Puzzle Corner
Allan J. Gottlieb

A Prisoner Escapes, a Nobelist Wins

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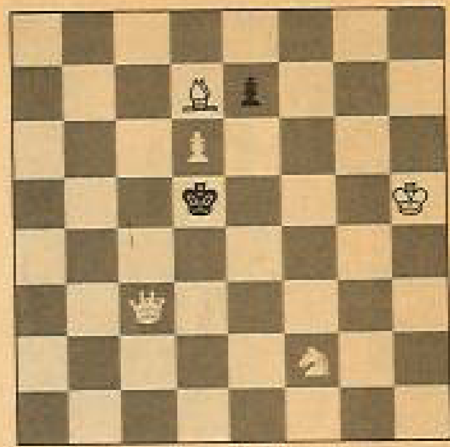
I have some very cheerful news to report from one of our faithful contributors. A few years ago I described a letter I received from a reader for whom an old mathematical spark was rekindled by "Puzzle Corner." He, therefore, decided to enter college after a prolonged absence from formal education. As a college professor, I have often taught adult students; but this was the only time to my knowledge that my role as puzzle editor contributed to their number. I can now congratulate that new Brown graduate, William Butler, S.B.

G. Sharmon has asked how a computer chess program would choose between several possible mating sequences. Generally short mates are preferred; and for sequences of the same length, whichever key move was tried first would be chosen.

Problems

A/S1 We begin this installment with a chess problem that Winthrop Leeds attributes to Geoffry Mott-Smith: given the diagram in the next column, White to mate in two.



A/S2 Frank Rubin has a number theory question: Consider a set of N distinct integers, the sum of any K of which is prime. What is the maximum possible value of N for K = 2, 3, 4, and 5?

A/S3 Here is a geometry problem that was posed as an M.I.T. undergraduate challenge a few years ago:
Given four distinct points that lie on (the boundary of) a square, construct the square. Under what conditions is the square uniquely determined?

A/S4 Theodore Goodman wants you to find all the solutions to this cryptarithmetic addition problem:

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THREE
  TWO
  TWO
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SEVEN
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A/S5 A novel algebraic problem from Draper Kaufman:
$(3 - \log x)/(2\log x) + \log d - \log b + 2\log c$
Solve for x; express your answer without any parentheses or numerals, using only one algebraic operation (addition, multiplication, exponentiation, etc.). You may use that operation as often as you like.

Speed Department

A/S SD1 Irving Hopkins has been playing around with the googol: Given the googol, $g = 10^{100}$, and the googolplex, $G = 10^g$, let $n! = g$ (approximately) and $N! = G$ (approximately). Find n and N.

A/S SD2 R. Crandall wonders why it is that any time a chessboard Knight moves on a path terminating at its original square, the number of moves is even.

Solutions

NS19 (nee 1979 D/J5) A solitaire game (called accordian, among other names) consists of dealing a deck, one card at a time, and then examining sets of four cards. If the four cards are of the same suit, the middle two are discarded. If the four cards are of the same value, all four are discarded. What are the odds of winning (no cards left)? What if the whole deck is laid out before starting?

This was an unusually easy NS problem, according to Jonathan Hardis, Steven Minsker,

Douglas Ell, Ed Friedman, Peter Steven, Harry Zaremba, and Judith Longyear. Mr. Ell notes that: If we let d_i = the amount the ith player starts with, after $i-1$ games he has $2^{i-1}d_i$. After i games he has $2^{i-1}d_i - (ND - 2^{i-1}d_i) = 2^i d_i - ND$. After N games he has $2^N d_i - 2^{N-i}DN$, which must equal D. Thus
$d_i = D/2^N(2^{N-i}N + 1)$.

M/A1 Find a hand where declarer's trumps are only A, 10 and dummy's only K, 7 but in which declarer can make 12 tricks against any opening lead except trump. You are allowed to specify all four hands.

