Wanted: Quickies Quick

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The problem is to prove or disprove that the series of leading 1's continues forever.

A/S 4. John Fogarty has sent us the "rug puzzle." You want to put wall-to-wall carpeting into a room that is 9x12 feet. You have two pieces of carpet, one 10x10 and the other 1x8. These do add to the correct square footage, but obviously the 10x10 must be cut. The challenge is to devise one continuous cut through the 10x10 piece such that the two resulting pieces will exactly fit the 9x12 area with one gap left over into which the 1x8 remnant can fit to complete the job.

A/S 5. Our last regular problem is reprinted from the M.I.T. Physics Department student newsletter edited by Minn Chung:

On a rainy day Fat Timothy was riding a donkey in the countryside ten miles away from the nearest shelter when he was caught in a strange rain which had a uniform mass density and fell straight down. Worried about being exposed to this strange precipitation, Timothy rode the faithful donkey as fast as he could to the nearest shelter. Assuming that the speed of the donkey was uniform and sufficiently high, find:

1. The amount of rain which fell on Timothy as a function of $\Theta$, the angle Timothy's body makes with the ground.
2. The minimum and maximum amount of rain which might fall on Timothy.
3. The ideal situation in which Timothy would be wet least.

Approximate Timothy's body as a box.

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Problems

A/S 1. Matthew Chen has a problem that uses less than one-third of a chess board and one-eighth of each army. Place white bishops on a1 and a3 and place black bishops on e1 and e3. By moving each color alternately, such that a black bishop can never take a white one and conversely, and restricting moves to ranks 1 to 4 and files a to e (i.e., to ten black squares), exchange the position of the bishops so that the black bishops end up on a1 and a3 and the white bishops on e1 and e3.

A/S 2. Emmet Duffy asks the following geometry question. An isosceles triangle has a bisector of one of the two equal angles that is 6 inches long. If the base is 5 inches, without using trigonometry, find the length of the two equal sides.

A/S 3. Steve Chilton asks an interesting question about primes. Write the prime numbers and take successive absolute differences

2 3 5 7 11 13 17 19 23 29 ...
1 2 4 2 4 2 4 2 4 ...
Note that the second row starts with a 1. Next repeat the process of taking successive absolute differences obtaining

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Speed Department

SD1. Daniel Seldman wants you to find the next term in each of the two (related) sequences.

3 4 8 9 14 23 ...
2 9 10 18 20 28 ...

August/September 1982
balance, the odd ball is the third ball, and it is heavier or lighter as already known. Now proceed to the full solution, where an asterisk (*) signifies this common "end game."

Identify the 39 balls as A through 39 and the four weighings as A through D, and proceed according to the table on the top of this page. (At the points marked with the double asterisk (**), use any balls known to be not odd.)

Clearly this is not a unique solution. For example, (D) following (A) =, (B) =, and (C) = can be solved simply by weighting (13) or (26) against one ball known not to be odd. Also, (B) following (A) = can lead to solution weighing six balls each side: 27-32 vs. 33-35 plus three balls known not odd; there are other arrangements which will solve (B) following (A) = as well.


M/A 3. Given the lengths of the sides of an irregular polygon, how should the sides be arranged and what should the angles be so as to maximize the area?

Henry Zaremba sent us the following correctly drawn solution:
The maximum area that can be enclosed by an irregular polygon occurs when each vertex lies on a circle circumscribing the polygon. The radius of the circle and polygonal area remain unchanged for any peripheral combination of the given sides of the inscribed polygon. The maximum area which can be confined within a given n-gon may be determined by the following analysis.

S1 = length of the nth side of the n-gon. (I = 1, 2, 3, . . ., n).

20 = angle subtended by S1 at center O of the circle.

For the indented lth triangle ABC, OC = R cos θ1 and S1 = 2R sin θ1 or

R = S1/(2 sin θ1) (1)

The area of ABC is

A = (1/2)R2 sin θ1 = (S1²/2) cot θ1.

Therefore, the total area of the polygon becomes

A = (1/4)S1² cot θ1 + S2² cot θ2 + . . . + Sn² cot θn.

(2)

Also from the figure, the sum of one-half of the central angles is

θ1 + θ2 + θn + . . . + θn = π.

Using equation (1), the ratio of the expression for the nth side that is tangent to the other sides yields

S1/Sn = (sin θ1)/(sin θn) or

θ1 = sin⁻¹(S1/Sn) sin θn.

(4)

Substitution of θ1 into (9) for i = 2, 3, . . ., n results in

θi = sin⁻¹(Si/Si sin θi) + . . . + sin⁻¹(S1/S1 sin θ1) + . . .

(5)

The solution to equation (5) can be obtained readily by iteration methods to any practical accuracy with the use of a computer or programmable calculator. With θ1 determined, all other angles θi, maximum area A, and radius R of the circle can be evaluated from equations (4), (2), and (1), respectively. For example, when n = 5 and sides S1 to S5 equal 5, 4, 3, and 2, respectively, the central half angles are

θ1 = 66°55'2.6".

θ2 = 47°23'15.9".

θ3 = 33°20'6.7".

θ4 = 21°35'27.1".

θ5 = 10°36'7.7".

and the area A = 13.60499 square units and the radius R = 2.717567.

Also solved by Matthew Fountain, Robert Bart, and the proposer, Irving Hopkins.

M/A 4. In how many ways can seven people be seated at a round table so that no person sits next to the same pair (unordered) of people twice?

Several readers pointed out that for n people, at most

n² - n = 2(n² - 1) arrangements are possible. Thus for n = 7 we obtain an upper bound of 15. Apparently the exact value is not known. Most respondents found 10 arrangements, but Ken Arbit found the following 11:

1, 7, 6, 5, 4, 3, 2 1, 6, 4, 7, 3, 2, 5

1, 5, 6, 4, 3, 2, 7 1, 3, 6, 7, 5, 4, 2

1, 4, 6, 3, 5, 2, 7 1, 3, 4, 2, 6, 5, 7

1, 7, 5, 6, 3, 2, 4 1, 6, 3, 7, 5, 2, 4

1, 7, 4, 5, 2, 3, 6 1, 3, 7, 6, 2, 5, 4

1, 6, 7, 4, 3, 5, 2

In the figure shown, R = radius of the circle circumscribing the n-gon.

August/September 1982