Can You Find the Bad Penny in the Bank?

Just as I am writing this the mailman has brought a letter from the New York State Crime Laboratory marked "official business." After two gulps and one reflection as to what I could have done that they found out about, I got up my courage to open the envelope and was relieved to find it was just a "Puzzle Corner" reader responding to several problems from the January issue.

As I have remarked previously, Nobuyuki Yoshigahara selected me as "World Puzzlist" No. 8, and now he has forwarded the issue of Quark in which this honor was officially bestowed. In addition to being flattered, I enjoyed hearing the transliteration of "Gottlieb" when one of my Japanese-speaking colleagues read the beginning of the column. Thank you again, Mr. Yoshigahara. I should also mention that long-time "Puzzle Corner" contributor, Richard Hess, was selected World Puzzlist No. 6.

Problems

APR 1. I read our first problem for this month in net.chess, an electronic newsgroup devoted to chess. Roughly speaking, these newsgroups consist of widely separated individuals who communicate with each other via electronic mail. I especially enjoyed the following two-part offering from Jeffrey Mattox, who noted that it is possible, albeit unlikely, for the position to occur in a game:

White is to play and mate in two. Mattox notes that at first glance there appears to be two possible solutions. You are to show that only one meets the need.

APR 2. Our next problem is from Phelps Meaker, who first asks you to study:

\[ 16601.92 + 14374.08 = 11334.4 + 19641.6 = 18521.44 + 12454.56 = 4147.36 + 26828.64 = \]

He then notes that each pair is a different way of evaluating the same equation and asks you to write the equation in the usual form. He also offers a hint, but you may wish to try the problem without this aid. [The hint is to note which three numbers are perfect squares.]

APR 3. Rich Decker wants you to find all maxima and minima of

\[ \ln(1 + e^{-x}) + \frac{x}{2} = \ln(1 + e^{-x}) - \frac{x}{2} \]

without using any calculus. This problem appeared in an Ohio State University prize exam for undergraduates.

APR 4. Allan Faller wants us to be penny wise and writes:

On each day of the year (not leap year) you are given a penny. On December 31 you are given your last penny and told that it was fresh from the U.S. Mint, but that one of the previous pennies may have been counterfeit, and therefore lighter or heavier than the standard penny. You are asked to determine the number of balancings, using a common pan balance, that would be necessary and sufficient to determine whether or not there is a counterfeit coin, and if there is, to tell whether it is heavier or lighter than the last penny that you received.

APR 5. Our final regular problem, from Martin Brock, is based on the familiar "crossed ladders" configuration at the bottom of the previous column: Mr. Block asks you to find X for two configurations. First when A = 15, B = 10, and H = 8; and second when A = 16 + 2\sqrt{2}, B = 16 - 2\sqrt{2}, and H = 2.

Speed Department

SD 1. A bridge quickie from Doug Van Patter:

North:
\[ \spadesuit \, K \, 8 \, 5 \, 4 \]
\[ \heartsuit \, Q \, 10 \, 7 \, 4 \]
\[ \clubsuit \, 5 \, 3 \]
\[ \spadesuit \, A \, Q \, 10 \]

South:
\[ \spadesuit \, A \, 9 \, 3 \]
\[ \heartsuit \, A \, Q \, 6 \]
\[ \spadesuit \, K \, 9 \, 8 \, 7 \, 6 \, 4 \, 3 \]

East: 1S 2C 3H 5H
West: P P P
North: 3C D P

Instead of defending the usual five-heart bid by East, you (South) make the aggressive bid of six clubs. West opens with ♥5, which draws the ♥10, ♥K, and a trump. You lead a club to dummy's ♥A and East shows out. Your finesse of the ♥Q loses to West's ♥K, and West returns a trump to the ♥10. You lead to the ♥A (East shows out), and ruff your third diamond with dummy's last club. Can you find a way to justify your overbid? (East is an excellent player, never known to psych).

SD 2. Joseph Horton writes: Great news! I have answered an age-old question: Which came first—chicken or egg?
Howard Stern found this problem to his liking: The following three plays represent the only possibilities resulting from White’s initial move:

**White:** Black:
Q-c2 K-b4
Kn-d3 K-a3
Q-b2 mate

**White:** Black:
Q-c2 K-d4
Kn-f3 K-x3
Q-d2 mate

**White:** Black:
Q-c2 K-b4
Kn-d3 K-a5
Q-a2 mate

Also solved by Eric Rayboy, David Delleweu, R. Bart, Benjamin Rouben, Matthew Foutain, Steve Feldman, Ronald Raines, Elliott Roberts, and the proposer, J. Weatherly.

