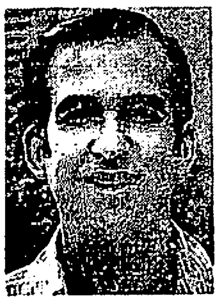
Puzzle Corner
Allan J. Gottlieb

A Small Loss of $26 \cdot 10^{-21}$



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Since the October issue begins a new volume of *Technology Review*, we take this chance to explain the ground rules of "Puzzle Corner" every year.

In each issue we present five regular problems (the first of which is chess- or bridge-related) and two "speed" problems. Readers are invited to submit solutions to the regular problems, and three issues later one submitted solution is printed for each problem; we also list other readers whose solutions were successful. In particular, solutions to the problems you see below will appear in the February/March issue. Since I must submit that column sometime in November (today is July 20), you should send your solutions to me during the next few weeks. Late solutions, as well as comments on published solutions, are acknowledged in the section "Better Late Than Never" in subsequent issues.

For "speed" problems the procedure is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same column in which the problem is published. For example, solutions to the October "speed" problems are given below. Only rarely are comments on "speed" problems published or acknowledged.

There is also an annual problem, published in the first issue of each new year; and sometimes we go back into history to republish problems which remained unsolved after their first appearance.

All problems come from readers, and all readers are invited to submit their favorites. I'll report on the size of the backlog, and on the criteria used in selecting problems for publication, in a future issue.

Problems

OCT 1 We begin this issue with a bridge problem from Doug Van Patter:

North:
♠ J 10 5 2
♥ 8 6 2
♦ 7 6 5 4
♣ K J

South:
♠ A K
♥ A K Q 5 4 3
♦ A 8 3
♣ A 9

East opens with a weak two-diamond bid (usually six cards, sometimes only five). Your (South) final contract is six hearts. The opening lead: ♦ 2. Problem: What line of play gives you the best chance of making this slam, after finding out that West has three trumps?

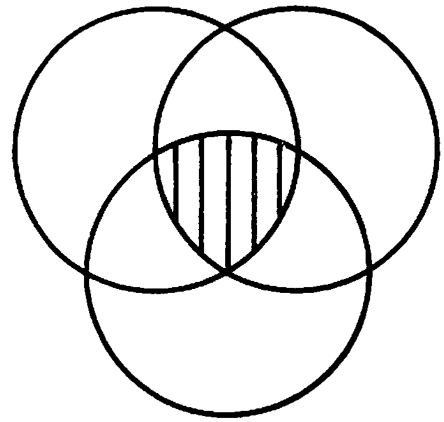
OCT 2 David Steyert would prefer a noncalculus solution to his cow problem: A cow is tethered to a circular silo in such a manner that the distance from the cow's mouth to the fixed tie-point is exactly 10 meters. If the circumference of the silo is exactly 20 meters, how much grazing area does the cow have?

OCT 3 Bill Bezdecheck wants to find an integer having the property that moving the rightmost digit to the leftmost slot yields a number exactly nine times the original number. For example, 2017 does not work since 7201 does not equal 9·2017.

OCT 4 In the best tradition of collegiate mathematics, Josh Cohen sends us a beer-theoretic problem posed by the M.I.T. Math Club: The familiar Ballantine Beer logo (pictured below) consists of three circles of equal radius placed so that the center of each circle lies on the boundaries of each of the other two. With elementary geometry or calculus it is easy to find that the area of the region of intersection is

$r^2(\pi - \sqrt{3})/2$,

where r is the radius of the circles. Consider the three-dimensional analogy: find the volume of the region of intersection of four spheres of radius r, each of which has its center on the surfaces of the other three.



OCT 5 Harry Zaremba is interested in an extension of the classical Pythagorean triple. He notes that a positive-integer solution to the pair of Pythagorean relations $A^2 + B^2 = C^2$ and $(A + 3)^2 + (B + 3)^2 = (C + 4)^2$ is the trivial triple $(A,B,C) = (0,1,1)$. Can you find two other positive-integer solutions? Determine if there are a finite or infinite number of such solutions.

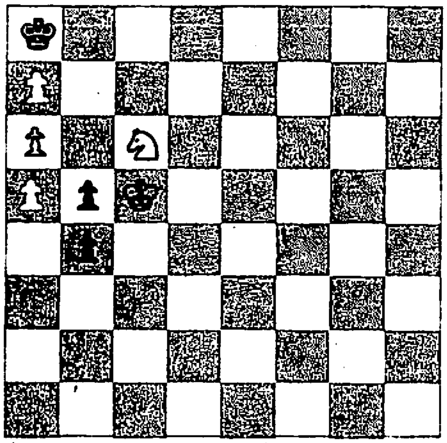
Speed Department

OCT SD 1 James Landau recalls his Army days: In the U.S. Army, a unit preparing to march forms a rectangular "rank and file" formation that is four files wide and as many ranks deep as needed. The distance between adjacent ranks is 30 inches. When marching, each man takes 120 30-inch steps per minute. Problem: the 1,764 men of his company were formed up on the parade ground at Fort Myer and Major Purdy, at the head of the formation, gave the order, "Forward, march!" As the company passed the reviewing stand, how long was the formation?

