Puzzle Corner
Allan J. Gottlieb

# Cracking the Secret of the Baltimore Hilton

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I learned with interest from H. Spacil that "Puzzle Corner" has been well received by Nobuyuki Yoshigahara, a Japanese author of mathematical puzzles, and Dr. Spacil was good enough to send one of Mr. Yoshigahara's articles in Japanese. *Technology Review*'s printer occasionally finds "Puzzle Corner" difficult to typeset, but it's really a piece of cake compared to Mr. Yoshigahara's article. The Roman character set is, of course, much simpler than the Japanese. Incidentally, when American computer scientists returned from a stay in (mainland) China, they commented on how hard it is to computerize the Chinese character set.

## Problems

**NS 18** (nee **1978 M/A 4**) We begin with a past problem that was **Never** (completely) **Solved**:
A solitaire game (called accordian, among other names) consists of dealing a deck, one card at a time, and then examining sets of four cards. If the four cards are of the same suit, the middle two are discarded. If the four cards are of the same value, all four are discarded. What are the odds of winning (no cards left)? What if the whole deck is laid out before starting?

**FEB 1** Our first new problem is a double chess challenge from Jerome Taylor:
White sets up pieces in standard form to start a game and has first move. Black sets his king in normal position; he may set what other pieces he uses on any unoccupied squares. For Black to have a forced win,
1. What is the least number of pieces that Black needs, what are they, and where are they placed?
2. If Black is restricted to pawns only, what is the least number needed, and where placed?

**FEB 2** Our second problem is from the late R. Robinson Rowe:
Up on my carport roof one day, a rope I had laid down carelessly, with part hanging over the edge, began to creep. As I reached for it the creep became a gallop, and it all slithered off and down to the driveway. This suggested a hypothetical problem: suppose the roof was horizontal and perfectly smooth, the rope a slippery, flexible, homogeneous line mass five meters long overhanging one centimeter, and the edge mechanically equivalent to a frictionless sheave of infinitesimal radius, how long would it take the rope to slither off the roof?

**FEB 3** Steve Gersuk wants us to help him break into the Baltimore Hilton. He writes:
On a recent business trip, I had occasion to stay at the Hilton Inn in Baltimore. In lieu of a conventional key lock, each room was equipped with a cipher lock that responded only to the four-digit code selected by the visitor when registering. In the course of playing with the lock (irresistable), I noted that the lock would open (indicated by a green LED) whenever the correct code was the last four digits of any sequence. In other words, any amount of garbage could be keyed in; if the last four digits matched, the bolt was energized. It occurred to me that the enterprising burglar would need to try many fewer than the 10,000 possible combinations if he could define a digit stream with the characteristic that each new digit entered resulted in a new four-digit sequence. The minimum number of entries must be 1,003 — four digits to enter the first number, with each of the subsequent 999 resulting in a new sequence. I see no obvious way to generate the most efficient sequence to minimize the number of keystrokes required. But surely this is child's play to *Technology Review* readers.

**FEB 4** David Gluss needs help to plan a 500-mile trip; as you'll see, he doesn't want to dig too deep a tunnel. Mr. Gluss writes:


It is known that the fastest way to get an object from point a to point b in a uniform gravity field is a cycloid. If a and b are 500 miles apart, the maximum depth of the cycloid would be 159 miles. What is the fastest curve if there is a more severe depth limitation — e.g., 50 miles?

