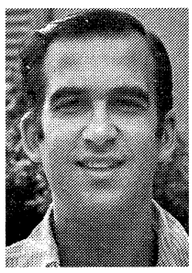
The Economic Theory of Baseball Stickers

Puzzle Corner/Allan Gottlieb



Allan J. Gottlieb, '67, is associate research professor at the Courant Institute of Mathematical Sciences of New York University; he studied mathematics at M.I.T. and Brandeis. Send problems, solutions, and comments to him at the Courant Institute, New York University, 251 Mercer St., New York, N.Y. 10012.

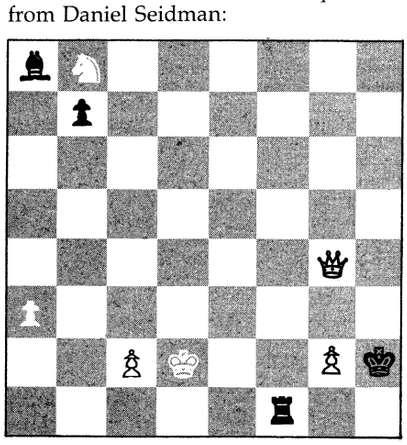
As you might know, essentially all problems that appear in "Puzzle Corner" are submitted by you, the readers of the column. So it's appropriate to report to you on the backlog of submitted problems awaiting publication. The largest backlog is for regular problems, where we have a one-and-a-half-year supply. For "speed" and bridge problems, the expected delay until publication is one year, and chess problems remain in short supply.

Let me repeat a comment about "speed" problems that I made in the October issue. These problems are often whimsical and should *not* be taken too seriously. If the proposer submits a solution with a problem, that solution appears at the end of the same column in which the problem is published. If the proposer does not submit a solution, normally one does not appear; only rarely are comments on speed problems published.

Finally, let me apologize to Steven Feldman for incorrectly listing his first name as Sidney.

Problems

F/M 1. We start with a chess problem from Daniel Seidman:



The problem: White to mate in two.

F/M 2. Here is an old chestnut from Matthew Chen: Three missionaries and three cannibals are on the left bank of a river and have to cross to the right bank using a boat that can hold two people. At least one person must be in the boat in order to move it from one side of the river to the other. On each bank, the missionaries must never be in a situation where they are outnumbered by the cannibals. How do all six people get across?

F/M 3. Lester Steffens wants to know the largest number of triangles that can be produced by passing a plane through a cube that is divided into 27 subcubes.

F/M 4. Robert Turner (son of $\int dt$) wants you to find a four-digit number whose square is an eight-digit number with middle four digits all 0; i.e. $(abcd)^2 = ef0000gh$

F/M 5. John Rudy wishes to better understand the economic theory of baseball cards (now converted to baseball stickers) and writes:
My nine-year-old son Brett collects baseball stickers. There are 260 in the set and they are purchased in groups of five for 20 cents. Topps also permits you to send away for groups of 10 (specific numbers) for $1 plus 20 cents postage (plus it costs 20 cents to mail). When is the optimal time to send away? What is the proper barter philosophy to have with friends?

Speed Department

SD 1. A bridge quickie from Doug Van Patter:

North:	*South:*
♠ A 10 9 7 6	♠ 5 4 3 2
♥ —	♥ A 9 8 7 6 5 2
♦ A Q 6 5 3	♦ 7
♣ 8 7 4	♣ 10 5

The bidding:

East	*South*	*West*	*North*
1 club	Pass	1 heart	2 hearts
3 clubs	3 spades	4 clubs	4 spades

(North's 2x bid shows five spades, five cards in a minor, and 10-15 points). By overly aggressive bidding, you (South)

become declarer in this four-spade contract. Since you can see that four clubs cannot make, you must try to make ten tricks. The defenders cash two club tricks, and you trump the third club. Can you find a reasonable line of play?

SD 2. Our final problem is from Phelps Meaker: An open-top cubical tank has an internal area in square decimeters equal to the weight of water it can hold in kilograms. How big is the tank?

