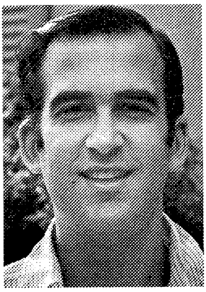
Can 1984 Be As Good as 1983?

Puzzle Corner/Allan Gottlieb



Allan J. Gottlieb, '67, is associate research professor at the Courant Institute of Mathematical Sciences of New York University; he studied mathematics at M.I.T. and Brandeis. Send problems, solutions, and comments to him at the Courant Institute, New York University, 251 Mercer St., New York, N.Y. 10012.

This being the first issue of another year, we again offer a "yearly problem" in which you are to express small integers in terms of the digits of the new year (1,9,8, and 4) and the arithmetic operators. The problem is formally stated in the "Problems" section, and the solution to the 1983 yearly problem is in the "Solutions" section.

Problems

Y1984 Form as many as possible of the integers from 1 to 100 using the digits 1,9,8, and 4 exactly once each and the operators +, -, × (multiplication), / (division), and exponentiation. We desire solutions containing the minimum number of operators; and, among solutions having a given number of operators, those using the digits in the order 1,9,8, and 4 are preferred. Parentheses may be used for grouping; they do not count as operators.

JAN 1 Doug Van Patter asks us a bridge problem based on the following hand that actually occurred in a tournament.

	♠ J 9 8 5 2	
	♥ A	
	♦ —	
	♣ A K Q 10 9 6 4	
♠ A Q 3		♠ K 7
♥ K J 3		♥ 9 6 5 4 2
♦ A J 10 8 6		♦ K Q
♣ J 2		♣ 5 3
	♠ 10 6 4	
	♥ Q 10 8 7	
	♦ 5 4 3 2	
	♣ 8 7	

South is declarer at a contract of four spades; West leads the ♦A. Would you choose to be declarer or defender?

JAN 2 A *Technology Review* reporter took a poll among a number of readers and found that 91.3 percent enjoyed "Puzzle Corner," a value accurate to three digits. What is the minimum number of readers the reporter could have polled for this value to be so accurate?

JAN 3 Winthrop Leeds has a three-part problem about the design of the soccer ball in current use. The ball appears to be made from 32 pieces of leather, 12 black pieces in the shape of regular pentagons and 20 white pieces in the shape of regular hexagons. Instead of forming a polyhedron, air pressure pushes the sides out into a circumscribing sphere. How many vertices does the basic polyhedron have? Do all of these vertices lie on the circumscribing sphere? If the edge of each pentagon and each hexagon is exactly 2 inches in length, calculate the diameter of the circumscribing sphere.

JAN 4 We end the regular section with a problem Dan Dewey sent to the M.I.T. Physics Department student newsletter. On the planet Trayshowed in a distant galaxy, an earth scientist was asked to measure the black body temperature of the sun. Their day/night cycle was 36 hours, so the scientist was somewhat frazzled; however, he was spared a little work when he noticed that sunlight would enter his appartus for a full 6 minutes without having to move it. As predicted, the blackbody equivalent temperature was found to be 5500 Kelvin. What kind of clothing did our scientist wear? (French scholars: take a guess. Physicists: calculate the average surface temperature of Trayshowed!)

Speed Department

SD1 Phelps Meaker offers a sidewalk speeder: A straight sidewalk is to be constructed of pre-cast concrete slabs—alternate isosceles trapezoids and rhombuses. There are ten trapezoids and eleven rhombuses. Two right-angle triangles are provided to dress up the ends. The altitude and the two parallel sides of the trapezoids are in the ratio 2:3:4. The rhombuses are 34 inches on a side. What are the width and length of the walk (within 1/4 inch)?

SD2 A gem (ruby) from Art DeLagrange. Here's a quickie: The Rubik's cube (3×3) has 26 sub-cubes, or "cubies," as the one in the center does not really exist. Each face has nine "facies," for 54 total. The six center facies do not move (only spin); the 48 remaining facies all have unique locations. They belong two to an "edge" cubie or three to a "corner" cubie. Do more belong to edge or corner cubies? Also, define the equivalent problem for the 4×4 cube and solve it.

