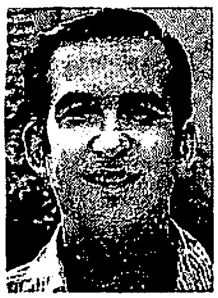
Puzzle Corner
Allan J. Gottlieb

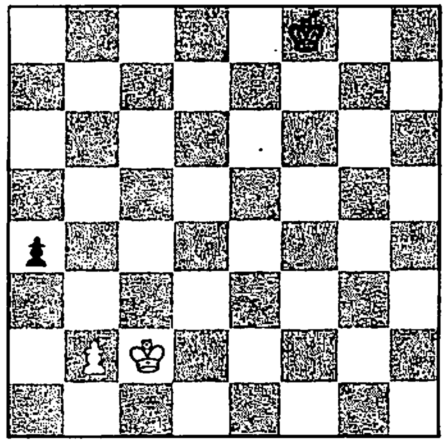
The Spider and the Fly



Allan Gottlieb is associate professor of mathematics at York College of the City University of New York; he studied mathematics at M.I.T. (S.B. 1967) and Brandeis (A.M. 1968, Ph.D. 1973). Send problems, solutions, and comments to him at the Department of Mathematics, York College, Jamaica, N.Y. 11451.

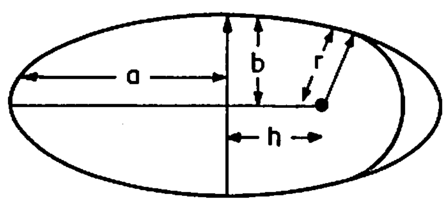
I have a story to tell. This year I have been working on the ultracomputer project at the Courant Institute of New York University (an ultracomputer is our mathematical model for a very-large-scale parallel computer), and every Thursday the faculty members go out for an ultralunch. Those of you familiar with Greenwich Village will not be surprised to hear that we are often leafletted as we walk. One Thursday a young man handed me a sheet of paper which I folded up to read later as we were talking a little shop. I was the only one in our group to receive the handout, so one of my colleagues asked me about it. Well, it turned out to be an announcement of a fraternity party, and I must have been the only one who looked young enough to be eligible. Of course I sort of floated through lunch; indeed, the event made my whole week. When I later recounted this tale to some of the graduate students, I was the object of some skepticism by a passing faculty member: Was the fraternity seeking youth or immaturity? Who knows?

FEB 1 We begin with a chess problem that Bob Kimble attributes to Robert Breiger: Given the diagram below, White is to move and win.



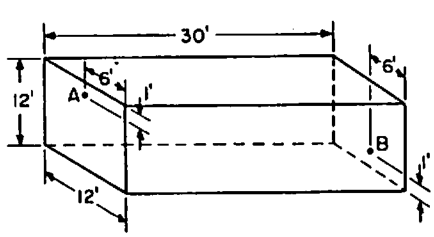
FEB 2 Next we have a number theory problem from Jerome Taylor: Does every prime number above 5 divide evenly into a one-less string of 1s? For example, 7 divides 111,111; 11 divides 1,111,111,111; and 13 divides 111,111,111,111. Does 17 divide 1,111,111,111,111,111? 19 divide 18 1s? Etc?

FEB 3 A geometry problem from Winslow Hartford, who writes: My daughter and son-in-law, who exhibit minerals, want to design a display shelf in the shape of an ellipse truncated by two circular arcs tangent to it, as illustrated.



1. Derive a general expression for the radius of the tangent circle in terms of the semi-axes of the ellipse, a and b, and the coordinate of the center of the circle h.
2. The largest circle that can be inscribed for a given ellipse has radius b. Show that there is also a smallest circle and find its radius and center in terms of a and b.

FEB 4 Will Lidell has sent us a copy of "the spider and the fly problem" that first appeared in *Link-Belt News* in August, 1935. Point A represents a spider and point B a fly, on opposite walls of a room. What is the length of the shortest path the spider can fol-



low to reach the fly? (The spider can only travel on solid surfaces — i.e., the walls, floor, and ceiling.)