Solutions

OCT 1. South is on lead and is to take all seven tricks against the best defense:

	♠ K ♥ 2 ♦ A 4 3 2 ♣ K	
♠ J 6 5 ♥ 3 ♦ 10 ♣ 8 3		♠ 9 8 2 ♥ — ♦ K Q ♣ Q J
	♠ A 10 7 ♥ K ♦ 5 ♣ A 10	

Dan Schwarzkopf and Amy Lowenstein first solved the unintended sub-problem (caused by the editor) of determining the trump suit and then proceeded to the main problem: If hearts are not trump, there is no solution; if spades are trump, West has a natural trump trick; if diamonds are trump, East has a natural trump trick; if the contract is no-trump, West can guard spades and East diamonds, eliminating the squeeze; and if clubs are trump, the XK cannot be used for ruffing purposes and the outcome is the same as in the no-trump situation.

With hearts as trump, South leads the ♠A at Trick 1, then continues with the ♠10. West has two alternatives:

1. If West fails to play the ♠J (or played it at Trick 1), Dummy discards a low diamond. South then trumps the remaining spade and overtakes, at Trick 4, the ♣K with the ♣A. South now plays the ♥K at Trick 5, squeezing East in the minor suits: if East releases a diamond, South leads a diamond to the ♦A and wins the last trick with the last diamond; if he releases a club, the last two tricks are won with the ♣10 and the ♦A.
2. If West plays the ♠J at Trick 2, Dummy ruffs, leads the ♣K, overtakes with the ♣A, and plays the ♥K at Trick 4, squeezing East in three suits: if East pitches a diamond, the ♦A and two low diamonds win the last three tricks; if East pitches a black-suit winner, South now has a winner in that suit and plays that card at Trick 5 (discarding a diamond from North). Again, if East discards the other black-suit winner, South plays the remaining black-suit card and wins the last trick with the ♦A, while a diamond discard results in two diamond winners in the North hand.

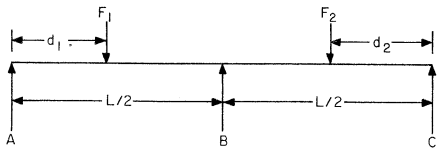
Also solved by Joel Feil, Robert Way, Matthew Fountain, Jerry Grossman, Winslow Hartford, D. Tumpler, Kenneth Bernstein, Richard Lesser, Lester Steffens, Richard Hess, and Doug Van Patter.

OCT 2. But of course, Fannie Dooley loves to baffle people. And Gefozzle them, too. You see, Fannie loves vanilla, but not chocolate. She loves Allan J. Gottlieb but not Robert Pease. She likes doors but not windows. She likes Mississippi, but not Alabama, California, or New York. . . . Also, she loves Arrowroot cookies . . . but doesn't care for tapioca. I told a couple of my friends, Lenna Mooree and William Llewellyn, and *they* figured it out. Fannie also loves the Massachusetts Institute of Technology and Harvard College, but not Princeton University . . . she likes cherry-wood furniture but not mahogany. Now do you see why? It's a fun game for people of 5th-grade mentality. . . . Of course, you have figured this out, but if you can't figure out why Fannie Dooley prefers all this hoop-la, to plain ordinary BLAH, ask a VOODOO doctor.

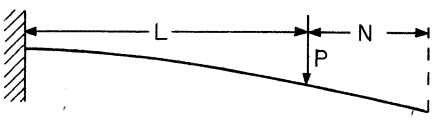
The key is that Fannie Dooley likes doubled letters. Warren Koontz believes that Fannie's name should be Doolee, noting that Dooley was the man who loved Laurie Foster but hated being hanged. Mr. Koontz also sent us a jingle from the PBS show "ZOOM" shown across the bottom of this page.

Also solved by Kenneth Bernstein, Karl Brendel, David Landress, Amy Lowenstein, Ronald Raines, Dan Sheingold, Ken Haruta, Harry Zaremba, Edgar Rose, Michael Yung, Dan Cooper, Steve Feldman, Matthew Fountain, Jerry Grossman, Emmet Duffy, Winslow Hartford, Richard Hess, John Woolston and D. Tumpler.

