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Puzzle Corner
Allan J. Gottlieb

The All-State Crossword

Allan J. Gottlieb, '67, is associate research professor of mathematical sciences at the Courant Institute of Mathematical Sciences, New York University; he studied mathematics at M.I.T. and Brandeis. Send problems, solutions, and comments to him at the Courant Institute, New York University, 251 Mercer St., New York, N.Y. 10021.

This month's problems were selected just as the Space Shuttle *Columbia* was making its historic first flight, and they include questions about digging a hole to the center of the earth and falling from space into the ocean. Fortunately, my only contacts with NASA have been as a computer expert and not as a soothsayer.

Problems

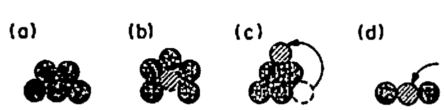
JUL 1 Emmett Duffy would like South to lead and make all seven remaining tricks:

	♠ J 8 6	
	♥ A J 4	
	♦ A	
	♣ —	
♠ K		♠ 10 7 2
♥ K 8		♥ 9 6
♦ 9		♦ 4
♣ K 8 6		♣ 9
	♠ A 9	
	♥ Q 7	
	♦ —	
	♣ J 10 3	

JUL 2 A two-part problem from Anthony Standen:
What is the smallest multiple of nine that has no odd digit? For the benefit of hair-splitters, I may say that it is to be in base 10; but if other bases are allowed, what then is the answer? And how many ways are there of doing it?

JUL 3 Eric Piehl wonders about the theoretical limit for high-diving competitions. A falling person reaches a terminal velocity of about 55 m/sec. Entering feet first, one can survive falls into water at speeds of up to 34 m/sec. Starting from rest, what is the maximum distance one can fall before entering water, and survive? Assume that drag is proportional to velocity and that gravitational acceleration is 9.8 m/sec.

JUL 4 This problem first appeared in the *Review* as part of a Calibron Products, Inc., advertisement in the 1930s:
Five coins, arranged as in (a) below, are to be shifted into arrangement (b), using *only four* accurate sliding moves [such as the move shown in (c)]. There is no restriction on the position of arrangement (b) relative to (a), but the new location of any coin moved must be fixed by *definite* contact with two other coins: *estimated* contacts [to form straight lines, as in (d)] are not allowed. Move only one coin at a time, without lifting. Our analysis of this old puzzle shows that there are no less than 24 straightforward solutions—but they are amazingly elusive! How many can you find?



JUL 5 Our last regular problem is from Richard Fisher:

What is the smallest square matrix in which one can write the names of 50 U.S. states, horizontally or vertically, in crossword-puzzle fashion? The ground rules are: words must be written from left to right, or from up to down; two-word states are to be written as if only one word—i.e., WESTVIRGINIA; each state must be written exactly once; no two-letters can be horizontally or vertically adjacent unless they form part of a state name; and the completed puzzle must be connected—i.e., you cannot tuck an orphan state in a corner unless it's connected to the rest of the puzzle.

Speed Department

JUL SD1 Here is a problem from Smith Turner that Jules Verne might have enjoyed: A person wanted to dig a hole to the center of the earth. He did not care how small it was at the bottom (a point would be fine) but realized that it must be wider at the top to permit workers to get in and that it must have some slope so that dirt would not slide back in too easily. He decided on a circular cone. To make the sides 1° off from vertical, what diameter of hole is required at the top?

JUL SD2 Finally, Joe Horton sends a quickie that may lead to a form of population control:

In an effort to ease the population explosion, it might be useful to decide whether to reject or adopt the sexist notion that a family should continue to make babies until one of the desired sex showed up. It has been argued that this must lead to large families. Question: If couples agreed to stop after a child of the desired sex had been born, what would be the average family size? For the sake of the problem, assume that boys and girls are equally likely to be born; and that twins don't occur.

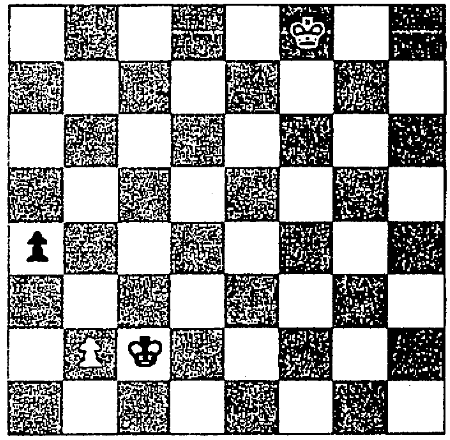
Solutions

FEB 1 Given the diagram at the top of the next column, White to move and win.