The following solution is from Susan Kolodkin; she assumes that spades are trump and that the declarer is South:

	North	
	♠ K 7 ♥ A Q ♦ A K Q J 10 9 8 ♣ 9 8	
West		**East**
♠ 6 5 4 3 2 ♥ 2 3 ♦ 2 3 4 ♣ 2 3 4		♠ Q J 9 8 ♥ 4 5 6 ♦ 5 6 7 ♣ 5 6 7
	South	
	♠ A 10 ♥ K J 10 9 8 7 ♦ — ♣ A K Q J 10	

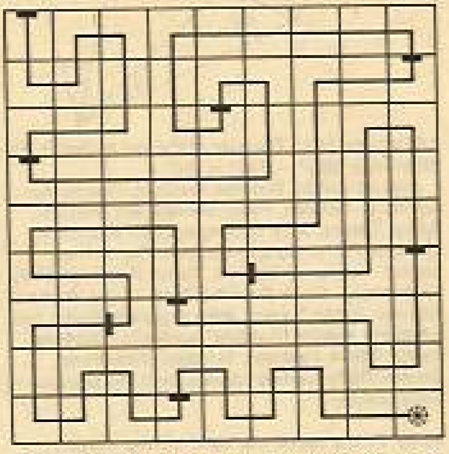
After any opening lead from East (other than trump), Declarer runs three diamonds from the dummy (only two if a diamond is led), sluffing two clubs and a heart from his hand. He then runs the ♥A and then the ♥Q, which he covers with the ♥K to get into his hand. (If a heart is led, however, the ♥A is played on the first trick.) Declarer then runs three clubs, sluffing one diamond off the board. (Again, if a club is led, only two clubs are played here and the ♥A is played to get back on the board.) Declarer then runs a third heart which is trumped by East and then overtrumped with the ♠7 on the board. Then run a diamond from the dummy which West should trump with the ♠8, allowing the Declarer to overtrump with the ♠10 and forcing East to undertrump. The Declarer then runs another heart, trumping on the board with the ♠K, and then a diamond back to the ♠A in the hand. This makes 12 tricks; the last trick is lost.

Also solved by Conrad Carlson, Carla Montgomery, John Woolston, Jack Mosinger, Kenneth Sawyer, Carey Rappaport, Harvey Fader, David Krohn, Smith Turner, Douglas Van Patter, Peter Steven, Winslow Hartford, Hal Hindman, Gardner Perry, and the proposer, Albert Fisher.

M/A2 A prisoner was thrown into a medieval dungeon with 145 doors. Nine, shown by black bars, are locked, but each one will open if before you reach it you pass through exactly eight open doors. You don't have to go through every open door, but you do have to go through every cell and all nine locked doors. If you enter a cell or go through a door a second time, the doors clang shut, trapping you.

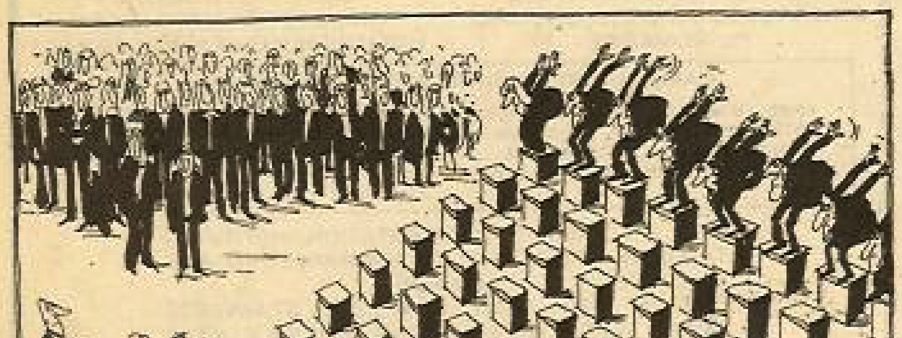
The prisoner (in the lower-right corner cell) had a drawing of the dungeon. He thought a long time before he set out. He went through all the locked doors and escaped through the last, upper-left corner one. What was his route?

For the solution to this problem, a picture is worth 1,000 words:



Solved by Gardner Perry, John Woolston, Jack Mosinger, Kenneth Sawyer, Peter Steven, Bruce Garetz, David Kates, Avi Ornstein, J. Moses, B. Rouben, Raphael Justewicz, Jordan Wouk, Naomi Markovitz, and the proposer, Joan Baum.

M/A3 Given the situation shown in the drawing (below), what is each competitor's chance of reaching the winner's platform? Assume that each interior contestant may jump to one of the two nearest forward squares (50 percent chance for each) and someone on the end must jump to the nearest forward square. When two contestants land on the same square they flip a fair coin and the loser is eliminated. If this is too easy, try to find "optional strategies" for jumping right and left (instead of 50-50).