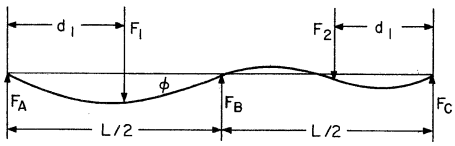
OCT 3. A massless beam of length L is supported by three stanchions A, B, and C at the ends and midpoint. The beam is loaded with point loads F_1 and F_2 at distances d_1 and d_2 from the ends. What is the downward force on each stanchion?



The following solution is from Matthew Fountain: This problem is best solved by the Myosotic method, named after the renowned Greek scholar Myosotic Palustris. We begin by quoting his formulas for the angular and linear deflections of a cantilever beam under a single concentrated load.



Angle: $PL^2/2EI$
Deflection: $PL^3/3EI$
These formulas apply at P. When the load P is a distance N from the free end, the angle of deflection is constant from P to the free end, making the deflection there
$PL^3/3EI - NPL^2/2EI$.



The beam in the problem is considered to be a pair of cantilevers fixed at B at angles ϕ and $-\phi$ with the line through A, B, and C. The forces F_1 and F_A cause a deflection of $(L/2)\phi$ at A to the left cantilever. The deflection at A due to F_A is
$-[F_A(L/2)^3]/3EI$
and the deflection due to F_1 is
$[F_1d_1(L/2)^2]/2EI + [F_1(L/2)^3]/3EI$.
The deflection at C of the right cantilever caused by F_C and F_2 is similar except that it is in the opposite direction. We can write
$-[F_A(L/2)^3]/3EI + (F_1d_1[(L/2) - d_1]^2)/2EI + (F_1[(L/2) - d_1]^3)/3EI = [F_C(L/2)^3]/3EI - (F_2d_2[(L/2) - d_2]^2)/2EI - (F_2[(L/2) - d_2]^3)/3EI$.

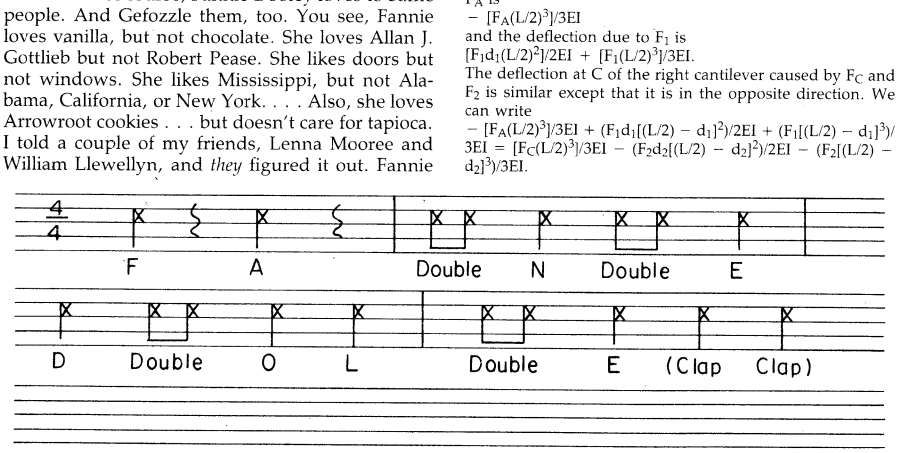
Taking moments about B, we can also write
$-F_A(L/2) - F_1[(L/2) - d_1] + F_C(L/2) - F_2[(L/2) - d_2] = 0$.
These two equations can be solved for F_A and F_C. F_B is obtained from the equation $F_A + F_B + F_C = F_1 + F_2$. With $R = (d_1/L)$ and $S = (d_2/L)$, the forces are:
$F_A = F_1[1 - (5/2)R + 2R^3] + F_2(2S^3 - \frac{1}{2}S)$
$F_C = F_2[1 - (5/2)S + 2S^3] + F_1(2R^3 - \frac{1}{2}R)$
$F_B = F_1(3BR - 4R^3) + F_2(3S - 4S^3)$
When $F_1 = F_2$ and $d_1 = d_2 = L/4$, $F_A = (5/16)F_1$ and $F_B = (11/8)F_1$. A beam hinged at B and similarly loaded would have $F_A = \frac{1}{2}F_1$ and $F_B = F_1$.

