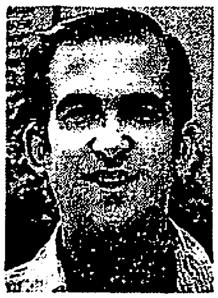
Puzzle Corner
Allan J. Gottlieb

Four Balls in One Box



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This being the first issue of another new year, we offer another of our "yearly problems" in which you are to express small integers in terms of the digits of the year (1982) and the arithmetic operators. The problem is formally stated in the "Problems" section, and the solutions to 1981 are in the "Solutions" section.

Although you are reading this issue at the end of December, I am writing it in mid-October while admiring the brilliant foliage outside our new house 50 miles north of New York City. On days like this I am reminded of the brilliant New England autumn colors I used to enjoy as an M.I.T. undergraduate.

Problems

Y1982 Form as many as possible of the integers from 1 to 100 using the digits 1, 9, 8, and 2 exactly once each and the operators +, −, · (multiply), / (divide), and exponentiation. We desire solutions containing the minimum number of operators, and solutions using the digits in the order 1, 9, 8, and 2 are preferred. Parentheses may be used for grouping, and they do not count as operators.

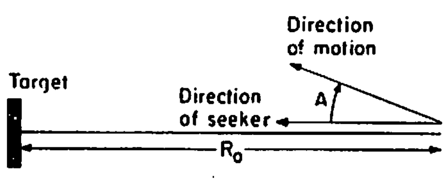
JAN 1 We begin our regular problems with a seven-card bridge problem from Emmet Duffy. South, on lead, is to make all seven tricks against any defense, with hearts as trumps:

	♠ —	
	♥ K Q	
	♦ 5	
♠ 8 7 6 5	♣ Q 10 9 5	♠ K 10
♥ J 5		♥ —
♦ —		♦ 10
♣ J	♠ J 9 3	♣ 8 7 6 4
	♥ —	
	♦ J	
	♣ A K 3	

JAN 2 Rob Cave has four identical balls that he must pack into a regular tetrahedral box. What is the minimum possible volume of the box?

JAN 3 Gertrude Fox suggests that we look for a minimal 3 x 3 magic square composed solely of prime numbers. For this problem, 1 is considered prime and a minimal square is one whose (equal) row, column, and diagonal sums are minimal.

JAN 4 Tom Hafer asks the following question, which he encountered during his work. A rocket is launched with constant speed from the point (R_0,O) toward a stationary target located at (O,O). The rocket has a seeker that is offset from the velocity vector of the rocket by a constant angle A. The seeker always points directly at the target. What is the flight path of the rocket and what is its acceleration?



Speed Department

SD 1 Joe Horton needs a small boat that can sail *directly* into the wind (not at an angle). Can you help him?

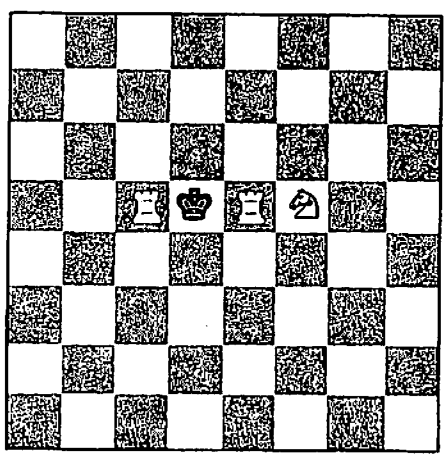
SD 2 Our final problem is from Winslow Hartford. A tourist buys four objects (all at different prices) at a bazaar. One object costs $1. The salesperson on duty totals the prices and announces that the combined cost is $6.75. The tourist, however, noticed that the salesperson used the multiply button instead of the add button while totalling the items. Surprisingly, this did not affect the result. What did the objects cost?

