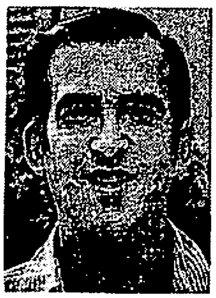
Puzzle Corner
Allan J. Gottlieb, '67

How to Play Red Dog



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As you may have known, I have been on leave from York College of the City University of New York since September, 1979, working on the ultracomputer project at the Courant Institute of Mathematical Sciences of New York University. Just yesterday I accepted an offer of a permanent position at Courant. This choice between C.U.N.Y. and N.Y.U. was one of the most difficult decisions in my life, for I have enjoyed working at both institutions. In the last analysis, however, the active graduate student and research programs at Courant tipped the scales their way.

Now that New York's financial situation has improved and the permanent York College campus is under construction, it looks as if the precarious times my colleagues and I suffered through in the late 1970s are finally over. I wish them all well, especially my fellow-members of the Mathematics Department; they certainly deserve it.

Problems

APR 1 We begin with a bridge problem from N. Piffenberger, who wants South to lead and make all remaining six tricks with hearts as trump:

	♠K 9 ♥K ♦4 ♣A Q	
♠S J 10 ♥— ♦2 ♣J 6		♠A 8 ♥— ♦— ♣K 5 4 3
	♠— ♥A Q J ♦3 ♣9 7	

APR 2 Smith Turner wants some gambling advice. In the game of Red Dog, a player is dealt four cards and bets that he can beat a fifth one by having a higher card in the same suit. Bets won or lost are taken from or added to the pot. What is the probability of winning (before looking at the four cards)? How many of the 48 outstanding cards should a player be able to beat in order to justify betting? (For simplification, assume a two-handed game in which each player, after looking at his/her cards, must either pass (without penalty) or bet the exact amount then in the pot.)

APR 3 Matthew Fountain proposes the following scheduling problem, which he writes is "a real-life puzzle submitted to me in 1946 by an official scorer of a bowling league in Washington, D.C.": Make a round-robin schedule for 12 teams which bowl on six double alleys, two teams to each double alley per night. Each team is to meet every other team once, with no team bowling either more than twice or twice in a row on any double alley.

APR 4 This problem first appeared in *Technology Review* for January 1939 in an advertisement for Calibron Products, Inc.:

Two well insulated compartments, filled with a "perfect" gas, are maintained at absolute temperatures T_1 and T_2, respectively. If a large tube connects the compartments, the pressures (P_1 and P_2) naturally tend to equalize. But (believe it or not) if the proportions of the tube are suitably reduced, the dynamical theory of gases indicates that a steady state will be reached in which the relationship $P_1/P_2 = (T_1/T_2)^{\frac{1}{2}}$ is approached. Can you verify and explain this formula?

APR 5 Our last regular problem is a cryptarithmetic problem from Abe Schwartz: Substitute digits for letters to make the following addition correct:

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  O N E
  T W O
+ S I X
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N I N E
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Some additional restrictions: "ONE" is divisible by 1, "TWO" by 2, "SIX" by 6, and "NINE" by 9; and "NINE" > "SIX" > "TWO" > "ONE."

Speed Department

APR SD 1 A musical "quicky," from Smith Turner, who writes: 50 years ago a rite of autumn at our country club in Texas was a treasure hunt. Couples were given clues that led to further clues of increasing difficulty. One year four couples at the last clue were utterly baffled when another couple arrived, took one look at the strip below, and were off. Where did they go to win the treasure?