Also solved by Karl Brendel, David Landress, Ken Haruta, Harry Zaremba, Edgar Rose, John Prussing, Richard Hess, and the proposer, Benjamin Madero.

OCT 4. Show that for all $N \geq 3$
$$\sum_{k=0}^{N-1} \cos(4\pi k/N) = 0.$$

Donald Trumpler found this problem simple—once he made it complex: The sum is the real part of the complex sum
$$\sum_{k=0}^{N-1} e^{i4\pi k/N} = \sum_{k=0}^{N-1} (e^{i4\pi/N})^k.$$
This is a finite geometric series whose sum is
$[1 - (e^{i4\pi/N})^N]/(1 - e^{i4\pi/N})$
As long as $e^{i4\pi/N} \neq 1$. This is true if $N \geq 3$. But then the numerator $1 - (e^{i4\pi/N})^N = 1 - e^{i4\pi} = 0$. Thus both the given sum and the corresponding one with sin for cos = 0.

Also solved by Robert Way, Matthew Fountain, Jerry Grossman, Winslow Hartford, Harry Zar-



emba, Kenneth Bernstein, Harry Lieberman, Frank Carbin, Marion Berger, Mark Pinsky, and the proposer, Greg Schaffer.

OCT 5. In six root extraction problems, each x is to be replaced with a digit (duplicates permitted), the numbers are base 10, no leading zeros are allowed, and no zeros are allowed in the roots themselves.

The following solution is from Harry Zaremba:

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      41                    511
√16 81               ∛133 432 831
 16                   125
  81                    8 432
  81                    7 651
                          781 831
     4 11                 781 831
√16 89 21
 16                             5 111
  89                  ∛133 511 182 631
  81                   125
   8 21                  8 511
   8 21                  7 651
                           860 182
      41 11              781 831
√16 90 03 21               78 351 631
 16                        78 351 631
  90
  81                                6111
   9 03           ∜1394 5983 1095 1041
   8 21            1296
    82 21            98 5983
    82 21            88 5841
                     10 0142 1095
                      9 1015 9041
                        9126 2054 1041
                        9126 2054 1041
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The roots were obtained by using the following relations:

$N^2 - a^2 = (2a + \Delta)\Delta$
$N^3 - a^3 = (3a^2 + 3a\Delta + \Delta^2)\Delta$
$N^4 - a^4 = (4a^3 + 6a^2\Delta + 4a\Delta^2 + \Delta^3)\Delta$

in which the integer a is a digit multiplied by appropriate power of 10 for an initial estimate of the root, and integer X consists of a digit multiplied by a suitable power of 10 to represent successive estimates of the remainder of the root.

Also solved by Robert Way, Matthew Fountain, Norman Wickstrand, Avi Ornstein, Kenneth Bernstein, Richard Hess, and the proposer, John Woolson.

Better Late Than Never

1981 OCT 4. As noted by Emmet Duffy, the formula given in the October 1983 issue contains a typo: the third team should be positive. Mr. Duffy also enclosed a highly accurate approximate solution.

M/J 4. As noted by Joel Brenner, the argument given incorrectly assumes that no prime is divisible by 3 neglecting the obvious (and unique) counterexample of 3 itself. Indeed, 3,7,11 is a three-term arithmetic progression of primes with common difference 4. The analysis given in the October issue can now be applied to show that this sequence is of maximal length.

M/J 5. Smith D. Turner and La Ting found simpler solutions.

JUL 3. Michael Jung has responded.

A/S 3. Emmet Duffy and Michael Jung have responded.

OCT SD2. Paul Hoffman, Judy Bergman, and Jerry Horton found a solution requiring one less operation.

Proposers' Solutions to Speed Problems

SD 1. West must have the ♦K. The diamonds need to split 4-3 and the trumps 2-2. Lead your singleton diamond and finesse the ♦Q. Trump a diamond, then a heart, then another diamond. If all goes well, Dummy's diamonds are now good. Lead your last spade to the ♠A, play another trump. Lo and behold—the trumps drop! (My wife *made* this 14-point game with *exactly* this line!)