**FEB 5** William Buttler has a warm-up for tax time. He notes that "the prospect of filling out 1040 forms and schedules, with their innumerable entries, can boggle the unprepared mind—unless one practices by working on a preliminary, less confusing task." He admits that the following problem is not as complex as our federal tax forms, but he hopes it will "help as an elementary warm-up exercise for stimulating our thinking caps": Five people had consecutive appointments with an income tax expert to help them fill out their 1040 forms and schedules. The electrical engineer had income from a savings account. The man who had a profit trading commodities was taking educational expenses as a deduction. When the man who contributed to a charity was leaving he met the taxpayer with dividend income. The biochemist is deducting interest on a mortgage. The computer programmer uses an SC-40 calculator. The man with three dependents is claiming storm damage as a deduction. The man with the charitable deduction followed the physicist. The man with five dependents exchanged amenities with the owner of the SR-50. When he looked at the tax expert's calendar, the man with the MX-140 noticed his name was next to that of the man with three dependents. The man with seven dependents sold some real estate for profit. The mathematician has six dependents. The income tax expert still had more than one scheduled appointment after he met the man with dividend income. Each man had a profession, owned a calculator, had a deductible expense, had some number of dependents, and had a second source of income. Who won money in a contest? Who owned an HP-45 calculator? (Note: the problem has a unique solution against 24,883,199,999 erroneous combinations.)

## Solutions

**NS 16** A palindrome is a number that reads the same left to right and right to left — e.g., 18781 and 372273. Take an arbitrary number and add it to its mirror image. If the sum is not a palindrome, add it to its mirror image. Keep going. Will a palindrome necessarily result? (It is reported that 196 is a particularly interesting one to try because it never yields a palindrome, but proof is lacking.)

Although we still have no proofs, several "experimental" results have been reported. Kathryn Bittman, E. Phillips, and Arthur Samuel have worked on the 196 sequence. Prof. Samuel (incidentally, the author of a celebrated computer program that played an excellent game of checkers 20 years ago) carried the analysis to 2,392 additions and 1,000 digits. Mr. Phillips supplied a lengthy argument to show that it is unlikely for a sequence to terminate after a large number of terms. A copy of the results of Phillips and Samuel can be obtained from the editor. Here I will reprint their summaries only. Mr. Phillips' results (all occurrence frequencies are relative to the universe of 900 three-digit starting points):

| Number of digits in palindrome | Typical starting point | Occurrence frequency (per cent) |
|---|---|---|
| 5 | 176 (or 79) | 11.8 |
| 6 | 188 | 2.8 |
| 7 | 589 | 0.4 |
| 8 | 167 | 1.0 |
| 10 | 177 | 1.2 |
| 12 | 998 | 0.2 |
| 13 | 187 (or 89) | 1.0 |
| No palindrome? | 196 | 1.4 |
| Total over 4 | | 19.7 |

Prof. Samuel's results:
Palindromes, when formed, seem to occur with a reasonably small number of additions. The number of different classes as well as the percentage of intransigency (numbers not forming palindromes within the calculations performed) seems to go up rapidly with the number of digits in the original numbers. The fact that intransigent cases exist with additions carried to 100 digits is no proof that some of these cases might not lead to palindromes if the search were extended. There are gaps in the tables; for example, no palindromes were found for three-digit numbers requiring adds of 12, 13, 16, and from 18 to 21, and there could be a longer gap from 24 to 223. This does seem unlikely, but we have no proof. I did carry the analysis for 196 to 2,392 additions and 1,000 digits, but even this is no proof. Results to 169 additions and 75 digits are attached.

I extended the analysis to six-digit numbers. About the only significant fact that seems to emerge is that the required number of adds seems to be going up:

| Number of digits | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| Intransigent classes | 0 | 3 | 11 | 246 | 937 |
| Intransigent cases | 0 | 13 | 233 | 5774 | * |
| Palindrome classes | 18 | 177 | 331 | 3174 | 5561 |
| Palindrome cases | 90 | 887 | 8767 | 84226 | * |
| Maximum adds for P's | 24 | 23 | 21 | 55 | 64 |

* Not computed, but the total would be 90,000.

**OCT 1** Suppose you and dummy have two seven-card fits (i.e., suits in which you are lacking six cards). The *a priori* odds for the division of six outstanding cards are well known at 48:36:15:1 for a 4-2:3-3:5-1:6-0 split. My question is, How do those odds change for the division of the second suit after you have played the first suit and established what the first split was? In particular, what are the odds for the division(s) of the second suit when the first suit is known to have split (a) 4-2 or (b) 3-3?