Solutions

Y1981 Integers from 10 to 100 are to be formed from the digits 1, 9, 8, and 1 according to the same rules as given above for **Y1982.**

Thomas Schonhoff and Harry Hazard were able to obtain 50 of the 100 integers. Everyone suspects that 1982 will be a better year (at least in this respect), since it has four distinct digits. An amalgam of their solutions follows:

1. 1^{981}	51.
2. $1^{98} + 1$	52.
3. $1 + 18/9$	53.
4. $8/(11 - 9)$	54. $9 \cdot [8 - (1 + 1)]$
5. $9 - 8/(1 + 1)$	55. $8^{(1+1)} - 9$
6. $8 + 9 - 11$	56. $(9 - 1) \cdot (8 - 1)$
7. $8 - 1^{19}$	57.
8. $1^{19} \cdot 8$	58.
9. $1 \cdot 81/9$	59.
10. $91 - 81$	60.
11. $19 - 8 \cdot 1$	61. $(8 \cdot 9) - 11$
12. $19 - 8 + 1$	62. $81 - 19$
13. $91/(8 - 1)$	63. $1 \cdot 9 \cdot (8 - 1)$
14.	64. $8^{(11-9)}$
15. $9 - [1 - (8 - 1)]$	65. $1 + (9 - 1) \cdot 8$
16. $8 \cdot (11 - 9)$	66.
17. $18 - 1^9$	67.
18. $1^9 \cdot 18$	68.
19. $1^9 + 18$	69.
20. $1^8 + 19$	70. $(1 + 9) \cdot (8 - 1)$
21.	71. $1 \cdot 81 - 9$
22.	73. $91 - 18$
23.	74. $1 + (9 \cdot 8) + 1$
24.	75.
25. $(1 + 1) \cdot 8 + 9$	76.
26. $19 + 8 - 1$	77.
27. $19 + 8^1$	78. $89 - 11$
28. $19 + 8 + 1$	79. $(8 \cdot 11) - 9$
29.	80. $81 - 19$
30.	81. $1^9 \cdot 81$
31.	82. $1^9 + 81$
32.	83. $(8 \cdot 9) + 11$
33.	84. $1 - 8 + 91$
34. $(1 + 1) \cdot (9 + 8)$	85.
35.	86.
36. $9 \cdot [8/(1 + 1)]$	87. $98 - 11$
37. $19 + 18$	88. $1 \cdot 89 - 1$
38.	89. $1 - 1 + 89$
39.	90. $1 \cdot 9 + 81$
40.	91. $1 + 9 + 81$
41.	92. $1^8 + 91$
42.	93.
43.	94.
44.	95.
45.	96. $98 - 1 - 1$
46.	97. $(1 \cdot 98) - 1$
47.	98. $1 + 98 - 1$
48.	99. $1 + 98^1$
49. $98/(1 + 1)$	100. $19 + 81$
50.	



Also solved by Jesse and Norman Spencer, David Freeman, David Buchanan, Peter Steven, Avi Ornstein, Neil Weiss, and Alan Katzenstein.

A/S 1 Place the Black King in the center of the board and then place two White Rooks and one White Knight so that the Black King is mated.

We have universal agreement that the solution is that shown at the bottom of the previous column.

Solved by Richard Hess, Matthew Fountain, Rick Shafer, Richard Kruger, Winslow Hartford, John Rollino, Jerry Grossman, Steve Feldman, Everett Leroy, Robert Sandow, Brett Rudy, Jack Mosinger, Arthur Hauser, Michael Jung, and the proposer, Roser Powell.

A/S 2 A cryptarithmetic problem, in which it is required that R = 1:

```
 ROOK
   TO
 KING
EIGHT
-----
CHECK
```

Dennis Sandow first found all 40 possible solutions to the bare problem (ten of which contain leading zeros) and then eliminated those not satisfying R = 1. The remaining two solutions are:

```
  1225        1772
    82          47
  3970        2809
 39068       58934
 -----       -----
 46345       63562
```

Also solved by William McGuinness, Richard Hess, Jack Mosinger, A. Fabens, and the proposer, William Butler.