Solutions

Y1983 Apparently 1983 *is* a very good year (remember, I write this in early October) since only two numbers cannot be formed as exemplified by Rik Anderson's solution:

1—1^{983}
2—38/19
3—18/(9 − 3)
4—9 + (8 − 13)
5—3 + (18/9)
6—18 − (3 + 9)
7—8 − 1^{93}
8—91 − 83
9—9 × 1^{83}
10—9 + 1^{83}
11—1^{9} × (8 + 3)
12—93 − 81
13—13 × (9 − 8)
14—19 − (8 − 3)
15—18 − (9/3)
16—1 − [9 − (8 × 3)]
17—1^{3} × (9 + 8)
18—9 + 8 + 1^{3}
19—38 − 19
20—1 × (9 + 8 + 3)
21—39 − 18
22—[3 × (9 + 1)] − 8
23—(8 × 3) − 1^{9}
24—19 + (8 − 3)
25—1 + [8 × (9/3)]
26—(9 × 3) − 1^{8}
27—81/(9/3)
28—38 − (1 + 9)
29—1 × (38 − 9)
30—19 + 8 + 3
31—1 × (39 − 8)
32—1 + (39 − 8)
33—3 × (19 − 8)
34—1 + 9 + (8 × 3)
35—1 × [8 + (3 × 9)]
36—9 + (81/3)
37—38 − 1^{9}
38—38 × 1^{9}
39—38 + 1^{9}
40—39 + 1^{8}
41—(8 × 9) − 31
42—81 − 39
43—19 + (8 × 3)
44—8 + [9 × (3 + 1)]
45—18 + (3 × 9)
46—9 + (38 − 1)
47—1 × (9 + 38)
48—1 + (9 + 38)
49—98/(3 − 1)
50—(1 + 9) × (8 − 3)
51—1 × [3 × (9 + 8)]
52—1 + [3 × (9 + 8)]
53—91 − 38
54—18(9/3)
55—8(3 − 1) − 9
56—8 × [9 (3 − 1)]
57—19 + 38
58—89 − 31
59—(8 × 9) − 13
60—[9 × (8 − 1)] − 3
61—[8 × (9 − 1)] − 3
62—
63—189/3
64—83 − 19
65—8 + (3 × 19)
66—198/3
67—98 − 31
68—(1 + 3) × (9 + 8)
69—81 − (9 + 3)
70—1 − [3 − (9 × 8)]
71—(9 × 8) − 1^{3}
72—1^{3} × (9 × 8)
73—83 − (1 + 9)
74—1 × (83 − 9)
75—93 − 18
76—89 − 13
77—[8 × (9 + 1)] − 3
78—81 − 9/3
79—
80—91 − (8 + 3)
81—3 × (19 + 8)
82—83 − 1^{9}
83—83 × 1^{9}
84—93 − (1 + 8)
85—98 − 13
86—1 + (93 − 8)
87—1 − (3 − 89)
88—89 − 1^{3}
89—89 × 1^{3}
90—89 + 1^{3}
91—9 + (83 − 1)
92—1 × (9 + 83)
93—1 + (9 + 83)
94—98 − (3 + 1)
95—19 × (8 − 3)
96—1 + (98 − 3)
97—98 − 1^{3}
98—98 × 1^{3}
99—98 + 1^{3}
100—98 + (3 − 1)

Also solved by Ron Newman, A. Holt, Allen Tracht, Kenneth Fawcett, Bill Dawson, Maria Petrocchi, Jay Roth, Harvey Fletcher, George Aronson, John Fine, Rik Anderson, Hal Steiner, Avi Ornstein, Harry Garber, Allan Katzenstein, Phelps Meaker, Harry Zaremba, David Evans, Linda Furrow, Burt Grosselfinger, and Rudy John.

A/S 1 In the situation shown at the top of the next column, White is to play and draw.

A detailed solution from David Evans. White cannot hope to win unless Black blunders. The passed KRP cannot be queened, and there is not enough time to clear the QR file before Black breaks through in the center. 1. K-K4 stops Black's pawn temporarily, but meanwhile Black merely picks off the KRP, then swings around with his king and eventually forces his way through in the center. White must therefore play for stalemate at QR5, with his QRP's at R4 and R6, his BP permanently immobilized, his KRP captured, and a Black pawn

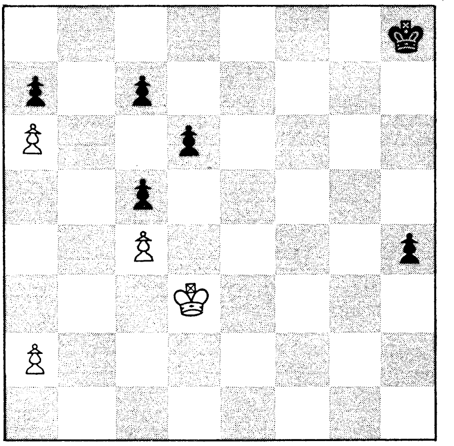


Diagram for A/S1 (see column 1)

at B3. The move P-B3 is natural for Black in his effort to clear the center, but he will avoid making it if he recognizes White's attempt to force a draw. Thus White loses if he merely pushes his KRP and heads for QR5: e.g., 1. P-R5, K-R2; 2. P-R6, K×P; 3. K-B3, K-N4; 4. K-N3, K-B5; 5. K-R4, K-K5; 6. K-R5, K-Q5; 7. P-R4 loses to 7. . . . P-Q4. If 8. P×P, P-B5; if 8. K-N5, P×P and in both cases Black queens first.