The following solution is from Alan LaVergne:
There is a certain ambiguity in the statement of the problem. At least when the first suit splits unevenly, it matters how one finds out about it. The odds on the various divisions of the second suit will depend on whether (1) one is simply told about the 4-2 split, or (2) the 4-2 split is discovered by playing three rounds of the first suit. In case (1), the declarer will know two cards in one opponent's hand, and four cards in the other's. In case (2), the declarer will know three cards in one hand and four in the other. Rather than trying to divine the proposer's intention, I will calculate the odds in case (1) and in case (2), given that the discard was not a card in the second suit. If there are N unknown cards in one opponent's hand and M in the other's, and min(M,N) ≥ max(A,B), the probability of an A-B division in a suit is

$$\frac{(A+B)!}{A!\,B!}\left[\frac{N!}{(N-A)!}\,\frac{M!}{(M-B)!}\,\frac{(N+M-A-B)!}{(N+M)!} + \frac{N!}{(N-B)!}\,\frac{M!}{(M-A)!}\,\frac{(N+M-A-B)!}{(N+M)!}\right]$$

unless A = B, when one of the terms in the sum is omitted.
If the first suit splits 3-3, then N = M = 10 in the above formula. If the first suit splits 4-2, then N = 11, M = 9 in case (1) above, and N = 10, M = 9 in case (2). The results for the 6-0, 5-1, 4-2, and 3-3 splits, respectively, are:
7:84:315:240 if the first suit splits 3-3,
91:924:3135:2310 if the first suit splits 4-2 (case 1),
7:84:315:240 if the first suit splits 4-2 (case 2),
12:117:390:286 if there is no information about any other suits.

In other words, the probabilities for the various divisions of the second suit are the same in these two situations: (I) Both opponents follow to three rounds of the first suit; (II) Both opponents follow to the first two rounds, but one opponent discards something from the third or fourth suit on the third round. I cannot explain why this should be so, but it is true in general: one gets the same results from the above formula by plugging in either N = M or N = M − 1.
Summarizing the results:

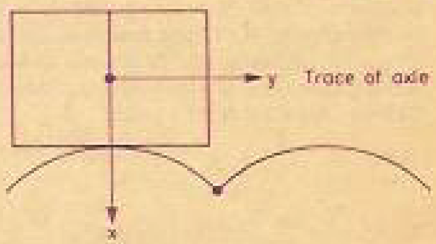
| split | no information | 3-3 or "4-3" | 4-2 |
|---|---|---|---|
| 6-0 | 1.491% | 1.084% | 1.409% |
| 5-1 | 14.534 | 13.003 | 14.303 |
| 4-2 | 48.447 | 48.762 | 48.529 |
| 3-3 | 35.528 | 37.152 | 35.759 |

Thus the 4-2 split in the first suit hardly changes the odds for the second suit at all, while the 3-3 and "4-3" splits slightly favor the flatter distributions in the second suit.

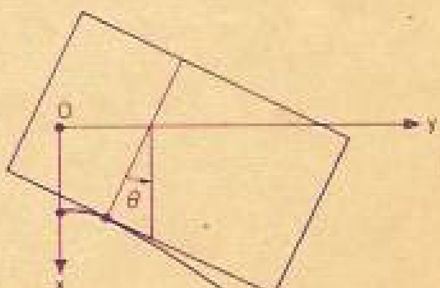
Also solved by Jerry Grossman.

**OCT 2** Describe a track on which a square wheel rolls smoothly without slipping.

Ten years ago Edwin McMillan wrote a paper on all sorts of "funny" wheels: lines, squares, and inclined planes, for examples. A copy of this paper can be obtained from the editor. In this case, we have several "conventional" solutions and one elaborate photographic demonstration. The following is from Doug Szper:
Consider a square wheel with side of length 2 units. We are asked to describe the track upon which such a wheel can roll smoothly without slipping. It suffices to describe the section of track over which the wheel turns through an angle $\theta$ from 0 to $\pi/4$, since the rest follows by symmetry, where $\theta = 0$ corresponds to one side parallel to the ground.