Gene Janulis cleverly avoided the drawing trap hidden in this problem. He writes:

It appears that White easily wins the Black pawn with K—B3, K—N4, KxP. But Black has the drawing resource 1. . . . P—R6!, which draws after both 2. PxP K—K2, 3. K—B4 K—Q3, and the Black king reaches QR1 and 2. P—N4 K—K2, 3. K—N3 K—Q3, 4. KxP K—B3, 5. K—R4 K—N3. However, White wins with 1. K—N1! P—R6 (the best), 2. P—N3! (P—N4 draws, as we shall see) K—K2, 3. K—R2 K—Q3, 4. KxP K—B4, 5. K—R4 K—N3, 6. K—N4 and wins (2. P—N4 would have made this move impossible). I did some research and found that F. Dedrle composed a similar problem with the Black king on KB3 in 1921.

Also solved by Matthew Fountain (who found a recent article by George Koltanowsky that featured Dedrle's problem), David Freeman, George Farnell, James Landau, Edwin McMillan, G. Sharman, Elliot



Roberts, Robert Bart, and the proposer, Bob Kimble.

FEB 2 Does every prime number above 5 divide evenly into a one-less string of 1s? For example, 7 divides 111,111; 11 divides 1,111,111,111; and 13 divides 111,111,111,111. Does 17 divide 1,111,111,111,111,111? 19 divide 18 1s. Etc.?

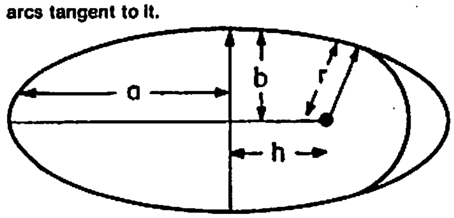
A joint effort by Steve Feldman and Pierre de Fermat shows that the answer is yes. Fermat's theorem (actually one of his many theorems) says that

$x^{p-1} = 1 \bmod p$

whenever x and p are relatively prime. Specifically, $10^{p-1} - 1$ is divisible by p whenever p and 10 are relatively prime (which is the case whenever $p > 5$). Thus p divides 9 . . . 9 (p-1 9s). Thus p divides 9 (1 . . . . 1) (p-1 1s). Therefore, since p does not divide 9, all prime numbers p ($p>5$) divide 1 . . . 1 (p- 1 1s).

Also solved by Robert Bart, Matthew Fountain, Harry Zaremba, David Freeman, Ernest Thiele, and Frank Carbin.

FEB 3 Given an ellipse truncated by two circular arcs tangent to it.



1. Derive a general expression for the radius of the tangent circle in terms of the semi-axes of the ellipse, a and b, and the coordinate of the center of the circle h.
2. The largest circle that can be inscribed for a given ellipse has radius b. Show that there is also a smallest circle and find its radius and center in terms of a and b.

Karl Schoenherr found this problem to his liking:

The equation of an ellipse with reference to a set of rectangular axes passing through the center O of the ellipse is

$$x^2/a^2 + y^2/b^2 = 1 \qquad (1)$$

The equation of a circle with reference to the same set of axes, whose radius is r and whose center is on the x-axis a distance h from O is:

$$r^2 = y^2 + (x - h)^2 \qquad (2)$$

The stipulation that the ellipse and the circle be tangent at a point C(x,y) requires that r be normal to the tangent to the ellipse at that point. The slope of the tangent is found by differentiating (1). Therefore,

$$m = dy/dx = -b^2/a^2 \cdot x/y, \qquad (3)$$

and the slope of the radius $y/(x - h)$ is, by a well-known relation,

$$m' = -1/m = a^2/b^2 \cdot y/x. \qquad (4)$$

Therefore, in addition to (1) and (2), the following relation between x and h holds:

$$x - h = (b^2/a^2)x. \qquad (5)$$

Combining (1), (2), and (5) and reducing, we get

$$r = b\sqrt{1 - h^2/(a^2 - b^2)}, \text{ and} \qquad (6)$$

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$$r = b/a\sqrt{a^2 - (1 - b^2/a^2)x^2}. \qquad (7)$$

