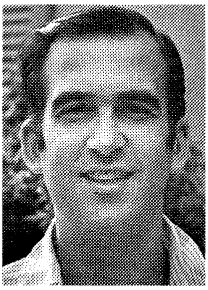
# Las Vegas Month

## Puzzle Corner/Allan Gottlieb



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I am honored to learn that the well-known Japanese puzzler Nob. Yoshigahara has selected me as World's Puzzler No. 6 in his puzzle column in *Quark*, a leading Japanese science magazine. Since the problems and solutions that appear in "Puzzle Corner" are contributions from you the readers, I know who deserves the honor.

### Problems

**JUL 1.** We begin with a bridge problem from Doug Van Patten:

North:
♠ J 10 9 8 6 3
♥ A J 5
♦ K
♣ A 9 3

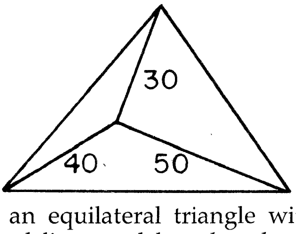
West:
♠ Q 4
♥ K 7 3
♦ 10 7 6 4
♣ Q J 10 7

East:
♠ 7 5 2
♥ Q 9 4 2
♦ Q 5 3 2
♣ 5 4

South:
♠ A K
♥ 10 8 6
♦ A J 9 8
♣ K 8 6 2

You are South and open one no-trump. Your partner falls in love with her hand, transfers in spades, and puts you in a highly optimistic six-spade contract. Can you justify her faith in your declarer play? Opening lead: ♣Q.

**JUL 2.** A geometry problem that Van Ling attributes to Walfred Lester:




Given an equilateral triangle with the indicated lines and lengths, determine the length of a side.

**JUL 3.** Dave Evans wants you to help him navigate in 4-space:
If a chess cube is placed in the corner square of a 3 × 3 chessboard, there are six distinct shortest paths (i.e., combinations of vertical and horizontal moves which do not backtrack) to the diagonally opposite corner. How many distinct shortest paths exist from a corner cell of a 3 × 3 × 3 cube to the cell opposite along a space diagonal, assuming movement parallel to the three axes? How about in a 3 × 3 × 3 × 3 hybercube?

**JUL 4.** Don't play craps with Albert Mullin! As a student of "loading up" the dice, he writes:
Show that it is physically impossible to "load" a pair of ordinary dice such that each of the eleven possible rolls is equally likely to occur. (Generalize to the other four regular polyhedra—e.g., the possible rolls of a pair of regular dodecahedra.)

**JUL 5.** This must be Las Vegas month, as Richard Jones wants to study betting on roulette:
Is it wise to bet conservatively (making many small bets) instead of making a few large bets? Specifically, a man gambles $100.00 a night by betting on black at roulette. Usually he places a single $100.00 wager and quits for the night. Since the wheel has only a single zero and pays even money, he has an 18/37 chance of winning $100.00. Suppose, instead, he starts with $100.00 but makes $1.00 bets until he runs out of money or is up $100.00. What is the probability that he will be a winner after a night of gambling?

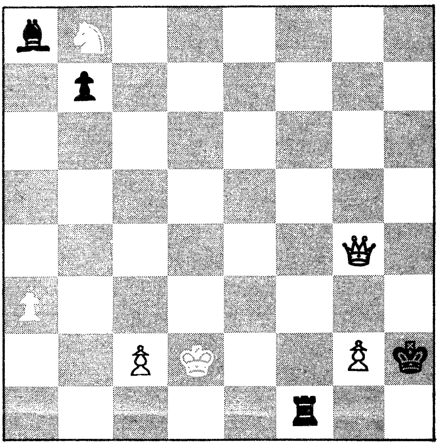
### Speed Department

**SD 1.** Inspired by Emmett Duffy, Phelps Meaker asks for the last four digits of $7^{7777}$, $7^{5555}$, and $7^{4444}$, and the last three digits of $13^{1313}$.

**SD2.** David Evans wonders if there are two people in San Francisco that have the same number of hairs on their heads. (As an aside, we just returned from a trip to San Francisco, which David Gottlieb pronounced KooKoo. He charmed the entire staff of a Chinese restaurant when he walked up to the owner, looked at me, and said, "This man KooKoo.")

## Solutions

**F/M 1.** White to mate in two:



John Bobbitt first shows that the world is upside down and then finds the appropriate underpromotion:
Why the peculiar P-B-N combination in the upper left corner? The answer is that it gives a clue as to what direction White and Black are moving. The only way the Black bishop could be trapped behind the pawn is for Black to be moving up the board. Given that information, the winning move for White is P×R with the pawn being replaced by a bishop. (A queen or rook leads to a stalemate, a knight leads to a chase around the board.) Black is forced to reply K-R1 and Whites's Q-N7 mate finishes the game.

Also solved by David Krohn, Richard Hess, Matthew Fountain, John Glenn, Ronald Raines, Craig Presson, Lester Steffens, and the proposer, Daniel Seidman.

