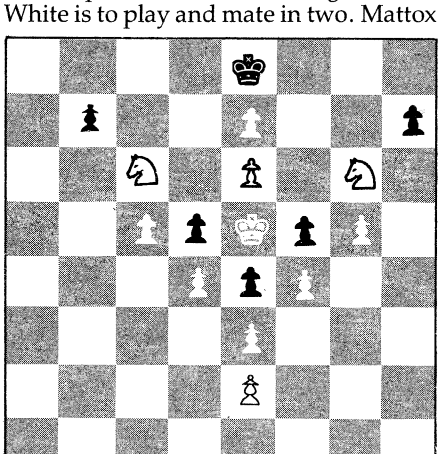
Can You Find the Bad Penny in the Bank?

Just as I am writing this the mailman has brought a letter from the New York State Crime Laboratory marked "official business." After two gulps and one reflection as to what I could have done that they found out about, I got up my courage to open the envelope and was relieved to find it was just a "Puzzle Corner" reader responding to several problems from the January issue.

As I have remarked previously, Nobuyuki Yoshigahara selected me as "World Puzzlist" No. 8, and now he has forwarded the issue of *Quark* in which this honor was officially bestowed. In addition to being flattered, I enjoyed hearing the transliteration of "Gottlieb" when one of my Japanese-speaking colleagues read the beginning of the column. Thank you again, Mr. Yoshigahara. I should also mention that long-time "Puzzle Corner" contributor, Richard Hess, was selected World Puzzlist No. 6.

Problems

APR 1. I read our first problem for this month in *net.chess*, an electronic newsgroup devoted to chess. Roughly speaking, these newsgroups consist of widely separated individuals who communicate with each other via electronic mail. I especially enjoyed the following two-part offering from Jeffrey Mattox, who noted that it is possible, albeit unlikely, for the position to occur in a game: White is to play and mate in two. Mattox notes that at first glance there appears to be two possible solutions. You are to show that only one meets the need.



APR 2. Our next problem is from Phelps Meaker, who first asks you to study:

16601.92 + 14374.08 =
11334.4 + 19641.6 =
18521.44 + 12454.56 =
4147.36 + 26828.64 =

He then notes that each pair is a different way of evaluating the same equation and asks you to write the equation in the usual form. He also offers a hint, but you may wish to try the problem without this aid. [*The hint is to note which three numbers are perfect squares.*]

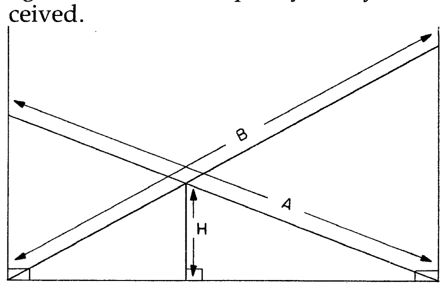
APR 3. Rich Decker wants you to find all maxima and minima of

$$\frac{\ln(1 + e^{-x}) + \frac{x}{2}}{\ln(1 + e^{x}) - \frac{x}{2}}$$

without using any calculus. This problem appeared in an Ohio State University prize exam for undergraduates.

APR 4. Allan Faller wants us to be penny wise and writes:

On each day of the year (not leap year) you are given a penny. On December 31 you are given your last penny and told that it was fresh from the U.S. Mint, but that one of the previous pennies may have been counterfeit, and therefore lighter or heavier than the standard penny. You are asked to determine the number of balancings, using a common pan balance, that would be necessary and sufficient to determine whether or not there is a counterfeit coin, and if there is, to tell whether it is heavier or lighter than the last penny that you received.



APR 5. Our final regular problem, from Martin Brock, is based on the familiar "crossed ladders" configuration at the bottom of the previous column:

Mr. Block asks you to find X for two configurations. First when A = 15, B = 10, and H = 8; and second when $A = 16 + 2\sqrt{2}$, $B = 16 - 2\sqrt{2}$, and H = 2.

Speed Department

SD 1. A bridge quickie from Doug Van Patter:

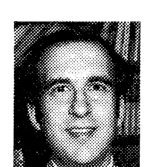
North:
♠ K 8 5 4
♥ Q 10 7 4
♦ 5 3
♣ A Q 10

South:
♠ A 9 3
♥—
♦ A Q 6
♣ K 9 8 7 6 4 3

East:	*South*:	*West*:	*North*:
1S	2C	P	3C
3H	5C	P	P
5H	6C	D	P

Instead of defending the usual five-heart bid by East, you (South) make the aggressive bid of six clubs. West opens with ♥5, which draws the ♥10, ♥K, and a trump. You lead a club to dummy's ♣A and East shows out. Your finesse of the ♦Q loses to West's ♦K, and West returns a trump to the ♣10. You lead to the ♦A (East shows out), and ruff your third diamond with dummy's last club. Can you find a way to justify your overbid? (East is an excellent player, never known to psych).

