Puzzle Corner
Allan J. Gottlieb

This month's problems were selected just as the Space Shuttle Columbia was making its historic first flight, and they include questions about digging a hole to the center of the earth and falling from space into the ocean. Fortunately, my only contacts with NASA have been as a computer expert and not as a soothsayer.

Problems
JUL 1 Emmett Duffy would like South to lead and make all seven remaining tricks:

- J 8 6
- A 4
- A
- K
- 10 7 2
- K 8
- 9
- 4
- 8 6
- A 9
- Q 7
- J 10 3

JUL 2 A two-part problem from Anthony Standen:
What is the smallest multiple of nine that has no odd digit? For the benefit of hair-splitters, I may say that it is to be in base 10; but if other bases are allowed, what then is the answer? And how many ways are there of doing it?

JUL 3 Eric Piehl wonders about the theoretical limit for high-diving competitions. A falling person reaches a terminal velocity of about 55 m/sec. Entering feet first, one can survive falls into water at speeds of up to 34 m/sec. Starting from rest, what is the maximum distance one can fall before entering water, and survive? Assume that drag is proportional to velocity and that gravitational acceleration is 9.8 m/sec.

JUL 4 This problem first appeared in the Review as part of a California Products, Inc., advertisement in the 1930s: Five coins, arranged as in (a) below, are to be shifted into arrangement (b), using only four accurate sliding moves [such as the move shown in (c)]. There is no restriction on the position of arrangement (b) relative to (a), but the new location of any coin moved must be fixed by definite contact with two other coins: estimated contacts [to form straight lines, as in (d)] are not allowed. Move only one coin at a time, without lifting. Our analysis of this old puzzle shows that there are no less than 24 straightforward solutions—but they are amazingly elusive! How many can you find?

(a)  
(b)  
(c)  
(d)  

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**JUL 5** Our last regular problem is from Richard Fisher:

What is the smallest square matrix in which one can write the names of 50 U.S. states, horizontally or vertically, in crossword-puzzle fashion? The ground rules are: words must be written from left to right, or from up to down; two-word states are to be written as if only one word—i.e., WEST VIRGINIA; each state must be written exactly once; no two letters can be horizontally or vertically adjacent unless they form part of a state name; and the completed puzzle must be connected—i.e., you cannot tuck an orphan state in a corner unless it's connected to the rest of the puzzle.

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**Speed Department**

**JUL 5D1** Here is a problem from Smith Turner that Jules Verne might have enjoyed: A person wanted to dig a hole to the center of the earth. He did not care how small it was at the bottom (a point would be fine) but realized that it must be wider at the top to permit workers to get in and that it must have some slope so that dirt would not slide back in too easily. He decided on a circular cone. To make the sides 1° off from vertical, what diameter of hole is required at the top?

**JUL 5D2** Finally, Joe Horton sends a quickie that may lead to a form of population control:

In an effort to ease the population explosion, it might be useful to decide whether to reject or adopt the sexist notion that a family should continue to make babies until one of the desired sex showed up. It has been argued that this must lead to large families. Question: If couples agreed to stop after a child of the desired sex had been born, what would be the average family size? For the sake of the problem, assume that boys and girls are equally likely to be born; and that twins don't occur.

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**Solutions**

**FEB 2** Given the diagram at the top of the next column, White to move and win.

Gene Janulis cleverly avoids the drawing trap hidden in this problem. He writes:


Also solved by Matthew Fountain (who found a recent article by George Koltanowski that featured Dederer's problem), David Freeman, George Farnell, James Landau, Edwin McMillan, G. Sharman, Elliott Roberts, Robert Bart, and the proposer, Bob Kimble.

**FEB 2** Does every prime number above 5 divide evenly into a one-less string of 1's? For example, 7 divides 111, 111; 11 divides 1,111, 111, 111; and 13 divides 1,111, 111, 111, 111, 111. Does 17 divide 1,111, 111, 111, 111, 111, 111, 111, 111, 111, 111, 111, 111, 111? Etc.

A joint effort by Steve Feldman and Pierre de Fermat shows that the answer is yes. Fermat's theorem (actually one of his many theorems) says that

\[ x^{p-1} \equiv 1 \pmod{p} \]

whenever \( x \) and \( p \) are relatively prime. Specifically, \( 10^{p-1} \equiv 1 \pmod{p} \) whenever \( p \) and 10 are relatively prime (which is the case whenever \( p > 5 \)). Thus \( p \) divides 9 . . . 9 (p-1 9's). Thus \( p \) divides 9 (1 . . . 1) (p-1 1's). Therefore, since \( p \) does not divide 9, all prime numbers \( p \) (p > 5) divide 1 . . . 1 (p-1 1's).

Also solved by Robert Bart, Matthew Fountain, Harry Zaremba, David Freeman, Ernest Thiele, and Frank Carbin.

**FEB 3** Given an ellipse truncated by two circular arcs tangent to it.

1. Derive a general expression for the radius of the tangent circle in terms of the semi-axes of the ellipse, \( a \) and \( b \), and the coordinate of the center of the circle \( h \).
2. The largest circle that can be inscribed for a given ellipse has radius \( b \). Show that there is no smaller circle and find its radius and center in terms of \( a \) and \( b \).

Karl Schoenhehr found this problem to his liking:

The equation of an ellipse with reference to a set of rectangular axes passing through the center \( O \) of the ellipse is

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

The equation of a circle with reference to the same set of axes, whose radius is \( r \) and whose center is on the \( x \)-axis a distance \( h \) from \( O \) is

\[ r^2 = y^2 + (x - h)^2 \]

The stipulation that the ellipse and the circle be tangent at a point \( C(x, y) \) requires that \( r \) be normal to the tangent to the ellipse at that point. The slope of the tangent is found by differentiating (1). Therefore,

\[ m = \frac{dy}{dx} = -\frac{b^2x}{a^3y} \]

and the slope of the radius \( y(x - h) \) is, by a well-known relation,

\[ m' = -\frac{1}{m} = \frac{a^2b^3}{y^2} \]

Therefore, in addition to (1) and (2), the following relation between \( x \) and \( h \) holds:

\[ x - h = \frac{b^2x}{a^3} \]

Combining (1) and (2) and (5) and reducing, we get

\[ r = b \sqrt{1 - \frac{h^2}{a^2}} \]

and