OCT SD 2 William Katz wants to know what is the smallest number of matches that will determine a winner in a club tennis tournament that has 43 entries, some of whom must be seeded because of ratings?

Solutions

MAY 1 White to play and mate in two moves:



Rick Tavan not only solved the problem but compared it with another from *Chess to Enjoy*. His solution is:

1. P x P P—N6
2. P—N7 mate

made possible because Black's last move had to be

0. . . . P—N4

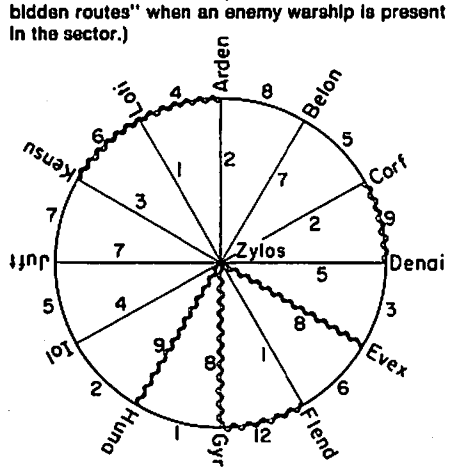
Rick then went on to write, "Interestingly, about an hour after I worked the problem I was reading Andy Soltis' *Chess to Enjoy* and encountered a similar problem in the last chapter of the book. The *en passent* capture appears at first glance to be the solution with identical reasoning as to Black's previous move. Further analysis shows that the two-square Pawn move could not have happened because a White Bishop is on QR1. In fact, Black could not have just moved. The problem is captioned "White mates in one" and the key is that Black must be on move! After Black's only legal move, a Queen mate is obvious. The theme of your problem is the red herring of this one."

Also solved by Robert Schmidt, John Cronin, Elliot Roberts, David Freeman, Daniel Seidman, Rufus Franklin, Larry Shiller, Richard Hess, Matthew Fountain, Winthrop Leeds, A. Ostapenko, and the proposer, George Farnell.

MAY 2 The Starship Enterprise is conducting negotiations on Arden with the Emperor of the Zylos System. Suddenly there is an alarm: a Klingon warship has entered the sector. The Enterprise must gather all of the leaders of the 12 outer planets and bring them to a conference on Zylos. It must use only the established space routes, whose travel times in zorpets are indicated in the chart. How soon can the conference be held?

Commander Michael Jung reports:

The first step is to locate the "forbitten" space routes, where a pair of connecting routes to the same destination may be traversed in fewer zorpets than the forbidden route itself. (Star Fleet Standing Order #HQ-3-2179-001 prohibits travel over "forbidden routes" when an enemy warship is present in the sector.)



Now our path is simple: we simply go clockwise around the rim, returning to the center (Zylos) wherever necessary to avoid a forbidden route. Thus the quickest sequence becomes the following, a total of 57 zorpets:

Arden	Huna
Belch	Gyr
Corf	Huna
Zylos	Iol
Denal	Juft
Evex	Kensu
Flend	Zylos
Zylos	Loti
Iol	Zylos

Since, as everyone knows, it takes a Klingon warship exactly 61.374268 zorpets to travel from the edge of a sector to the central planet, there will be plenty of time to plan strategy.

Also solved by Matthew Fountain, Jordan Wouk, David Freeman, Avi Ornstein, Harry Zaremba, Naomi Markovitz, W. Smith, Marion Weiss, Richard Hess, Larry Shiller, A. Ostapenko, and Winthrop Leeds.

MAY 3 What is the sum of $1^{-1} + 2^{-2} + 3^{-3} + \ldots$?

Apparently no one could find a closed-form solution. Of all the approximations given, John Wrench claims the most accuracy (25 decimal places):
1.29128 59970 62663 54040 72826.
Richard Hess only calculated 21 places but got:
1.29128 59970 62663 54038 1.
Well, gentlemen, what about the missing $26 \cdot 10^{-21}$?

Also solved by Winslow Hartford, Matthew Fountain, Winthrop Leeds, and Larry Shiller.

MAY 4 The proposer explained his interest in constructing N-by-N matrices of letters such that each row is an N-letter English word (read from left to right) and so is each column (read from top to bottom). Such a matrix is called N-perfect if the 2N words are all distinct. Problem: What is the largest possible N-perfect matrix?