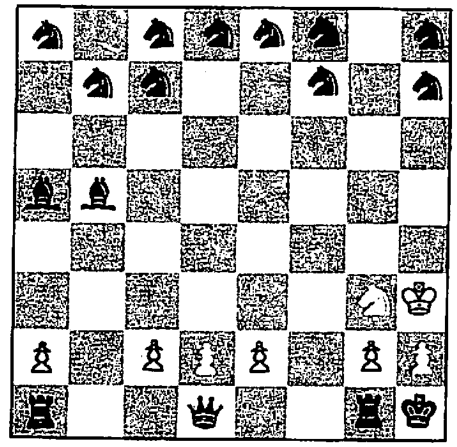
APR SD 2 We close with an amusing problem from James Landau: "Puzzle Corner" is occasionally accompanied by a box labelled "This Space Available for Your Listing." Since I work the problems on sheets of paper torn from old computer listings, I wish to take advantage of the publisher's kind offer. The box is 2¼ inches wide and 2⁹⁄₁₆ inches tall. My computer paper is 14⅞ inches wide and 11 inches tall. Question: How many times do I have to fold the paper in order to make it fit in the box?

Solutions

NOV/DEC 1 Which reachable chess positions require the greatest number of moves to be

reached?

The proposer submitted a position which he claims requires at least 102½ moves to reach. The position is:



and his argument is as follows: since Black has ten knights, eight pawns must have knighted (this requires at least 40 moves). All eight of the Black pawns must have arrived at either QN7 or KB7, which implies that the pawns captured at least eight White men en route to the seventh rank. But White has lost only eight men. Thus all eight knightings must have occurred on QN8 or KB8. Now consider the current position of the ten knights. The two original knights started on KN1 and QN1; the other eight started on the White squares QN8 or KB8. For these latter knights, three moves are required to reach a Black square on the second rank, four to reach a White square on the second or first ranks, and five to reach a Black square on the first rank. It is not hard to see that the minimal number of knight moves is 5 + 5 + 4 + 4 + 4 + 4 + 4 + 3 + 2 + 2 (the two 2s are for the original knights to reach KB1 and QB1). This gives a total of 75 moves for the Black pawns and knights. The rooks require three moves each, the bishops two each, the queen three, and the king 12 (the king can only approach the White pawns via QR6). Thus Black has made at least 102 moves. Since Black is in check, White made the last move. (Unfortunately, Clyde Kruskal and I noticed that the position is in fact unreachable, since the White King's bishop could neither have moved nor have been captured on its home square. However, we still feel that the solution is quite clever and hope that a small change can repair it.)

Winslow Hartford has also responded.

NOV/DEC 2 Suppose we are given the sequence
$S0 = \{1^n, 2^n, 3^n, \ldots\}$
and form the difference sequence
$S1 = \{2^n - 1^n, 3^n - 2^n, \ldots\}$
by taking the difference between consecutive elements in S0. Now form S2 as the difference sequence of S1, etc. Show that
$Sn = \{n!, n!, n!, \ldots\}$.

I have no choice but to use Joseph Keilin's solution, since he cites a book written by two Courant-trained mathematicians:

This is a standard result of numerical methods (see, for example, Isaacson and Keller, *Analysis of Numerical Methods*, pp. 261-62). For equally spaced data points $x_0, x_1, \ldots x_n$, let
$D^1f(x_0) = f(x_1) - f(x_0)$
$D^nf(x_0) = D^{n-1}f(x_1) - D^{n-1}f(x_0)$
$f^{(n)}(x) = df^n/dx^n$. Then
$D^nf(x_0) = (x_1 - x_0)^n f^{(n)}(x)$ for some x, $x_0 < x < x_n$.
For the special case of $f(x) = x^n$ and $x_i - x_{i-1} = 1$, the result is
$D^nf(x_0) = n!$

Also solved by David Freeman, Winslow Hartford, Gerald Blum, and the proposer, Ten Clinkenbeard.