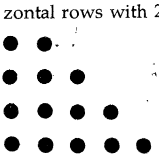
Therefore White must force Black to play P-B3. This is accomplished with:
1. K-K4
The threat is 2. K-Q5 followed by 3. K-B6. Black cannot ignore this threat: e.g. if 1. . . . K-R2; 2. K-Q5, K-R3; 3. K-B6, K-R4; 4. K×P(B7) wins since of 4. . . . K×P; 5. K×P followed by 6. K×P and White queens; or if 4. . . . P-Q4; 5. K×QP and Black's QBP falls quickly. Black's king cannot help since it must guard White's KRP: e.g., 1. . . . K-N2; 2. K-Q5, K-B3; 3. K-B6, K-K4 loses to 4. P-R5, for if now 4. . . . P-Q4; 5. P-R6 and Black must retreat to the corner, after which White mops up in the center as before.

Thus Black must prevent White's 2. K-Q5. The reply 1. . . . P-Q4 ch obviously loses quickly, so 1. . . . P-B3 is forced. Now White must force immobilization of his BP, since he cannot allow Black to force an exchange at White's Q5: e.g., 2. K-Q3, K-R2; 3. K-B3, K-R3, 4. K-N3, K-R4; 5. K-R4. K×P; 6. K-R5, K-N5; 7. P-R4 and Black still wins with 7. . . . P-Q4; 8. P×P, P×P and stalemate is broken. Then 9. K-N5 (forced), P-B5!; 10. K-N4 (forced—else 10. . . . P-B6 and queens), K-B5; 11. K-B3, K-K6 and Black queens. Thus White must force Black to play P-Q4 and P-Q5, after which White can reach his stalemate at QR5. This is accomplished by:
2. K-B5. . . .
If now 2. . . . K moves; 3. K-K6, K moves; 4. K×P and White wins, for Black cannot save the BP's and simultaneously stop the queening threat on the QB and KR files; or 3. . . . P-Q4; 4. P×P, P-B5 (not 4. . . . P×P: 5. K×P followed by 6. K×P and White wins); 5. P×P and White queens first, capturing at QB1 when Black queens. So, 2.. . . P-Q4 is forced. Now:
3. K-K5
If 3. . . . P×P, 4. K-K4 and Black cannot queen—e.g., 4. . . . P-B6; 5. K-Q3, P-B7; 6. K×P (or 5. . . . P-B5 ch; 6. K×P (B3)) and Black loses all his QBP's, enabling White to queen on one or the other of the rook files. If instead 3. . . . K moves; 4. P×P followed by 5. P×P as above and White wins. Therefore Black must play 3. . . . P-Q5, and White has achieved his objective. Black, of course, cannot advance his QP without his king's support, and this gives White the time he needs to reach stalemate: 4. K-K4, K-R2; 5. K-Q3, K-R3, 6. K-B2, K-R4; 7. N-N3, K×P; 8. K-R4; or 4. . . . K-N2; 5. K-Q3, K-B3; 6. K-B2, K-K4; 7. P-R5 and Black must return to the corner. In either case, White reaches QR4 before Black can advance QP; e.g., continuation of the former line of play gives: 8. . . . P-Q6; 9. K-R5, P-Q7; 10. P-R4 stalemate. The latter line would continue: 7. . . . K moves (not 7. . . . P-Q6 ch; 8.

K×P and White reaches QR5 and pulls up the QRP before Black can elminate the KRP and break the logjam on the QB file); 8. P-R6 and Black must retreat. Of course, if Black plays P-Q6 before White reaches QR4, White simply captures, then queens on either the QB or KR file.

Also solved by Robert Way, Matthew Fountain, Ronald Raines and the proposer, Bob Kimble.

A/S 2 A total of 14 coins are arranged in four horizontal rows with 2,3,4, and 5 coins, respectively:



Two opponents, "A" and "B," take turns picking up any one or more coins from any one horizontal row until one opponent wins by leaving the last coin for the other opponent to pick up. If "A" starts, there will be no way for "A" to win regardless of his first move, unless "B" fails to make the right moves thereafter. The problem is to identify how few and what configurations "B" can leave for "A" on "B"'s first move (after any starting move by "A") so that "B" can win, regardless of any subsequent move by "A."