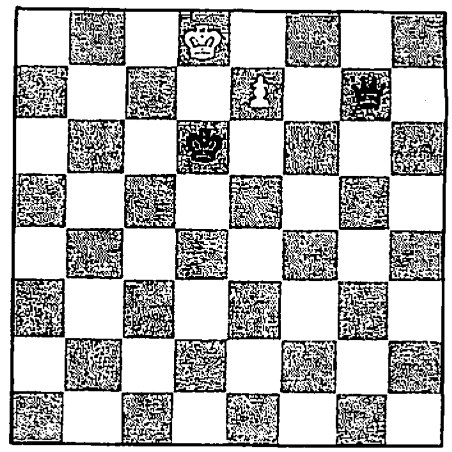
FEB 5 We end with a problem from J. L. Friedman that Calibron Products, Inc., printed in Technology Review in 1938: A rectangular leaded glass window was made up of 12 rectangular pieces of glass having the following dimensions in inches:

3 × 10	10 × 12
4 × 20	10 × 15
5 × 7	10 × 15
5 × 20	10 × 20
7 × 23	13 × 15
8 × 23	13 × 15

What were the proportions of the window, and how were the pieces of glass arranged? (There were no gaps or overlappings.)

Speed Department

FEB SD 1 A chess speed problem from Peter Sorant, a dorm-mate of mine back at M.I.T.:
White is to move and draw.



FEB SD 2 This cryptarithmetic quicky is from Winthrop Leeds: Replace each letter by a unique digit so that a valid multiplication and addition is obtained. Only the nine digits may be used, and *zero is excluded.*

```
  U P
× A
  D O
–R E
  M I
```

Solutions

OCT 1 South is on lead with hearts trump and is to take six of the remaining seven tricks against any defense:

North:
♠ 8 3
♥ J 5
♦ 6 4 2
♣ —

West:
♠ J 7 6
♥ 9
♦ 9 5
♣ J

East:
♠ 5
♥ 6
♦ Q 10
♣ K 8 4

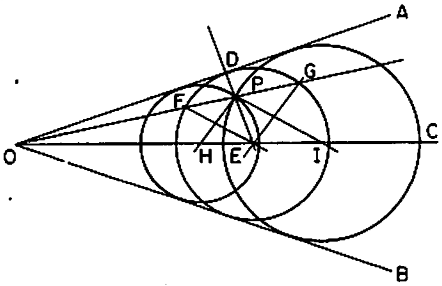
South:
♠ K 4
♥ —
♦ A J
♣ Q 7 6

John Boynton sent us the following plan of attack:
The only workable initial lead is to ruff a club in dummy. Of the possible second leads (from dummy), only the trump lead carries; but the slough in South is the interesting maneuver. By discarding the ♠K, declarer effectively unblocks his hand so he will be later locked in dummy where the good tricks are. Thus, after ruffing the initial club and drawing a round of trump (discarding the ♠K), declarer takes the diamond finesse and plays the remaining diamond (winning both tricks, depending on East's play). Declarer is now in South with only a small spade and the clubs. A spade lead now either wins in dummy, if West ducks, or loses to the ♠J, whereupon West leads to the promoted ♠8 in dummy. Finally, the good diamond becomes the sixth trick (two hearts, three diamonds, and a spade).

Also solved by George Holderness, Doug Van Patter, Al Salish, William Katz, R. Bart, Matthew Fountain, John Woolston, Chester Claff, Robert Park, Winslow Hartford, Manuel Matnick, Dorothy Bryant, Smith Turner, and the proposer, Emmet Duffy.

OCT 2 Given an angle and a point within an angle. Using only a straightedge and compass, construct the two circles that pass through the point and are tangent to the lines forming the sides of the angle.

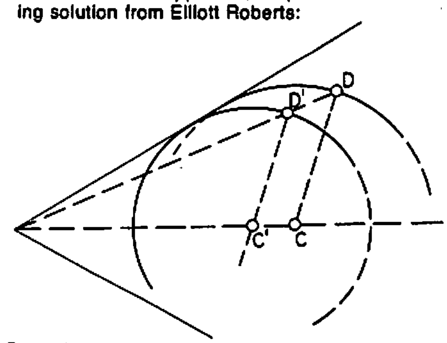
Emmet Duffy (by coincidence the proposer of the last problem) has sent us several carefully drawn constructions for this geometry problem, as well as noting a connection between this problem and 1978 J/J 5. I print below the solution that he prefers:
Referring to the drawing, let the vertex be O, the sides OA and OB, and the point P. Construct the bisector OC of angle AOB. Through point P, construct a line perpendicular to OA, intersecting OA at D and OC at E. With ED as a radius and E as a center, draw a circle tangent to OA at D and also tangent to line OB. Draw a straight line through O



and P intersecting the circle at F and G. Draw EF and EG. After drawing EF and EG, simply draw through point P lines parallel to EF and EG, intersecting OC at H and I. With H as a center and HP as a radius and I as a center and IP as a radius, draw the desired circles.