NOV/DEC 3 Five people had consecutive appointments with an income tax expert to help them fill out their 1040 forms and schedules. The electrical engineer had income from a savings account. The man who had a profit trading commodities was taking educational expenses as a deduction. When the man who contributed to a charity was leaving he met the taxpayer with dividend income. The biochemist is deducting interest on a mortgage. The computer programmer uses an SC-40 calculator. When the man with the medical expense left, he met the man with the educational deduction. The man with four dependents owns a C1400 calculator. The man with three dependents is claiming storm damage as a deduction. The man with the charitable deduction followed the physicist. The man with five dependents exchanged amenities with the owner of the SR-50. When he looked at the tax expert's calendar, the man with the MX-140 noticed his name was next to that of the man with three dependents. The man with seven dependents sold some real estate for profit. The mathematician has six dependents. The income tax expert still had more than one scheduled appointment after he met the man with dividend income. Each man had a profession, owned a calculator, had a deductible expense, had some number of dependents, and had a second source of income. Who won money in a contest? Who owned an HP-45 calculator?

The following is from C. Muench:

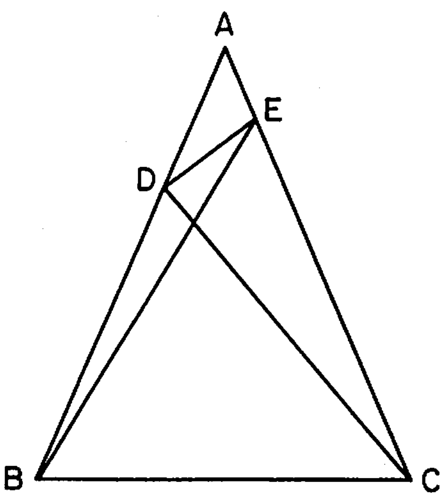
The physicist won the money in the contest and the mathematician owned the HP-45 calculator. A complete list of the order of visits and who owns what follows:

1. The physicist owns the SR-50, is deducting storm damage, has three dependents, and won money in a contest.
2. The electrical engineer owns the MX-140, is deducting charitable contributions, has five dependents, and has an income from savings.
3. The biochemist owns the C-1400, is deducting interest on a mortgage, has four dependents, and has income from dividends.
4. The computer programmer owns the SC-40, is deducting medical expenses, has seven dependents, and is reporting income from real estate.
5. The mathematician owns the HP-45 and has an educational deduction, six dependents, and profits from commodities trading.

But why are they all, including the income tax expert, men?

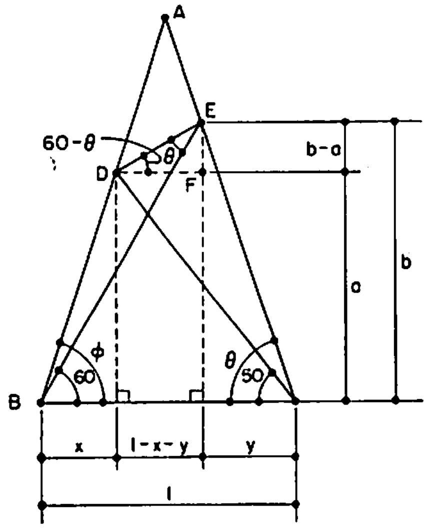
Also solved by Joseph Keilin, Harry Zaremba, David Freeman, R. Zlatoper, Michael Jung, Marcia Martin, Gardner Perry, Chris Ziegler, Steve Feldman, Marion Weiss, Mike Bercher, Harry Hazard, Philip Lang, W. McGuinness, and Winslow Hartford.

NOV/DEC 4 Given angle EBC = 60°, angle DCB = 50°, and AB = AC. Find angle DEB:



I received good looking solutions from Harry Zaremba and Norman Wickstrand, who obtained formulas that at first glance appeared different. However, they both agree on the special case solved by the proposer, Sheldon Katz, who determined that the answer is 30° when angle BAC is 20°. The solutions of Zaremba and Wickstrand follow in order:

Zaremba: Since AC = AB, triangle ABC is isosceles.