**F/M 2.** Three missionaries and three cannibals are on the left bank of a river and have to cross to the right bank using a boat that can hold two people. At least one person must be in the boat in order to move it from one side of the river to the other. On each bank, the missionaries must never be in a situation where they are outnumbered by the cannibals. How do all six people get across?

Naomi Markovitz has sent us the following solution:

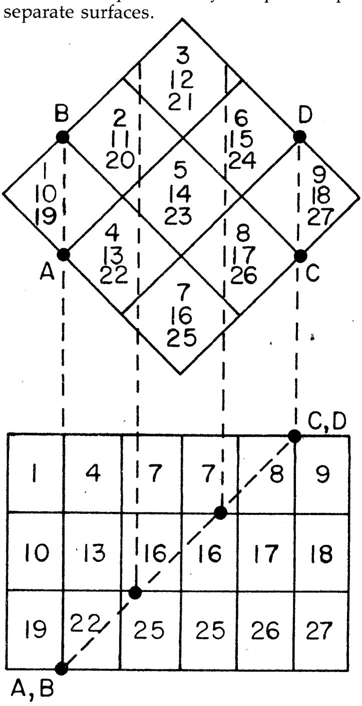
| *Left bank* | *Boat* | *Right bank* |
|---|---|---|
| MMMCCC | | |
| MMMC | CC → | |
| MMMC | | CC |
| MMMC | ← C | C |
| MMMCC | | C |
| MMM | CC → | C |
| MMM | | CCC |
| MMM | ← C | CC |
| MMMC | | CC |
| MC | MM → | CC |
| MC | | MMCC |
| MC | ← MC | MC |
| MMCC | | MC |
| CC | MM → | MC |
| CC | | MMMC |
| CC | ← C | MMM |
| CCC | | MMM |
| C | CC → | MMM |
| C | | MMMCC |
| C | ← C | MMMC |
| CC | | MMMC |
| | CC → | MMMC |
| | | MMMCCC |

Also solved by Ronald Raines, Matthew Fountain, Richard Hess, David Krohn, Angel Silva, Harry Zaremba, John Bobbitt, Winslow Hartford, John Woolston, Avi Ornstein, Alan Robock, Mary Lindenberg, and Mitchell Gaynor.

**F/M 3.** What is the largest number of triangles that can be produced by passing a plane through a cube that is divided into 27 subcubes?

Although not stated, the intent, as noted by Matthew Fountain, was that the original cube be divided into 27 *equal* subcubes. Harry Zaremba and Tom Harriman each found 100 triangles. Mr. Zaremba's pictorial-tabular solution is given below; if the printer could handle it (which he can't, because it involves three colors as well as several dashed and dotted lines in black) we'd also publish Mr. Harriman's picture.

As shown in the figures, the subcubes are numbered from 1 to 9 at the top, 10 to 18 at the center, and 19 to 27 at the bottom. To obtain the number of triangles, it is assumed all abutting faces of the subcubes and sloped cuts by the plane represent two separate surfaces.




Let the plane pass through points A, B, C, and D. Nineteen subcubes will be cut by the plane. The number of triangles produced on the top, bottom, side, and sloped faces of the cubes is tabulated below. A total of *100 triangles* is formed.

| *Cube number* | *Top* | *Bottom* | *Sides* | *Slope* | |
|---|---|---|---|---|---|
| 3 | | 1 | 2 | 2 | |
| 5 | | 1 | 2 | 2 | |
| 6 | | 1 | 4 | | |
| 7 | | 1 | 2 | 2 | |
| 8 | | 1 | 4 | | |
| 9 | 2 | | 2 | 2 | |
| 11 | | 1 | 2 | 2 | |
| 12 | 1 | 1 | 4 | | |
| 13 | | 1 | 2 | 2 | |
| 14 | 1 | 1 | 4 | | |
| **Total** | **4** | **9** | **28** | **12** | **53** |
| 15 | 1 | | 2 | 2 | |
| 16 | 1 | 1 | 4 | | |
| 17 | 1 | | 2 | 2 | |
| 19 | | 2 | 2 | 2 | |
| 20 | 1 | | 4 | | |
| 21 | 1 | | 2 | 2 | |
| 22 | 1 | | 4 | | |
| 23 | 1 | | 2 | 2 | |
| 25 | 1 | | 2 | 2 | |
| **Total** | **8** | **3** | **24** | **12** | **47** |
| | | | | | **100** |

*Average number of purchases required to decrease N over range shown, as determined by 1,000 trials*

| | | | | | | |
|---|---|---|---|---|---|---|
| N starts at | 80 | 79 | 69 | 68 | 57 | 56 |
| N ends at or below | 70 | 70 | 60 | 60 | 50 | 50 |
| Number of purchases | 7.196 | 6.540 | 7.670 | 6.782 | 7.227 | 6.144 |
| Cost of purchases | $1.439 | $1.308 | $1.534 | $1.356 | $1.445 | $1.229 |

Also solved by Richard Hess and the proposer, Lester Steffens.

**F/M 4.** Find a four-digit number whose square is an eight-digit number with the middle four digits all 0; i.e., $(abcd)^2 = ef000gh$.

Angel Silva found $6245^2 = 39000025$.