Define a coordinate system with the Y-axis parallel to the ground, along the trace of the axle, and the X-axis increasing downward from the origin, at the axle position for $\theta = 0$. It is assumed that the axle is always directly above the point at which the wheel touches the track, as shown below.




Indicate the tangent point as $(x(\theta), y(\theta))$ in the coordinate plane. The distance along the track to $(x(\theta))$ is $\tan\theta$, since there is no slipping. Thus we have:

$$\int_0^\theta \left[ \left( \frac{dx}{d\theta} \right)^2 + \left( \frac{dy}{d\theta} \right)^2 \right]^{\frac{1}{2}} d\theta = \tan \theta \quad (1)$$

Since the axle moves along the line X = 0, the value $x(\theta)$ equals $1/\cos \theta$. Thus $dx/d\theta = \sin \theta / \cos^2\theta$. Differentiating formula (1) with respect to $\theta$, we obtain:

$$\left[ \left( \frac{dx}{d\theta} \right)^2 + \left( \frac{dy}{d\theta} \right)^2 \right]^{\frac{1}{2}} = \sec^2\theta \quad (2)$$

Substituting $\sin \theta \cdot \sec^2\theta$ for $dx/d\theta$ and solving, we obtain:

$$dy/d\theta = (\sec^4\theta - \sin^2\theta \sec^4\theta)^{\frac{1}{2}}$$
$$= \sec^2\theta \cdot \cos \theta = \sec \theta$$

Thus we find:

$$y(\theta) = \int_0^\theta \sec \theta \, d\theta = \log (\sec \theta + \tan \theta)$$

The track segment is thus defined by the set of points:

$$\{(x,y) = [\sec \theta, \log (\sec \theta + \tan \theta)]\}$$
$$0 \le \theta \le \pi/4$$

Or:

$$y = \log (x + (x^2 - 1)^{\frac{1}{2}})$$
for: $1 \le x \le \sqrt{2}$

Also solved by Harry Zaremba, Alan LaVergne, Raphael Justewicz, James Landau, Edmond Nadler and Allen Tracht.

**OCT 3** Find the smallest number N which can be partitioned into seven distinct positive integers such that the sums of any six is a perfect square; then try for eight positive integers with the sum of any seven a square, then any eight out of nine, and finally any nine out of ten.

Harry Zaremba sent us a concise solution:

The smallest integers N, the partitioning, and the corresponding squares in each case are as follows:

*Seven-integer partition:*
N = 2,236.
Integers = 27, 120, 211, 300, 387, 555, and 636.
Squares = $40^2$, $41^2$, $43^2$, $44^2$, $45^2$, $46^2$, and $47^2$.

*Eight-integer partition:*
N = 3,156.
Integers = 20, 131, 240, 347, 452, 555, 656, and 755.
Squares = $49^2$, $50^2$, $51^2$, $52^2$, $53^2$, $54^2$, $55^2$, and $56^2$.

*Nine-integer partition:*
N = 4,908.
Integers = 8, 147, 284, 419, 552, 683, 812, 939, and 1,064.
Squares = $62^2$, $63^2$, $64^2$, $65^2$, $66^2$, $67^2$, $68^2$, $69^2$, and $70^2$.

*Ten-integer partition:*
N = 8,656
Integers = 7, 192, 375, 556, 735, 912, 1,087, 1,260, 1,600, and 1,932.
Squares = $82^2$, $84^2$, $86^2$, $87^2$, $88^2$, $89^2$, $90^2$, $91^2$, $92^2$, and $93^2$.

Also solved by Frank Rubin, Alan LaVergne, Neil Hopkins, L. Postas, and Al Weiss, who claims that I am "a capitalist tool in the employ of the computer manufacturers" because of the large quantities of machine time my problems require.

