, $H = 8$), the left hand side of (1) will always be greater than $1/6 = 1/15 + 1/10$. But the right hand side is 1/8. Therefore, (1) can never be satisfied; so these "crossed ladders" represent an impossible configuration. The highest point of crossing is 6, never 8.

The second set of parameters do allow for a physically possible solution. After substituing $A = 16 + 2(2)^{1/2}$, $B = 16 - 2(2)^{1/2}$ and $H = 2$ in (1), the solution is $X = (168)^{1/2} = 12.96$.

Also solved by Allen Tracht, Avi Ornstein, Frederic Jelen, Harry Zaremba, Matthew Fountain, Peter Card, Ross Hoffman, Steve Feldman, Winslow Hartford, and the proposer, Martin Brock.

Better Late Than Never

1984 N/D 1. Neil Hochstedler notes that the third move in the third variation should be Q-a4 mate.

1985 JAN 3. Randall Whitman has responded.

F/M 3, F/M 4. George Parks has responded.

APR SD2. E.P. Schact wants to rescind Horton's Nobel Prize believing that the *chicken* came first.

Proposer's Solutions to Speed Problems

SD 1. Zero.

SD 2. Don't cash your fourth heart! Lead a club and trust that declarer now thinks that it is safe to take the club finesse into your hand.

	♠ K 9 3 ♥ 6 5 3 ♦ A J 9 2 ♣ Q 7 4	
♠ J 5 4 2 ♥ 10 9 8 ♦ 8 6 3 ♣ 8 6 5 3		♠ 10 7 6 ♥ A K Q 7 ♦ Q 7 5 ♣ J 10 9
	♠ A Q 8 ♥ J 4 2 ♦ K 10 4 3 ♣ A K 3	