Also solved by R. Duff Ginter, Richard Hess, Alan Robock, Avi Ornstein, John Woolston, Winslow Hartford, John Bobbitt, Harry Zaremba, David Krohn, Matthew Fountain, John Glenn, Jeff Schoenwald, G. Zartarian, Richard Marks, John Jordan, Frank Carbin, Jesse Spencer, Joseph Horton, Frank Norton, George Piotrowski, Donald Rosen, Steve Feldman, and the proposer, Smith Turner.

**F/M 5.** There are 260 baseball stickers in a set and they are purchased in groups of five for 20 cents. You can also send away for groups of 10 (specific numbers) for $1 plus 20 cents postage (plus it costs 20 cents to mail). When is the optimal time to send away? What is the proper barter philosophy to have with friends?

The following solution is from Mattew Fountain: The collector who does not trade and simply wishes to advance toward completion of one full set should send away for one when he has acquired 186 different stickers. If he intends to complete the set, he should send away when his total reaches 180. If his total skips 180, he should wait till 190. If his total also skips both 190 and 191, he should wait until 200. If his total also skips 200, 201, 202, and 203, he should wait till reaching 210 before sending away.

The optimum barter philosophy with friends is that of cooperation and patience. Friends should exchange stickers, one for one, whenever it helps one and does not hurt the other. The one with the fewest stickers should make the next purchase. As long as they as a group lack 75 different stickers they should not send away. Even then they can postpone sending away by inducting new members into their group. When they do send away, orders are placed to cover the needs of the group, with the cost of the unused portion of the last group of ten stickers divided evenly among the group of members.

The above statements are based on the following reasoning. The phrase "optimum time to send away" is taken to apply to the purchase of one group of 10 stickers, and "plus it costs 20 cents to mail" refers to the cost of requesting one group. This makes the unit cost of the specific stickers 14 cents. When N stickers are missing, the number of missing stickers expected in a 20-cent purchase of five different, unspecified stickers is (5/260)N, making their expected unit cost 20/(N/52) cents. When N = 74.2 their expected unit cost is 14 cents, the same as those bought by mail. When N = 80 their cost is 13 cents. When N = 70 their cost is 14.9 cents. As buying 74 specific stickers by mail entails buying 80 stickers, one can save about 78 cents on the average by starting mail purchases when N is either 80 or 70, instead of 74.

While of trivial importance, it is interesting to determine which value of N is best for starting buying by mail. I programmed my computer to run a Monte Carlo test of 1,000 trials of how many 20-cent purchases would be required to decrease N from 80 to 70. As the computer tallied each simulated purchase, it decreased the trial N by an integer ranging from 0 to 5, the integer being selected by a random process weighted to assure that each integer had the same probability of being selected as that the integer number of wanted stickers would appear in a 20-cent purchase, the probabilities changing as N decreases. Each trial ended when N decreased to no more than 70. I then repeated the test starting with other values of N, but always ending each trial when N decreased to or below an even multiple of ten. The results are shown above (*box*). When the lone collector sends away at N = 80, his expected savings are not the difference between $1.439 and $1.40, for in doing so he forfeits his chance of skipping N = 70 and N = 69. Should he reach N = 68, he would continue on to N = 60 with the expected saving of 4.4 cents. And should he skip to N = 67 or N = 66, his expected savings would be about 22 cents or 40 cents, respectively. To determine the frequency that he would skip certain values of N, I had my computer rerun my program, starting with N = 80, but increasing the number of trials to 10,000. The frequencies that N reached 70, 69, 68, 67, and 66 were .5832, .3104, .0927, .0122, and .0015, respectively. The chance of skipping past N = 70 is therefore worth about $(4.4)(.0927) + (22)(.0122) + (40)(.0015) = 0.736$ cents. The expected savings from sending away at N = 80 is $4.4 - .74 = 3.66$ cents.

A cheap "fair" way of acquiring a collection is to offer collectors who have just made their first 20-cent purchases, "If you let me buy up to four of your stickers, I will pay you 4 cents each. To make it more than fair, I'll pay you 4 cents also for any sticker you get in your next five-sticker purchase that gives you a duplicate. You'll end up with at least five, and probably more, different stickers at a cost of 4 cents each." This scheme is not entirely fair, as those who practice it do not acquire the normal number of duplicates that are useful to others. This is the reason why when friends cooperate they should trade rather than sell, and why the one with the least stickers should do the buying. It evens up the distribution of excess duplicate stickers.

Also solved by David Krohn, Richard Hess, Howard Wagner, and Frank Carbin.



**Better Late than Never**

**1982 OCT 3.** Howard Wagner has responded.

**N/D 2, N/D 3, N/D 4.** Naomi Markovitz has responded.

**1984 JAN 1.** Tom Harriman and G. Zartarian have responded.

**JAN 2, JAN 3.** Tom Harriman has responded.

**F/M SD2.** Jeff Schoenwald, Angel Silva, and George Downie note that the side of the tank is 5 decimeters in length.

**Proposers' Solutions to Speed Problems**

**SD1.** 9207, 1943, 6401, and 253.

**SD2.** Yes, by the pigeon-hole principle, since the number of hairs is under 100,000.