**N/D 2.** Find a number that equals its own logarithm. There is no positive real number x such that either \( \log_{10}(x) = x \) or \( \log_{10}(x) = -x \). Several readers, including the proposer, Smith D. Turner (jdj), went on to consider \( \log_{10} \). However, I do not feel this meets the conditions of the problem (note that logarithm was used in the singular). Tim Macloney (and others) made another generalization; he writes:

First, one must recognize that the solution is a complete number, call it \( Z = u + iw = re^{\theta} \).

Then

\[
\begin{align*}
\ln(r) & = \ln(re^{\theta}) \\
r\cos \theta + ir\sin \theta & = ln(r) + i0, \text{ or} \\
\ln(r)/r & = \cos \theta \\
r & = \theta / \sin \theta.
\end{align*}
\]

We must therefore solve

\[
\ln(\sin \theta) = \theta / \cos \theta.
\]

An iterative solution gives

\[
\theta = 1.337236 \ldots, \text{ for } e^{0.337236} = (3.181313 \ldots) + i(1.337236 \ldots).
\]

I must admit that I first saw this problem in my junior year at M.I.T. (1970), when someone proposed the problem in a lunchtime discussion in Professor Daniel Kleppner’s research group. Kleppner immediately drew a graph on the board to show the solution could not be real, asserted that it must be complex, and left us all speechless.

Also solved by Eric Rayboy, R. Bart, Matthew Foutain, Ronald Raines, Winslow Hartford, John Spalding, John Woolston, Naomi Markovitz, Mike
On January 1, the bank issued a new bill with the picture of Prince Centime replacing that of the late Queen Peseta. After one year, they found that 10/27 of the bills in circulation were the new variety. After two years, 2/3 of the bills; and after four years all the bills were the new type. What is the life expectancy of a dolat bill?

Harry Zarembe sent us a lucid solution:

Assume N to be constant number of bills in circulation. In terms of N and their common denominator, the fractional amounts of new bills in circulation during the successive four years were 10N/27, N/27, 28N/27, and 27N/27. The number of old-variety bills that were replaced in each of the four years was 10N/27, 8N/27, 6N/27, and 3N/27, and their respective average years in circulation were 1/2, 3/2, 5/2, and 7/2 years per dolat. Let A be the weighted average life expectancy of the old bills. Then:

A = 10N/27/1/2 + 8N/27/3/2 + 6N/27/5/2 + 3N/27/7/2,

A = 1.574 years per dolat.

Also solved by Eric Rayboy, R. Bart, Matthew Fountain, Winslow Hartford, Frederic Jelen, and the proposer, Frank Rubin.

Better Late Than Never

Y 1984. Claes Wihlborg and Mats Ohlin have responded.

M 1. R. Bart found two alternative solutions.

JUL 3. Smith D. Turner (Jdt) found a simpler solution strategy.

A/S 1. Samuel Levitin and Benjamin Rouben have responded.

A/S 2. Samuel Levitin has responded.

OCT 1. R. Bart, W. Smith, and Richard Hess have responded.

OCT 2. R. Bart has responded.

OCT 3. R. Bart, Samuel Levitin, Richard Hess, and R. Morgan have responded.

OCT 4. R. Bart, Richard Hess, Samuel Levitin, Phelps Meaker, and Altamash Kamal have responded.

OCT 5. R. Bart has responded.

N/D SD 1. Michael Strieby and Dick Robnett found alternate solutions.

N/D SD 2. The contest answer is 8, as noted by Eric Rayboy, David DeLeuww, R. Bart, Winslow Hartford, John Spalding, John Woolston, Naomi Markovitz, Pat Kinney, Mike Strieby, Frederic Jelen, Howard Stern, Yokichi Tamaka, and Joe Feil.


Proposers’ Solutions to Speed Problems

SD 1. East has 12 cards in the majors, most likely 6-6-1-0. She needs the ♥A for her opening bid. In order to get back to your hand to draw the last trump, you must not play a spade! (1) It may be trumped (true), and (2) You need the ♥A to execute the so-called “simple” squeeze play. (What squeeze is simple at the table?). Ruff a heart and play all your trumps but one, then remove this facing card position:

North:
♥ K 8 3
♦ ♥ ♥
East:
Q Q J 10
South:
♥ ♥ A 9 3 ♥
♥ 3
After you play your last club, discarding a spade from dummy, East is rendered helpless.

SD 2. The egg. The first egg from which a chicken hatched had to have been laid by the immediate evolutionary ancestor of the chicken. On the other hand, the first chicken laid something like what we have for breakfast—what else, a diinosaur egg? Please forward all correspondence to the Nobel Institute in Stockholm, Sweden. I’ll be waiting for it there.