A/S 3 All the houses on a street are identical except for their addresses, which are consecutively numbered 1, 2, 3, . . ., m. One house, n, had the property that the sum of all the addresses less than n was equal to the sum of all the addresses greater than n. For what m and n is this possible?

Several different solutions were submitted for this problem. Many, including Richard Hess's reprinted below, rest upon the so-called Pell equation which is described in Dickerson, *Introduction to the Theory of Numbers.* Mr. Hess writes:

The numbers below n add to $B_n = n(n - 1)/2$.
The numbers above n add to $A_n = (m - n)(m + n + 1)/2$.
Since A_n and B_n are equal, it follows that $8n^2 = (2m + 1)^2 - 1$.
Let $u = 2n$ and $v = 2m + 1$, obtaining $2u^2 = v^2 - 1$.
This is Pell's equation and has solutions $(u,v) = (0,1), (2,3), (12,17), \ldots$
$(u_i = 6u_{i-1} - u_{i-2}, v_i = 6v_{i-1} - v_{i-2})$.
Converting back to m and n gives
$m_i = 6m_{i-1} - m_{i-2} + 2$.
$n_i = 6n_{i-1} - n_{i-2}$.

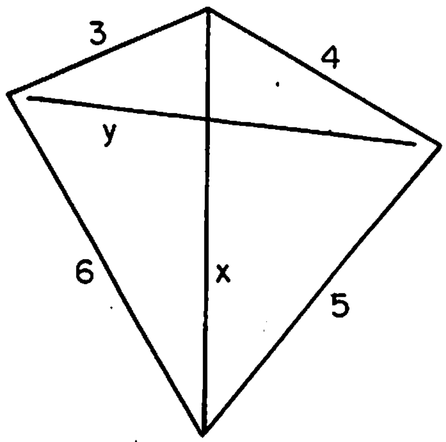
m	n
1	1
8	6
49	35
288	204
1681	1189
. .	. .

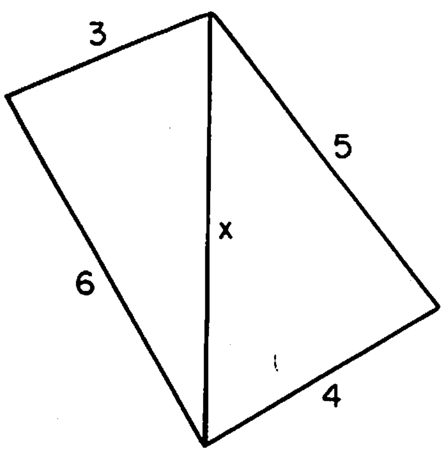
Also solved by Susan Henrichs, George Flynn, Greg Schaffer, Douglas Merkle, Avi Ornstein, Steve Feldman, John Rollino, Winslow Hartford, Richard Kruger, William Stein, Jerry Grossman, Matthew Fountain, Harry Zaremba, William Schoenfeld, Ronald Goldman, Edwin McMillan, Frank Carbin, Mike Bercher, Peter Coffee, Irving Hopkins, Arthur Hausner, Naomi Markovitz, Jack Mosinger, Emmet Duffy, Daniel Lufkin, A. Fabens, and Gerald Leibowitz.

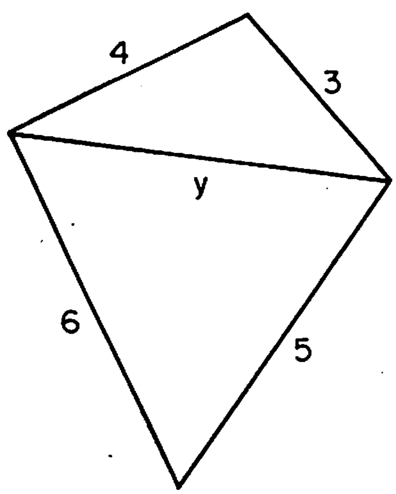
A/S 4 Four pieces of fencing, lengths 3, 4, 5, and 6 units, are to be arranged to enclose the largest pos' ea. What is the best configuration, and what is the maximum area?