Equation (6) is the general equation for r in terms of the independent variable h, and (7) is the general equation for r in terms of x as independent variable. In order to solve the second part of the problem, consider (7): inspection shows that r will be maximum when the second term under the square root is zero. This will be true when x equals zero. Similarly, inspection shows that r will be a minimum when x equals a. Introducing these values into the equation, we get

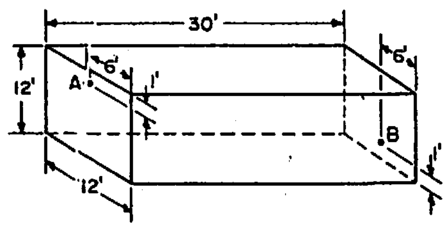
$r_{max} = b$, and

$r_{min} = b^2/a$,

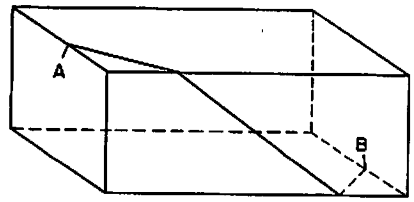
the latter of which is the solution to the second part of the problem.

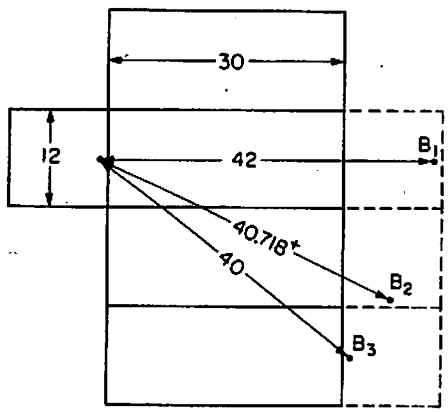
Also solved by David Freeman, Harry Zaremba, Matthew Fountain, John Prussing, Richard Kruger, P. Bonomo, Irving Hopkins, Joel Storch, James Lefferts, Norman Wickstrand, Winthrop Leeds, and the proposer, Winslow Hartford.

FEB 4 Point A represents a spider and point B a fly, on opposite walls of a room. What is the length of the shortest path the spider can follow to reach the fly? (The spider can only travel on solid surfaces—the walls, floor, and ceiling.)



Robert Bart noted that if you "unfold" the room you get the first diagram below. Thus there are three straight-line solutions (corresponding to three ways of entering the opposite face), the shortest of which is 40 feet. The second diagram shows this solution when the room is "refolded."





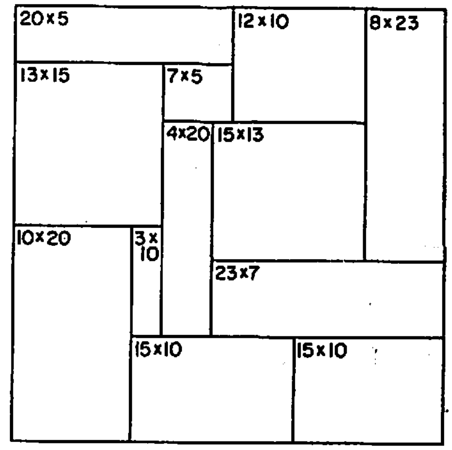
Also solved by Richard Kruger, John Prussing, Matthew Fountain, Harry Zaremba, Albert Hayes, Jordan Wouk, Raymond Gaillard, Joseph Friedman, Dennis Wood, Marion Weiss, David Freeman, Alexander Borsanyi, and Elliott Roberts.

FEB 5 A rectangular leaded glass window was made up of 12 rectangular pieces of glass having the following dimensions in inches:

3 × 10	5 × 20	10 × 12	10 × 20
4 × 20	7 × 23	10 × 15	13 × 15
5 × 7	8 × 23	10 × 15	13 × 15

What were the proportions of the window, and how were the pieces of glass arranged? (There were no gaps or overlappings.)

Dennis Sandow and Robert Bart found a solution for a square 40" × 40" and claim that no solutions exist for rectangles 50" × 32", 64" × 25", and 80" × 20". A partial solution was submitted by Winslow Hartford. Mr. Sandow's solution follows; Mr. Bart's is a mirror image.



Better Late Than Never

1980 A/S 1 George Farnell has responded.
N/D 5 Naomi Markovitz has responded.

Proposers' Solutions to Speed Problems

SD1 No way. The sides of the hole will be vertical no matter how big the hole is at the top.

SD2 The average number of children per family is $\Sigma nP(n) = \Sigma n/2^n = 2$, where n is the number of children in a given family and P(n) is the probability of that happening.

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