**OCT 4** What are the dimensions of Smith's ranch, described in the following conversation?

SMITH: Down in Todd County, which is a 19-mile square, I have a ranch — rectangular, not square, in shape — measuring a whole number of miles each way.

JAMES: Hold on a minute. I happen to know the area of your ranch; let me see if I can figure out its dimensions. (He figures furiously.) I need more information. Is the width more than half the length?

*Smith answered the question.*

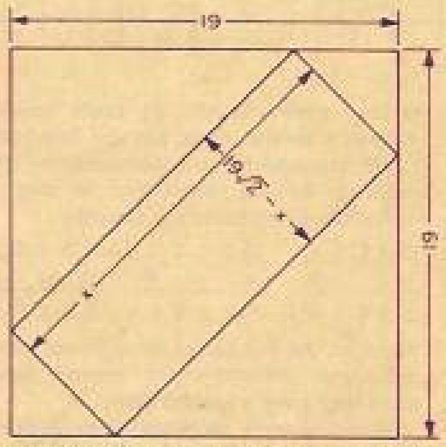
JAMES: Now I know the dimensions of your ranch.

BROWN: I, too, know the area and, although I did not hear your answer to James' question, I, too, can tell you the dimensions.

GREEN: I did not know the area of your ranch but, since I have heard this conversation, I can deduce it.

Harry Garber avoided the trap of assuming that all sides are limited to 19 miles. This trap caused several readers to go astray. Mr. Garber's solution follows:

One key to this problem is to realize that rectangles with length greater than 19 miles can still fit inside a 19-mile-by-19-mile square. For a rectangle with length x miles, the largest possible width is $19\sqrt{2} - x$ miles:




Recalling that we are dealing with only whole-number dimensions, I constructed a table containing the areas resulting from all possible pairs of dimensions, shown across the bottom of this page. The "staircase" is a boundary; all areas above it have width > length/2 while all areas below it have width ≤ length/2. I arranged the areas in ascending order, and after each value indicated how many times that value appeared above (Y) or below (N) the staircase. I won't bore you with my entire result, but a coded portion of my list looks like this:

| | | | |
|---|---|---|---|
| 2 N | 6 NY | 18 NNN | 144 NY |
| 3 N | 7 N | 24 NNNY | 180 YY |
| 4 N | 8 NN | 119 N | 255 Y |
| 5 N | 12 NNY | 120 NYY | 342 Y |

For example: an area of 24 square miles can result from four different pairs of dimensions, three below the staircase and one above the staircase.

Now we can deduce the area logically, as Green did. Since James required more information, we eliminate any value which occurs only once (e.g., 7, 119, or 255). Not