Matthew Fountain has sent us the following solution:

The maximum area is 18.9737 and is independent of the order of the sides. Any two configurations with this area have one diagonal equal.





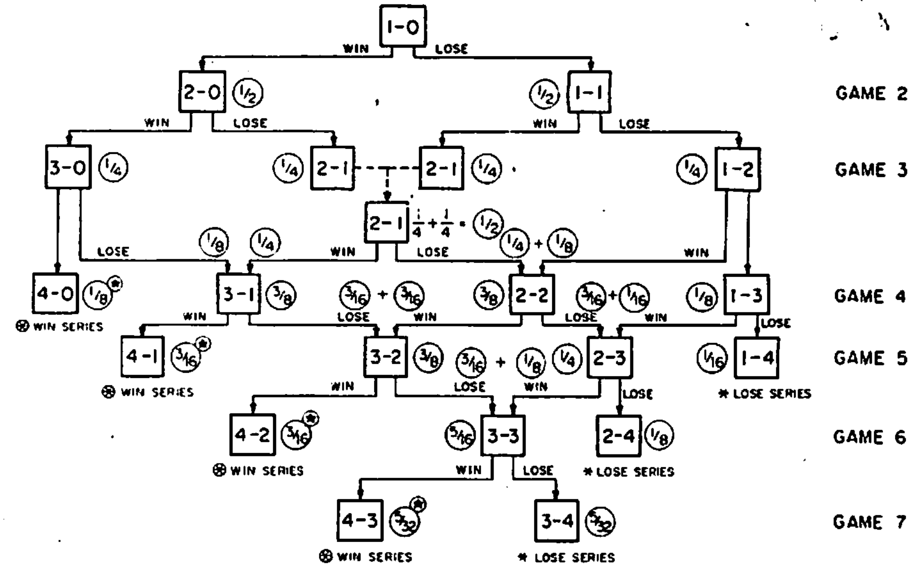


The drawings show all possible fencing configurations except for mirror images of those shown. The first and second configurations each contain triangles with sides 3, 6, x and 4, 5, x and total area A. The first and third each contain triangles with sides 3, 6, y and 3, 4, y and total area A.

Using Heron's formula for the area of a triangle

$$A = \sqrt{(9+x)/2 \cdot (9-x)/2 \cdot (x+3)/2 \cdot (x-3)/2} + \sqrt{(9+x)/2 \cdot (9-x)/2 \cdot (x+1)/2 \cdot (x-1)/2}$$

$$= \frac{1}{4}\sqrt{81-x^2}\left(\sqrt{x^2-9} + \sqrt{x^2-1}\right)$$



Solution by David DeLeeuw to A/S 5. The numbers in the boxes represent Team A's win-lose record; the numbers in the circles represent the probability of occurrence of the associated box or event. The probabilities associated with the four winning events at the lower left corner are $\frac{1}{8} + \frac{3}{16} + \frac{3}{16} + \frac{5}{32} = \frac{21}{32}$ = 65.6 percent.

$$\frac{dA}{dx} = \frac{1}{4}\,\frac{-x\left(\sqrt{x^2-9}+\sqrt{x^2-1}\right)\left(\sqrt{(x^2-9)(x^2-1)}-81+x^2\right)}{\sqrt{(81-x^2)(x^2-9)(x^2-1)}}$$

$dA/dx = 0$ when $x = 0$ and area is minimum or $\sqrt{(x^2-9)\,(x^2-1)} - 81 + x^2 = 0$ and $x = \sqrt{819/19} = 6.56546$ and area is maximum.