Thomas Stowe correctly determined that there are four configurations "B" can leave for "A":
Piles containing 1, 2, and 3 coins.
Piles containing 1, 4, and 5 coins.
Piles containing 2, 2, 3, and 3 coins.
Piles containing 2, 2, 4, and 4 coins.

Henry Curtis analyzed the entire game:
A foolproof approach for B to win the "2-3-4-5 coin game" is to present player A with an "even configuration" on each play until near the end. To determine whether an "even configuration" exists, follow these two steps:
1. Express the number of coins in each row as a binary number. for the starting position, this would be:

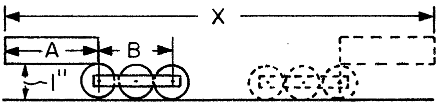
Row number	*Number of coins: Decimal*	*Binary*
1	2	10
2	3	11
3	4	100
4	5	101

2. Count the number of non-zero digits in each column of the binary listing. If the number of non-zero digits in every column is even, then the array is "even." For the starting position, there are two non-zero digits in the units column, two in the 2's column, and two in the 4's column, making the starting array "even."

Note that a player confronted with an "even" configuration at the beginning of his turn will have to leave an "odd" configuration for his opponent, no matter what move he chooses to make. On the other hand, a player confronted with an "odd" configuration can leave an "even" or "odd" configuration for his opponent, depending on the move he chooses to make. To force player A to take the last coin, B should then take all of the coins in the row containing more than one coin, leaving A with an odd number of rows, each containing one coin. As an example, suppose A starts by taking one coin from the longest row, leaving rows of 2, 3, 4, and 4 coins. B can make this an "even" configuration by taking one coin from the second row, leaving rows of 2, 2, 4, and 4 coins. For a second example, assume A takes all coins from the longest row, leaving rows of 2, 3, and 4 coins. B can leave an "even" configuration by taking 3 coins from the third row, leaving rows of 2, 3, and 1 coins, and so on. One can see that B can be the master of the game—i.e., he may choose in advance to force A to take the last coin or to prevent A from getting the last coin. For the latter option, B simply presents an "even" configuration to A throughout the game. The method outlined here works not only for the "2-3-4-5 coin game" but also for an expanded game of *any* number of rows with *any* number of coins in each row. B can win all the time, provided he has an opportunity to present an "even" configuration to A sometime during the course of the game.

Also solved by Robert Way, David Evans, Matthew Fountain, John Woolston, Harry Zaremba, Richard Hess, Winslow Hartford, Emmet Duffy and the proposer, Donald Richardson.

A/S 3 A member A moves on rollers, without slipping, from the solid-line position to that shown in dotted lines. What is the value of X in terms of the lengths A and B?



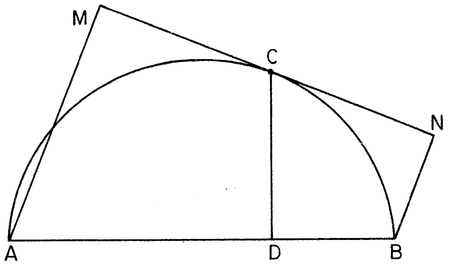
Karl Brendel shows us that the answer is
X = 3A + 2B

By inspection, if the rollers did not move relative to the floor, X would be equal to the distance moved by a point on the member, relative to the rollers, plus the length A. That distance is equal to A + B, so
X [Member Relative to Rollers] = 2A + B
However, we are given rollers move without slipping. Therefore, the rollers move, relative to the floor, the same distance moved by the point on the member, relative to the rollers: A + B
X [Rollers Relative to Floor] = A + B
The X we want is the summation:

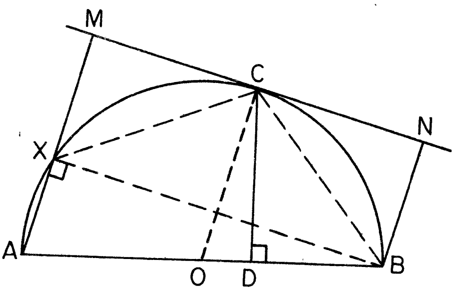
X [Member Relative to Floor]
= X [Member Relative to Roll] + [Roll Relative to Floor]
= 2A + B + A + B.
X = 3A + 2B

Also solved by David Evans, George Piotrowski, Thomas Stowe, Ronald Raines, Norman Wickstrand, John Woolston, Richard Hess, Harry Zaremba, Howard Wagner, Mary and Martin Lindenberg, James Abbot, Robert Way, Michael Neschleba, and Matthew Fountain.