| Length \ Width | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2 | 2 | | | | | | | | | | | | | | | | | | | | | | | | | |
| 3 | 3 | 6 | | | | | | | | | | | | | | | | | | | | | | | | |
| 4 | 4 | 8 | 12 | | | | | | | | | | | | | | | | | | | | | | | |
| 5 | 5 | 10 | 15 | 20 | | | | | | | | | | | | | | | | | | | | | | |
| 6 | 6 | 12 | 18 | 24 | 30 | | | | | | | | | | | | | | | | | | | | | |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | | | | | | | | | | | | | | | | | | | | |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | | | | | | | | | | | | | | | | | | | |
| 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | | | | | | | | | | | | | | | | | | |
| 10 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | | | | | | | | | | | | | | | | | |
| 11 | 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | | | | | | | | | | | | | | | | |
| 12 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | | | | | | | | | | | | | | | |
| 13 | 13 | 26 | 39 | 52 | 65 | 78 | 91 | 104 | 117 | 130 | 143 | 156 | | | | | | | | | | | | | | |
| 14 | 14 | 28 | 42 | 56 | 70 | 84 | 98 | 112 | 126 | 140 | 154 | 168 | 182 | | | | | | | | | | | | | |
| 15 | 15 | 30 | 45 | 60 | 75 | 90 | 105 | 120 | 135 | 150 | 165 | 180 | 195 | 210 | | | | | | | | | | | | |
| 16 | 16 | 32 | 48 | 64 | 80 | 96 | 112 | 128 | 144 | 160 | 176 | 192 | 208 | 224 | 240 | | | | | | | | | | | |
| 17 | 17 | 34 | 51 | 68 | 85 | 102 | 119 | 136 | 153 | 170 | 187 | 204 | 221 | 238 | 255 | 272 | | | | | | | | | | |
| 18 | 18 | 36 | 54 | 72 | 90 | 108 | 126 | 144 | 162 | 180 | 198 | 216 | 234 | 252 | 270 | 288 | 306 | | | | | | | | | |
| 19 | 19 | 38 | 57 | 76 | 95 | 114 | 133 | 152 | 171 | 190 | 209 | 228 | 247 | 266 | 285 | 304 | 323 | 342 | | | | | | | | |
| 20 | 20 | 40 | 60 | 80 | 100 | 120 | - | - | - | - | - | - | - | - | - | - | - | - | - | | | | | | | |
| 21 | 21 | 42 | 63 | 84 | 105 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | | | | | | |
| 22 | 22 | 44 | 66 | 88 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | | | | | |
| 23 | 23 | 46 | 69 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | | | | |
| 24 | 24 | 48 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | | | |
| 25 | 25 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | | |
| 26 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | |

knowing Smith's answer to James's question, Brown couldn't be so certain of the dimensions if the correct value were an NY entry, so we eliminate those (e.g., 6, 144). And James also couldn't be certain if the area involved all one letter such as NNN or YY, so we eliminate those (e.g., 8, 18, 180). This narrows the original list of 192 coded area values down to just thirteen values:

| | | | |
|---|---|---|---|
| 12 NNY | 40 NNY | 72 NNY | |
| 20 NNY | 42 NNY | 80 NNY | |
| 24 NNNY | 48 NNNY | 84 NNY | 120 NYY |
| 30 NNY | 60 NNNY | 90 NNY | |

Now, Smith's answer to James's question couldn't allow Green to even be certain of the area, much less the dimensions, if that answer were Y. But there *is* one area where an N answer does uniquely determine the dimensions. That area is 120 square miles; hence the dimensions of the ranch are 20 miles by 6 miles.

Responses received from Jordan Wouk, Ronald Ort, L. Postas, Robert Zimmerman, Harry Zaremba, Alan LaVergne, Smith Turner, and P. Jung.

**OCT 5** Replace each letter with a unique digit:

FOUND × A = SKILL

What is the value of SKILL? This is the *only* short multiplication in base 10 which has a unique solution and which involves as few as eleven digits in total. That is, this is the best possible problem, in the sense of least redundancy.

Many readers used a computer search for this problem. Those who did not characterized their method as brute force. Although several facts can be deduced, many cases and much trial-and-error are required. Several responders felt that a multiplicity of solutions are possible:

| | | |
|---|---|---|
| 0 3 1 7 4 | 0 9 1 3 7 | 3 0 7 9 4 |
| × 9 | × 6 | × 2 |
| 2 8 5 6 6 | 5 4 8 2 2 | 6 1 5 8 8 |

However, the first two of these do not meet the usual criptarithemetic ground rules since they have a leading zero.

Solutions were received from Stephen Washburne, Robert Zimmerman (from Guadalajara), Harry Garber, G. Ropes, James Landau, Harry Hazard, Ronald Ort, Peter Sorant (a Baker House colleague of mine), Harry Zaremba, Alan LaVergne, Avi Ornstein, and Naomi Markovitz.

**Better Late Than Never**

**J/J 2** Neil Hopkins has submitted the following comments:

I was one of the original solvers of this problem, and I believe that Mr. Furthmyer's concern warrants a reply. I'm happy to oblige.