In the figure, if we let angle DEB equal θ, then angle EDF equals $60° - \theta$. Also, let BC = 1 and angles ABC and ACB equal ϕ. Then,

$b = y \tan\phi = (1 - y)\tan 60$, from which

$y = (\tan 60)/(\tan\phi + \tan 60) = \sqrt{3}/(\tan\phi + \sqrt{3})$, and

$b = (\sqrt{3}\tan\phi)/(\tan\phi + \sqrt{3})$.

Similarly,

$a = x\tan\phi = (1 - x)\tan 50$, from which

$x = (\tan 50)/(\tan\phi + \tan 50)$, and

$a = (\tan\phi \tan 50)/(\tan\phi + \tan 50)$.

Thus,

$$b - a = [\tan^2\phi\,(\sqrt{3} - \tan 50)]/(\tan\phi + \sqrt{3})(\tan\phi + \tan 50) \qquad (1)$$

and

$$1 - x - y = (\tan^2\phi - \sqrt{3}\tan 50)/(\tan\phi + \sqrt{3})(\tan\phi + \tan 50). \qquad (2)$$

From the figure,

$$\tan(60 - \theta) = (b - a)/(1 - x - y). \qquad (3)$$

Substituting (1) and (2) into (3) and using the following identity,

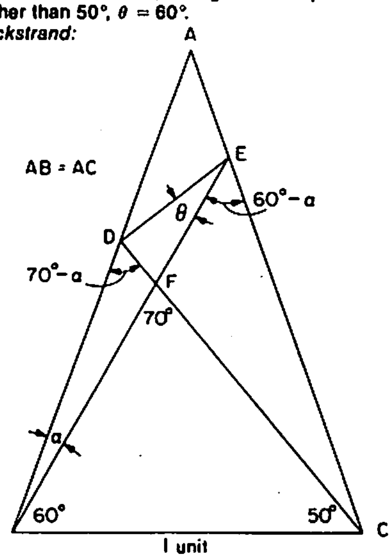
$\tan(60 - \theta) = (\tan 60 - \tan\theta)/(1 + \tan 60 \tan\theta)$,

we get for angle DEB:

$$\theta = \tan^{-1}\left[\frac{(\tan^2\phi - 3)\tan 50}{\tan^2\phi(4 - \sqrt{3}\tan 50) - \sqrt{3}\tan 50}\right]$$

Hence the value of θ is a function of the base angles ϕ, and we note that when $\phi = 60°$, $\theta = 0°$; and when $\phi = 90°$, $\theta = \tan^{-1}[(\tan 50)/(4 - \sqrt{3}\tan 50)] = 31°37'3.97''$. Also, if angle DCB equalled 60° rather than 50°, $\theta = 60°$.

Wickstrand:



BF = (sin 50)/(sin 70) = .815208

α	0	5°	10°	15°	20°	25°	30°*
θ	0	13°4'	19°28'	27°37'	30°	31°14'	31°37'

CF = (sin 60)/(sin 70) = .921605
DF = .815208 sin x/sin (70 − α)
EF = .921605 sin (10 + α)/sin (60 − α)
$DE^2 = DF^2 + EF^2 - 2DF \cdot EF \cos 70$
$\sin \theta = \sin 70 \cdot DF/DE$.

Hence the table at the top of this column, including α = 30° as the extreme case where AB = AC → ∞. From that we determine that angle DEB (or θ) can be anything from 0° to nearly 31°37′.

Edward Dawson obtained yet another formula (this one in terms of B = angle ABC) that gives 30° when angle BAC = 20° (B = 80). Dawson's formula states that the cotangent of angle DEB is
[sin 60 · sin (B + 50) · sin (B − 50)]/
[sin 50 · sin 70 · sin (B + 60)] − cot 70.

Also solved by Winslow Hartford, Steven Goyeche, Andrew Combie, Joseph Keilin, Eugene Sard, David Freeman, Valeta Wheeler, John Fogarty, L. Upton, and Gerald Blum.