$A_{max} = \frac{1}{4}\sqrt{81 - 819/19}\left(\sqrt{819/19 - 9} + \sqrt{819/19 - 1}\right) = 18.9737$

The value of y at maximum area is obtained similarly, beginning with

$A = \sqrt{(11+y)/2 \cdot (11-y)/2 \cdot (y+1)/2 \cdot (y-1)/2} + \sqrt{(7+y)/2 \cdot (7-y)/2 \cdot (y+1)/2 \cdot (y-1)/2}$

and ending with $y = \sqrt{247/7} = 5.9402$

Substitution of this value of y in the equation for A yields 18.9737 as expected.

Also solved by Peter Coffee, Arthur Hausner, Harry Zaremba, Jerry Grossman, Richard Hess, Winslow Hartford, Avi Ornstein, Albert Nuttall, Dennis Wood, Everett Leroy, William Schoenfeld, Norman Wickstrand, and the proposer, Irving Hopkins.

A/S 5 What is the probability that a World Series will be won by the team that wins the first game, assuming two teams are playing, the first to win four games is the victor, and tie games are impossible?

David DeLeeuw diagrammed all possible outcomes and supplied a little baseball history to show that theory agrees moderately well with reality. His solution (66 percent) is based on the assumption that in any given game the odds are even that a given team will win. Mr. DeLeeuw's diagram is shown at the top of these columns. He writes:

Although this is a trivial problem, it is interesting to note that, based upon actual results, the team that won the first game of the World Series went on to win the series 59 percent of the time (41 out of 70). This does not include those series which required five victories and/or those which contained a tie game. Consider now another problem: What is the probability that a team will win the World Series after winning the first *two* games? Answer: 81.25 percent (26 times out of 32). In actuality, this has happened 25 out of 31 times! Amazing! The six teams that blew the World Series after winning the first two games: Yankees (1955), Dodgers (1956 and 1978), Braves (1958), Twins (1965), and Orioles (1971).

Also solved by Richard Duffy, Richard Hess, Richard Kruger, Winslow Hartford, Steve Feldman, Jerry Grossman, Arthur Hausner, Jack Mosinger, Naomi Markovitz, Frank Carbin, Edwin McMillan, Matthew Fountain, Michael Jung, Rich Shafer, Smith Turner, Daniel Lufkin, Harry Movitz, Emmet Duffy, Harry Zaremba, and the proposer, Jack Parsons.

Better Late Than Never

APR 3 David DeLeeuw sent us the official American Bowling Congress 12-team bowling schedule.

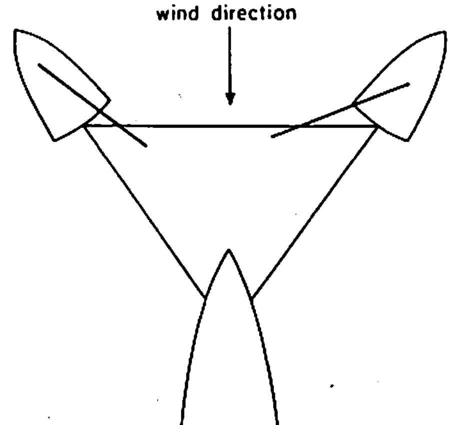
APR 4 Richard Kruger's name was inadvertently omitted from the list of solvers.

APR SD 1 The proposer believes that the excerpted phrase does indeed say, "Tea for two."

JUL 1 James Dotson has responded.

Proposers' Solutions to Speed Problems

SD 1 The boat is actually defined as the sum of the three vessels joined as shown. Only the two forward ones have sails; they tack and synchronize but in opposite directions. The net motion can be directly into the wind.



SD 2 The objects' costs are $1, $1.50, $2, and $2.25. Since one object (say D) costs $1, we have

$A \cdot B \cdot C = 6.75$

$A + B + C = 5.75$

But $675 = 5 \cdot 5 \cdot 3 \cdot 3 \cdot 3$. Thus $5 \cdot 0.3 = 1.50$ and $5 \cdot 5 \cdot 0.3 \cdot 0.3 = 2.25$ are logical choices for an object. Either of these choices leads to a simple quadratic for the other two values.