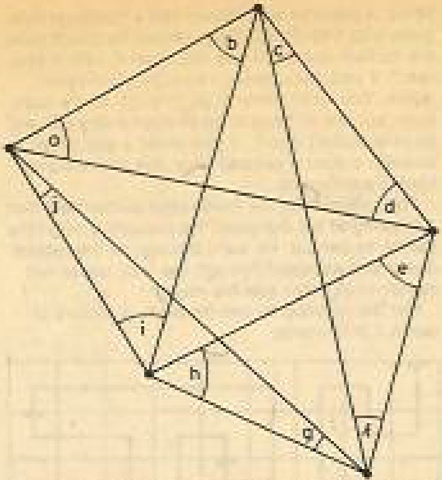


Selecting the Nobel winners (see M/A 3 above). (Niculae Asciu from the New York Times)

This problem is still open for eight contestants and 50-50 probability. Winslow Hartford calculated the probabilities assuming that at any stage either all interior contestants jumped left or they all jumped right. Kenneth Sawyer performed the calculations for seven contestants. Finally, Glen Stoops discussed an improved strategy and calculated the probabilities for eight contestants with the modified strategy and six with the 50-50 strategy. A copy of Mr. Stoops' analysis may be obtained from the editor.

M/A4 Drawing the diagonals in a convex pentagon results in a five-pointed star. What is the sum of the measure of the angles exterior to the star and bounded by the pentagon?

The following solution is from Kenneth Sawyer: The ten labelled angles lie within five triangles. The sum of those ten angles plus the other five included angles must be 5 times 180, or 900°. But those five other angles are congruent with the five interior angles of the inside pentagon. The sum of those five angles must be (n − 2)180 or 3 times 180 equals 540°. The sum of the ten angles is the difference: 900° − 540° = 360°.



Also solved by Winslow Hartford, Naomi Markovitz, B. Rouben, Peter Steven, Avi Ornstein, Bruce Garetz, Jack Mosinger, John Woolston, Gardner Perry, Conrad Carlson, Carey Rappaport, Raymond Gaillard, Farrel Powsner, Mary Lindenberg, Frank Schafer, Patty Doyle, Richard Skinner, Smith Turner, John Prussing, Reino and Christina Hakala, Raphael Justewitz, LouAnne Nesta, Steve Feldman, Norman Wickstrand, Emmet Duffy, James Landau, and the proposer, Gary Nelson.

M/A5 Find a rectilinear solid having integer-length sides, face diagonals, and space diagonal. That is, find integers A, B, and C such that $A^2 + B^2$, $A^2 + C^2$, $B^2 + C^2$, and $A^2 + B^2 + C^2$ are all perfect squares.

I had planned to print one reader's proof of the stronger fact that there do not exist positive integers A, B, C, and K such that $A^2 + B^2 + C^2 = K^2$. But now I find that 1 + 4 + 4 = 9. I. Iverson, who is planning a book on Pythagorean triangles and Archimedean ellipses, had submitted a solution to the original problem. Perhaps s/he will resubmit that solution. As of now the problem is open.

Better Late Than Never

1979 OCT5 Dennis Sandow found solutions to similar problems, such as (FOUND)(A) = KITTY.
NOV5 James Landau points out that his solution was a collaboration with Allen Beadle.
1980 D/J3 Robert Pease found a simpler solution.
D/J4 James Lefferts found an alternate form for the solution.
FEB1 Jerome Taylor has responded.
FEB3 Peter Steven has responded.
M/A SD1 Jordan Wouk and Iri Smith noted that the first phrase should be ABCD∞.
MAY SD2 Half the equation was omitted. The equation should have been:
$I[x/(1 - x)] = (2x - 1)y$.
PERM 3 Harry Hazard still maintains that the year 1789 yields 77 numbers and notes that 28 = 8/[(9/7) − 1]. Overall, he adds, "Harry Hazard has responded, cheerfully conceding that Mr. Gerling's computer is more accurate than his own pencil."

Proposers' Solutions to Speed Problems

SD1 Finding n is easy. Using a calculator (the TI SR50A or 57), we find that 69! is $1.711224524 \times 10^{98}$, and 70! is $1.197857167 \times 10^{100}$; so we may say 70! ≈ g.
For G, we may use Stirling's formula:
$X = K(2\pi N)^{\frac{1}{2}} (N/e)^N \approx N! \approx G$, where for K = 1, X < N! and for $K = [1 + 1/(12N - 1)]$, X > N!. Then
$\log X = \log K + \log (2\pi N)^{\frac{1}{2}} - N\log (N/e)$.
By cut-and-try, we find that logK and $\log(2\pi N)^{\frac{1}{2}}$ become negligible and that the nearest approximation to N is $N = 1.024838384 \times 10^{98}$, or 0.01024838384 g.

SD2 The color of the square on which the knight resides must change with each move.