The following artifice eliminates much geometry. Assume that a photograph of the drawing has been made and a positive transparency is placed in an enlarger-reducer. Assume a projection is placed over the original drawing keeping point O and the lines of the angle in the projection coincident with the original drawing. As the projection is enlarged, point F in the projection can be made to coincide with point P of original drawing, and point E of the projection will move to the right along line OC. Mark this point as I on original drawing. It will be the center of the desired large circle, the radius of which will be the distance IP. In a similar manner if the projection is reduced until point G of the projection coincides with point P of original drawing, point E of the projection will move left along line OC. Mark this position as point H on original drawing. It will be the center of the desired small circle and radius will be distance HP. Points I and H can be found geometrically using Proposition IX of Book III: If a line is drawn through two sides of a triangle parallel to the third side, it divides the sides proportionally. Construction is as follows: draw a line through P, parallel to EF and intersecting OC at I, and draw a line through P, parallel to EG and intersecting OC at H. Then OP/OF = OI/OE; OP/OG = OH/OE; and I and H are the centers and IP and HP are the radii of the desired circles.

For a different approach, we present the following solution from Elliott Roberts:



Draw the center line of the angle. Inscribe a random circle with its center at C′. Find P′ on a radial line from the vertex of the angle. Draw P′C′. Draw PC parallel thereto. C is the center of one circle. Proceed similarly for the other circle.

Also solved by D. Freeman, Harry Maynard, Matthew Fountain, William Rapp, John Woolston, John Joseph, Raymond Gaillard, R. Bart, Harry Zaremba, Norman Wickstrand, Richard Hess, John Gray and J. Younkin, and the proposer, Jon Davis.

OCT 3 Consider summing each of the following eight arithmetic progressions:

$1 + 2 + 3 + \ldots + n$
$1 + 3 + 5 + \ldots + (2n - 1)$
$1 + 4 + 7 + \ldots + (3n - 2)$
$1 + 5 + 9 + \ldots + (4n - 3)$
$1 + 6 + 11 + \ldots + (5n - 4)$
$1 + 7 + 13 + \ldots + (6n - 5)$
$1 + 8 + 15 + \ldots + (7n - 6)$
$1 + 9 + 17 + \ldots + (8n - 7)$

For each of these series, when n = 1 the sum is 1 = 1^2. Find the one series for which there is no $n > 1$ so that the sum is a perfect square. You might then want to find the two series for which there is an n so that the sum is the perfect number 2305843008139952128.

Douglas Szper solved the first part as follows: Let

$S(1, n) = 1 + 2 + \ldots + n$
$S(2, n) = 1 + 3 + \ldots + (2n - 1)$
. .
$S(8, n) = 1 + 9 + \ldots + (8n - 7)$

Then we have for any d from 1 to 8

$$S(d, n) = 1 + (d + 1) + \ldots + ((n - 1)d + 1) = (1 + 1 + \ldots + 1) + (d + 2d + \ldots + (n - 1)d) = n + n(n - 1)d/2$$

Thus

$S(1, 8) = 6^2$
$S(2, 2) = 2^2$
$S(3, 81) = 99^2$
$S(4, 25) = 35^2$
$S(5, 6) = 6^2$
$S(6, 9) = 15^2$
$S(7, 2) = 3^2$

and hence the first seven series all yield perfect squares. Now assume that there is an n such that $S(8, n)$ is a perfect square, say k^2. Then $4n^2 - 3n - k^2 = 0$.

So for n to be integral, $9 + 16k^2$ must be a perfect square, say d^2, and thus $d^2 - (4k)^2 = 9$. But no two perfect squares differ by 9.

Frank Carbin found one series that gave the perfect number desired. The Euclid-Euler theorem states that any even perfect number equals

$2^{p-1}(2^p - 1)$,

so it seems prudent to try to factor our perfect number in that fashion. Indeed, it is equal to $2^{30}(2^{31} - 1)$, or

$$2^{61} - 2^{30} = 2.2^{60} - 2^{30} = 2n^2 - n, \text{ for } n = 2^{30}.$$

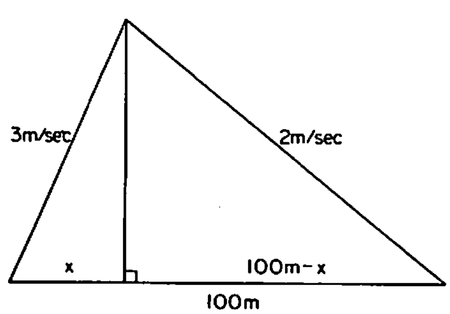
But $2n^2 - n = S(4, n)$, so the fourth series gives the number sought.