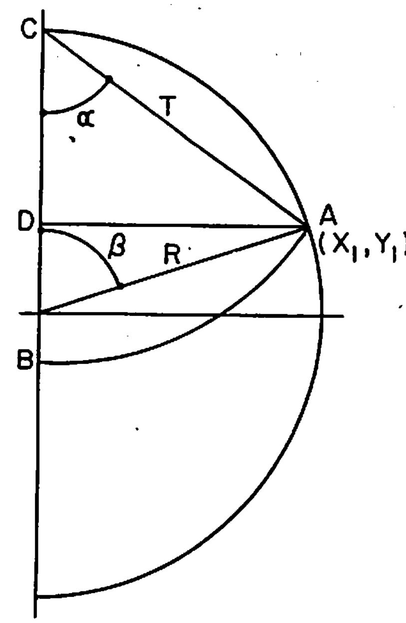
Marion Weiss found several four-perfect squares and Winthrop Leeds found a five-perfect square. Larry Shiller refers us to *Games* magazine and Edwin McMillan and Matthew Fountain suggest *Language on Vacation* by Dmitri Borgman. Finally, Mary Hazard sent us the following:

"Word squares" such as Barie Gilbert describes as "N-by-N matrices" have been a pastime for word-puzzle addicts known as "formists" (those who create word forms—a square is just one of many forms these crossword-like structures can take; there are diamonds, rhomboids, pyramids, etc.) for at least 100 years. Recently, a young Minnesotan by the name of Blake Greenlee created a 7-by-7 double word square (a double square is one in which the words across are different from those down, thus making 14 different words in the square) which was published in *The Enigma* (1980). Unfortunately, this word genius died at the untimely age of 25 this year; we regretfully will see no more such output from his active mind. Greenlee's 7-by-7 square is:

S	C	A	B	A	R	D
M	A	R	I	N	E	R
A	M	I	R	A	T	E
S	P	E	C	T	E	D
H	A	T	H	I	N	G
E	N	T	E	N	T	E
S	E	E	R	E	S	S

All of these words are in the *Oxford English Dictionary* or the second or third editions of *Webster's New International*. Other formists have in the past made 8-by-8 double squares, but some of the components are two-word phrases (dictionary entries, all right, but usually pretty odd words). Mr. Greenlee did many 6-by-6 double squares, and his is not the only 7-by-7 double square extant.

MAY 5 A farmer rents out one half of a circular, fenced pasture. A cow is to be tethered to a point on the fence so that she is able to graze on exactly one half of the pasture area. What is the length of the tether (T) if the radius (R) of the pasture is 100 feet?



The following solution is from William Moody. Referring to the accompanying sketch, the area of segment BAD: $\frac{1}{2}\alpha T^2 - \frac{1}{2}x_1(R - y_1)$, and area of segment CAD: $\frac{1}{2}\beta R^2 - \frac{1}{2}x_1y_1$, where $\alpha = \sin^{-1}(x_1/T)$, $\beta = \sin^{-1}(x_1/R)$, and $x_1 = T\sqrt{1 - T^2/(4R^2)}$.
Twice the sum of the segments must equal $\pi R^2/2$, so that $T^2 \sin^{-1}\sqrt{1-T^2/(4R^2)} + R^2 \sin^{-1}(T\sqrt{1-T^2/(4R^2)}) - RT\sqrt{1-T^2/(4R^2)} - \pi/2 = 0$.
To simplify the writing, let $T = uR$ and $v = \sqrt{4-u^2}/2$, so that: $u^2 \sin^{-1}v + \sin^{-1}(uv) - uv - \pi/2 = 0$. The solution to this transcendental equation is approximately: $u = 1.158728473$. With $R = 100'$ the tether length is a bit less than 115′ 10½″. Practically, of course, the tether must be somewhat shorter since the cow will be able to reach a foot or so beyond the end of her rope as she grazes.

Also solved by Michael Jung, Richard Farber, L. Upton, Larry Shiller, Edwin McMillan, Mary Lindenberg, Edward Dawson, Matthew Fountain, John Wrench, Richard Hess, Jordan Wouk, David Freeman, Herman Friedlaender, John Prussing, Norman Wickstrand, Larry Bell, Raphael Justewicz, Winslow Hartford, Harry Zaremba, and Emmet Duffy.

Better Late Than Never

FEB/MAR 1 Gerald Blum, Richard Hess, and Winslow Hartford have responded.
FEB/MAR 2 Richard Hess, Gerald Blum, Winslow Hartford, Harry Zaremba, and Donald Savage have responded.
FEB/MAR 3 Gerald Blum and Richard Hess have responded.
FEB/MAR 4 Michael Jung, Gerald Blum, Richard Hess, and Winslow Hartford have responded.
FEB/MAR 5 Richard Hess and Gerald Blum have responded.
APR 1,4 Winslow Hartford has responded.
APR 5 Naomi Markovitz, Winthrop Leeds, and Winslow Hartford have responded.

Proposers' Solutions to Speed Problems

SD 1 There are two possible answers. *First*, 1,105 feet. The 1,764 men form a first rank and 440 following ranks; hence in theory the formation should be 440 • 2.5 feet long, or 1,100 feet. However, due to speed-of-sound lag, the last rank will hear the order one second late and will therefore always be two steps or five feet behind their proper location. *Second*: Indeterminate. The head end of the company will be in the mess hall before the rear is past the reviewing stand; thus the answer depends on the efficiency of the serving line.

SD 2 Since 42 people have to be eliminated no matter how you seed or arrange, there will be 42 matches (one person being eliminated in each match).