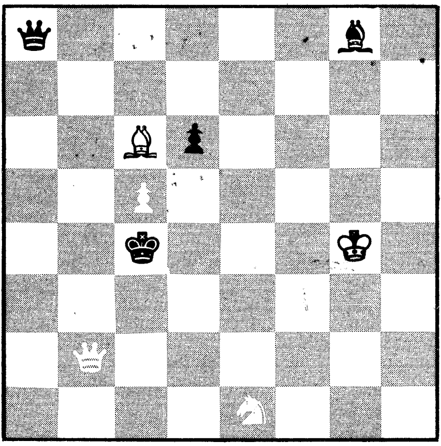
SD 2. Joseph Horton writes: Great news! I have answered an age-old question: Which came first—chicken or egg?



SEND PROBLEMS, SOLUTIONS, AND COMMENTS TO ALLAN J. GOTTLIEB, '67, ASSOCIATE RESEARCH PROFESSOR AT THE COURANT INSTITUTE OF MATHEMATICAL SCIENCES, NEW YORK UNIVERSITY, 251 MERCER ST., NEW YORK, N.Y., 10012.

Solutions

N/D 1. White, moving first, is to mate in three moves:



Howard Stern found this problem to his liking: The following three plays represent the only possibilities resulting from White's initial move:

White:	*Black*:
Q-c2	K-b4
Kn-d3	K-a3
Q-b2 mate	

White:	*Black*:
Q-c2	K-d4
Kn-f3	K-e3
Q-d2 mate	

White:	*Black*:
Q-c2	K-b4
Kn-d3	K-a5
Q-a2 mate	

Also solved by Eric Rayboy, David DeLeeuw, R. Bart, Benjamin Rouben, Matthew Fountain, Steve Feldman, Ronald Raines, Elliott Roberts, and the proposer, J. Weatherly.

N/D 2. Find a number that equals its own logarithm.

There is no positive real number x such that either $\log_{10}(x) = x$ or $\log_e(x) = x$. Several readers, including the proposer, Smith D. Turner ($\int dt$), went on to consider $\log_a$. However, I do not feel this meets the conditions of the problem (note that logarithm was used in the singular). Tim Maloney (and others) made another generalization; he writes:

First, one must recognize that the solution is a complete number, call it $Z = u + iw = re^{i\theta}$

Then

$re^{i\theta} = \ln(re^{i\theta})$

$r \cdot \cos\theta + ir \cdot \sin\theta = \ln(r) + i\theta$, or

$\ln(r)/r = \cos\theta$

$r = \theta/\sin\theta$.

We must therefore solve

$\ln(\theta/\sin\theta) = \theta \cdot \cot\theta$.

An iterative solution gives

$\theta = 1.337236\ldots$, so

$Z = (1.374557\ldots)e^{(i1.337236\ldots)}$

$= (.3181313\ldots) + i(1.337236\ldots)$.

I must admit that I first saw this problem in my junior year at M.I.T. (1970), when someone proposed the problem in a lunchtime discussion in Professor Daniel Kleppner's research group. Kleppner immediately drew a graph on the board to prove the solution could not be real, asserted that it must be complex, and left us all speechless.

Also solved by Eric Rayboy, R. Bart, Matthew Fountain, Ronald Raines, Winslow Hartford, John Spalding, John Woolston, Naomi Markovitz, Mike

Hennessey, Frank Carbin, Greg Huber, Avi Ornstein, Oren Cheyette, John Prussing. Edwin McMillan, Charles Sutton, and the proposer, Smith D. Turner (∫dt).

N/D 3. Given a floor lamp with two bulbs, in which each socket has a chain which when pulled will change the on/off state of only the bulb in that socket. When the lamp is on, it is difficult to determine whether one (and if so which one) or both bulbs are on. The problem is to find the shortest sequence of pulls that will turn the lamp completely off sometime during the sequence (e.g, if chains are labeled A and B, AABBAB fits the requirements, but a shorter sequence may be found). Can you generalize your solution to a lamp of three bulbs, four bulbs, etc.? Is your solution unique?

The following solution is from John Spalding:

The solutions can be generated recursively starting with the solution for the trivial case of one bulb. Suppose we number the bulbs 1, 2, 3 . . . instead of lettering them. Then the solution for n bulbs could be given as the following function f(n) giving a sequence of numbers:

$f(1) = 1$

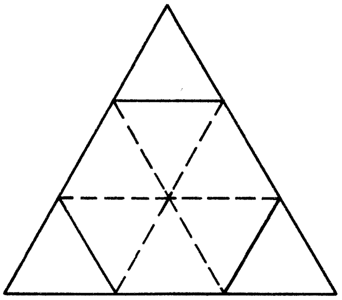
$f(n) = f(n - 1), n, f(n - 1)$

For two bulbs, the solution is thus 1,2,1 or ABA: for three bulbs, the solution generated would be ABACABA; for four, ABACABADABACABA, etc. When the chains are pulled in this sequence, the bulbs will cycle through what I think is called a Gray Code, in which all the possible n-digit binary numbers are generated by only changing one digit at a time. The number of steps required for n bulbs is thus $2^n - 1$, which is what we would expect. Is the solution unique? Well, no. Multiple working sequences may be generated by simply permuting the bulbs' labels—e.g., ABA and BAB. In addition, labels may be permuted at intermediate levels of the recursion—e.g., ABACABA and ABACBAB are working sequences for three bulbs.