NOV/DEC 5 Replace each letter by a unique decimal digit to obtain a correct multiplication:

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      S I N K
      T H E M
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D E E P D E E P
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Emmet Duffy among others found this problem relatively easy:
Dividing the product by DEEP results in the number 10001 whose factors are the prime numbers 73 and 137. Then one of the numbers SINK or THEM is a multiple of 73 and the other is a multiple of 137. As K when multiplied by M results in P, differing from K and M, neither K nor M can be 0 or 1. Also, no digit is repeated in SINK or THEM. A list will be made of multiples of 73 and 137, which have four digits, none repeated, and do not end in 0 or 1. A hand calculator with repetitive addition is convenient for this purpose. There will be 27 multiples of 137 and 44 multiples of 73. Now multiply any of the 27 multiples by any of the 44 multiples, provided that all eight digits of the two four-digit numbers are different and that the two digits at the right when multiplied result in digit P which differs from the other eight digits. There will be 17 multiplications, of which only one is the answer:
SINK = 5069; THEM = 2847; and DEEPDEEP = 14431443.

Also solved by Avi Ornstein, Steven Goyeche, Winslow Hartford, Gerald Blum, David Freeman, Joseph Keilin, Dennis Sandow, Frank Carbin, Harry Garber, Allen Wiesner, Lester Nathan, W. McGuinness, Steve Feldman, and Harry Hazard.

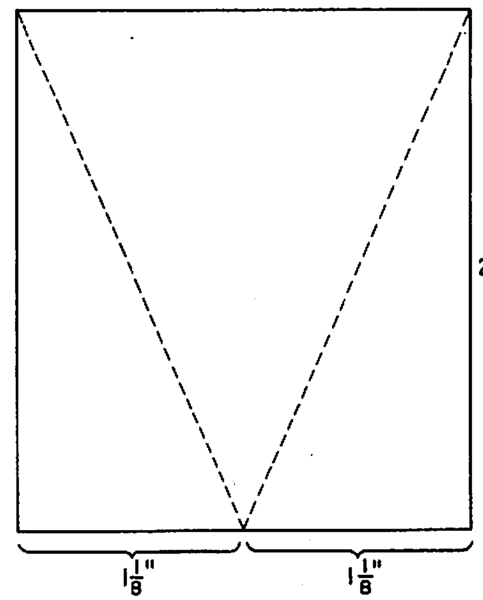
Better Late Than Never

MAY 1 Martin Lubell notes that this problem was first composed by Arthur MacKenzie in 1904.
MAY 3 Michael Jung and Frank Rubin offer "amyotonia." Rubin also found "oogonia," "aposiopesia," and "alopecia areata." Gerald Blum offers "aalii."
A/S 3 Joel Freilich claims that the square is unique if an obtuse angle can be found with two of the points on one ray and the other two points on the other ray.
A/S 4 Michael Jung has responded.
OCT 4 Hugh Tett has responded.
Y1970 Mike Bercher has responded.
NOV/DEC SD 1 Dennis Sandow feels that a self-employed master carpenter would have cut the wood to leave the most useful remaining scrap.

Proposers' Solutions to Speed Problems

APR SD 1 The music was recognized as from "Tea for Two," in the then-current musical comedy "No, No Nanette," and the words corresponding to the seven notes are, "tea for two and two for tea." Thus the couple drove off to the tee for hole two (of the country club's golf course). Since this was before wood and plastic tees, each tee had a box of sand used for scrubbing the ball and then teeing it up. The couple proceeded to dig in the sand for the treasure.

APR SD 2 Twice. Fold it in half twice, top to bottom each time, giving a rectangle 14⅞ inches wide by 2¾ inches tall. Let it open into a V-shape and stand it on end. It will fit into the box.



Each of the dotted lines is 2.79+ inches long, or about 1/20 inch longer than the computer paper. Each sheet has a thickness of about 1/200 inch.