Finally, Matthew Fountain noted that series 1 gives all the partial sums that series 4 does, since $1 + 5 + 9 + \ldots = 1 + (2 + 3) + (4 + 5) + \ldots$

Also solved by Neil Hopkins, Edwin McMillan, Jim Landau, Harry Zaremba, Steve Feldman, Winslow Hartford, Emmet Duffy, D. Friedman, Richard Hess, and R. Bart.

OCT 4 A steam railroad runs parallel to a river 100 meters away. Unfortunately, the engine boiler has developed a small leak so that there is a loss of one liter of water per second for every ten meters per second the train travels. The conductor is dispatched to refill the train's water reservoir using a ten-liter bucket. He can walk three meters per second with the empty bucket but only two meters per second with the full bucket. What is the fastest constant rate of travel that the train can maintain under these circumstances?

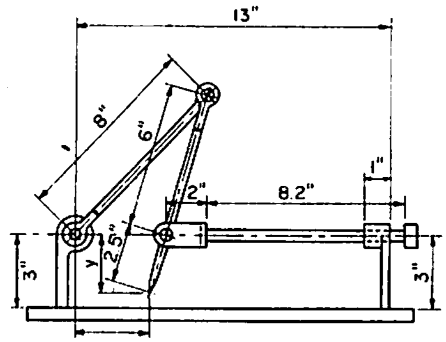
The following solution is from Winslow Hartford:



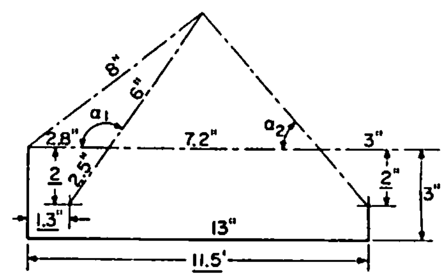
To keep the train moving at the fastest possible speed, it must use up 10 liters of water in the time required for the conductor to go to the river and return with 10 more liters. Since the train uses 1 liter per second for each 10 meters per second of speed, the train will travel 100 meters to use the 10 liters. The total time for the conductor is thus $(\sqrt{10{,}000 + x^2})/3 + [\sqrt{10000 - (100 - x)^2}]/2$. By setting the derivative equal to zero and solving for x, we find a minimum of 92.708 seconds for x = 62.320 meters. Thus the train travels 100 meters in 92.708 seconds or 1.07865 meters per second, and the conductor travels 224.69 meters in the same time — true dedication.

Also solved by Harry Zaremba, Marlon Weiss, Victor Newton, Douglas Szper, R. Bart, Steve Feldman, Richard Kruger, Smith Turner, Matthew Fountain, Robert Park, D. Freeman, Richard Hess, and Jim Landau.

OCT 5 Find the values of x and y at the two extreme positions for the device shown below.



Norman Wickstrand had very little trouble with this one:



$$\cos \alpha_1 = \frac{2.8^2 + 6^2 - 8^2}{2 \cdot 2.8 \cdot 6} = -.6 = \cos 126.87°$$

$x_1 = 2.8 + 2.5 \cos \alpha_1 = 1.3"$

$y_1 = 2.5 \sin \alpha_1 = 2"$.

$$\cos \alpha_2 = \frac{10^2 - 6^2 - 8^2}{2 \cdot 10 \cdot 6} = .6 = \cos 53.13°$$

$x_2 = 10 + 2.5 \cos \alpha_2 = 11.5"$

$y_2 = 2.5 \sin \alpha_2 = 2"$.

Also solved by Harry Zaremba, Douglas Szper, Frank Carbin, Avi Ornstein, R. Bart, Emmet Duffy, Edward Dawson, Richard Hess, Michael Jung, and the proposer, L. Steigler.

Better Late Than Never

MAY 3 Dan Pratt offers "amyotonia" and "oidia."
J/J 3 H. Moore has responded.
A/S 3 Mary Lindenberg has submitted a construction.
A/S 4 Robert Lutton and Dennis Sandow have responded.
A/S 5 Robert Lutton has responded.

Proposer's Solutions to Speed Problems

FEB SD 1 P—K8 promoting to a Knight (making a Queen loses to Q—QB2).

FEB SD 2

```
  17
 × 4
  68
 +25
  93
```