Also solved by Eric Rayboy, R. Bart, Matthew Fountain, Winslow Hartford, John Woolston, Naomi Markovitz, Mike Hennessey, Frank Carbin, Oren Cheyette, Pat Kinney, Mike Strieby, Harry Zaremba, Howard Stern, Yokichi Tamaka, and Joe Feil.

N/D 4. A regular hexagon can be inscribed in an equilateral triangle so that its alternate sides coincide with the sides of the triangle. What is the ratio between the areas of the hexagon and the triangle?

Walter Cluett has little trouble with this one: Divide the hexagon into six equilateral triangles and the answer is 6 to 9 or 1 to 1 1/2.



Also solved by Eric Rayboy, R. Bart, Steve Feldman, Howard Stern, Ronald Raines, Winslow Hartford, John Spalding, John Woolston, Naomi Markovitz, Mike Hennessey, Frank Carbin, Greg Huber, Avi Ornstein, Pat Kinney, Harry Zaremba, Frederic Jelen, Ruben Cohen, Stefan Habsburg, Peter Silverberg, George Byrd, James Reswick, Dick Robnett, Raymond Gaillard, Michael Lamoureux, Mary Lindenberg, Smith D. Turner (∫dt), and the proposer, Phelps Meaker.

N/D 5. In the country of Moolah, the national bank issues new dola bills to replace each bill that wears out or is lost or destroyed, so there is always a constant number of dolas in circulation. On January 1, the bank issued a new bill with the picture of Prince Centime replacing that of the late Queen Peseta. After one year, they found that 10/27 of the bills in circulation were the new variety. After two years, 2/3 of the bills; and after four years all the bills were the new type. What is the life expectancy of a dola bill?

Harry Zaremba sent us a lucid solution: Assume N to be constant number of bills in circulation. In terms of N and their common denominator, the fractional amounts of new bills in circulation during the successive four years were 10N/27, 18N/27, 24N/27, and 27N/27. The number of old-variety bills that were replaced in each of the four years was 10N/27, 8N/27, 6N/27, and 3N/27, and their respective *average* years in circulation were 1/2, 3/2, 5/2, and 7/2 years per dola. Let A be the weighted average life expectancy of the old bills. Then,

$A \cdot N = 10N/27 \cdot 1/2 + 8N/27 \cdot 3/2 + 6N/27 \cdot 5/2 + 3N/27 \cdot 7/2$, or

$A = 1.574$ years per dolla.

Also solved by Eric Rayboy, R. Bart, Matthew Fountain, Winslow Hartford, Frederic Jelen, and the proposer, Frank Rubin.

Better Late Than Never

Y 1984. Claes Wihlborg and Mats Ohlin have responded.

M/J 1. R. Bart found two alternative solutions.

JUL 3. Smith D. Turner (∫dt) found a simpler solution strategy.

A/S 1. Samuel Levitin and Benjamin Rouben have responded.

A/S 2. Samuel Levitin has responded.

OCT 1. R. Bart, W. Smith, and Richard Hess have responded.

OCT 2. R. Bart has responded.

OCT 3. R. Bart, Samuel Levitin, Richard Hess, and R. Morgan have responded.

OCT 4. R. Bart, Richard Hess, Samuel Levitin, Phelps Meaker, and Altamash Kamal have responded.

OCT 5. R. Bart has responded.

N/D SD 1. Michael Strieby and Dick Robnett found alternate solutions.

N/D SD 2. The correct answer is 8, as noted by Eric Rayboy, David DeLeeuw, R. Bart, Winslow Hartford, John Spalding, John Woolston, Naomi Markovitz, Pat Kinney, Mike Hennessey, Mike Strieby, Frederic Jelen, Ruben Cohen, Stefan Habsburg, Peter Silverberg, George Byrd, James Reswick, Dick Robnett, Mike Ober, Walter Cluett, Roger Allen, Harry "Hap" Hazard, Peter DeFoe, Victor Christiansen, W. Katz, and Phelps Meaker.

Proposers' Solutions to Speed Problems

SD 1. East has 12 cards in the majors, most likely 6-6-1-0. She needs the ♥A for her opening bid. In order to get back to your hand to draw the last trump, you must *not* play a spade! (1) It may be trumped (true!), and (2) You need the ♠A to execute a so-called "simple" squeeze play. (What squeeze is simple at the table?). Ruff a heart and play all your trumps but one, reaching this four-card position:

North:
♠ K 8 5
♥ Q

East:
♠ Q J 10
♥ A

South:
♠ A 9 3
♣ 3

After you play your last club, discarding a spade from dummy, East is rendered helpless.

SD 2. The egg. The first egg from which a chicken hatched had to have been laid by the immediate evolutionary ancestor of the chicken. On the other hand, the first chicken laid something like what we have for breakfast—what else, a dinosaur egg? Please forward all correspondence to the Nobel Institute in Stockholm, Sweden. I'll be waiting for it there.