A/S 4 In the drawing, C is a point on the semicircle with AB as diameter. MN is tangent to the semicircle at C. AM and BN are perpendicular to MN, and CD is perpendicular to AB. Show that CD = CM = CN and that $CD^2 = (AM)(BN)$



The following solution is from Henry Lieberman.



Let the intersection of AM and the semicircle be X and draw BX. Now angle BXA is a right angle since it is inscribed in a semicircle. Thus MNBX is a rectangle. Now let O be the center of the circle and draw CO. Since MN is tangent to the circle at C, CO is perpendicular to MN. Therefore, CO, MA, and NB are parallel. Since CO bisects AB, CO also bisects MN whereby CM = CN. Since CO is perpendicular to MN and MN is parallel to BX, CO is perpendicular to BX. Moreover, CO bisects BX since it bisects MN. So, CO is the perpendicular bisector of BX. Hence, chords CX and CB are equal. Therefore, arc CX = arc CB and so angle COD = angle XAB. So, triangle COD is similar to triangle XAB. The similarity of these two triangles implies

CD/XB = CO/AB = 1/2.

Therefore, CD = XB = MN = CM = CN, one of the desired results.

Observe that angles CAB and NCB are both measured by arc CB, and hence they are equal. Moreover, angle BCD = angle CAB since they are both compliments of the same angle. Hence, angle BCD = angle NCB and therefore triangle BCD = triangle BCN. Then DB = NB. Similarly, AD = AM. But, $CD^2 = (AD)(DB)$ and therefore $CD^2 = (AM)(BN)$, the second desired result.

Also solved by Steve Feldman, Avi Ornstein, Farrel Pownser, G. Yin, Winslow Hartford, Phelps Meaker, Karl Brendel, David Evans, George Piotrowski, Richard Hess, Harry Zaremba, Martin Lindenberg, James Abbot, Robert Way, Matthew Fountain, Raymond Gaillard, Apulia Servi, Norman Wickstrand, Emmet Duffy, Naomi Markovitz, and the proposer, Mary Lindenberg.

A/S 5 Which integers X have the property that 9X is the same as X with the digits in reverse order? There is one other integer multiplier (besides 9 and trivially 1) that reverses digits for an infinite number of integers. What is this multiplier and what are the multiplicands?

Robert Way found the solutions:
+11 × [integer part of (9.99 . . . × 10^a)]
where a is a positive number. He also determined that the other multiplier is 4 with mulitiplicands.
+100 × [integer part of 2.1999 . . .×10^a)] + 78.
David Evans noted that the multiplicands for 4 are twice those for 9.

Also solved by Matthew Fountain, Richard Hess, Harry Zaremba, Winslow Hartford, Emmet Duffy, and the proposer, Susan Henrichs.

Better Late Than Never

FM 2 John Langhaar believes the area is 33.512.

FM 3 John Langhaar notes that in 1957 Sidney Clark submitted these equations to a brainteaser column edited by Mr. Langhaar. The solutions were

$x = \pm\ [(3^{1/3} + 3)^{1/2} \pm (3^{1/3} - 1)^{1/2}]$

$y = \pm\ [3^{1/3}(3^{1/3} + 3)^{1/2} \pm (3.9^{1/3} - 1)^{1/2}]$

where x and y must be taken of the same parity.

M/J 5 William Peirce found a simpler solution.

JUL 1 Mearle Smith, Alan Robok and Pi-Jan Sheu, John Woolston, and Emmet Duffy have responded.

JUL 3 John Woolston has responded.

JUL 4 John Woolston, Emmet Duffy, James Abbot, and Karl Brendel have responded.

JUL 5 Karl Brendel, Michael Jung, and Emmet Duffy have responded.

JUL SD 1 James Abbot tried the solution given, using two mirrors, and does not believe that it works.

A/S SD 2 George Holderness and Robert Way believe that there are better ways to play the hand.

Proposers' Solutions to Speed Problems

SD 1 W = 33", L = 959.75" (80 feet).

SD 2 For the 3×3, there are 8 corners with 3 facies and 12 edges with 2, for an equal number of 24. For the 4×4, things are a bit more complicated. There are 56 cubies, with 2×2 subset in the center nonexistent. Some are duplicates. All move. There are 96 facies; their locations are not necessarily unique. There are now 24 facies on center cubies, 48 facies on edges, and still 24 facies on corners.