1. One has to proceed with the assumption that there is a solution to the problem. Frank Rubin is an honorable man.

2. We are asked to find the length of the bridge. Nothing else. Six equations and six unknowns provide two solutions. One solution results in a good round number to the length of the bridge, strongly suggesting that this is the solution the author had in mind. Consequently I submitted the following: "The bridge is ten miles long, and the foot-traveller establishes new records for walking and running speeds."

3. Information regarding the passage of the diesel obviously involves only the length of the diesel, not a solution requirement. It cannot alter the results already obtained under (2) above. It cannot add new sets of values. It cannot eliminate both solutions based on the original premise per (1) above. It can eliminate one of the two solutions; that is all. I paid no attention to this distraction, confident that I had arrived at the solution the author had in mind — correctly, it seems. In the real world, the engineer involved with railroad bridges takes nothing for granted. He has to be on guard against venality, rascality, and stupidity. He must check everything, including the length of the diesel. In the happy never-never land of Aardvark and Zymurgy, where railroad bridges are ten miles long and a foot traveller skips along at record-breaking speeds, we get into the spirit of the place and ignore such niceties as the speed of sound in air, starting and stopping velocities of trains, and the like. I had no difficulty with this aspect of the problem.

**Y1979** and **PERM 3** John Gratwick and George Gerling have responded, and the latter also found an interesting "solution" to the John Rule challenge. Mr. Gerling writes:

While solving **Y1979** I became curious as to whether any four-digit combination produced all of the first 100 integers. Under the rules, there is none. The most prodigious are 1,389, 2,347, and 2,379, each of which generates 98 integers, and 1,379, 2,356, and 2,457, which generate 97. Fourteen combinations generate 96 integers, and eight more produce 96. The digit combinations which produce the most successive integers are 1,278 (all integers through 92 and a total of 96) and 2,367 (all through 90 and a total of 96). You mentioned curiosity about how 1,999 and 2,222 will fare. The sequence 1,999 generates only 32 integers, failing first on the integer "4." The sequence 2,222 produces 27 integers failing first on "7." My investigation of all the possible four-digit combinations suggested that if the field of search could be increased by an order of magnitude, some combinations producing 99 integers would be found and, with a little luck, there would be one or two that produced all 100. (There appears to be considerable independence between the integers that are generated by using four digits and three operators and those generated by juxtaposition of digits and fewer operators). I increased the field of search by allowing the additional assumption that the digits can be assumed to be in any given base in which they have validity (i.e., use of digits 0-7 in base 8 and use of all 10 in any base larger than 9). The results were quite interesting. I found many sequence-base combinations which yielded 99 integers but only one that produced all 100. It is the digit combination 2,345, in the base 19. A selection of optimal solutions is shown on the next page (optimized for a number of operators but with no preferential digit sequence). This sequence is also interesting in that it produces only the first 100 integers and does not generate either "101" or "102." I have ceased work on a rather inelegant proof that this is the only such sequence that exists, but I would be surprised if a second example (or more) were found. Perhaps your readers would enjoy the challenge of finding this unique solution under the expanded condition of permitting bases other than 10 to be assumed. Needless to say, my work was computer-aided.

**FEB 4** Smith Turner defended and explained his solution: "The E refers to the EE key on the SR-50. The integers produced after EE are displayed at the right, and — so help me — you will find $3^2 = 7$. Ha!"

**MAY 4** Richard Askey notes that relevant work in this area was done by Gauss in the last century and by Jim Wilson last year.

**A/S 1** P. Jung, Michael Kay, and Winthrop Leeds responded. Frank Rubin feels that Black, by moving K—QN8 and R—QN7, can draw against White's queen and king. However, I believe that White can win that position and that the computer program BELLE proved this.

**OCT SD 1** G. Michael, R. Cralle, and a CDC-7600 found the following solutions: bilihumin, dislikelihood, Elihu, goodlihead, kinglihood, knightlihood, Lihyonite, likelihead, likelihood, livelihood, lonelihood, Y 2345 (Base 19)

The following are optimal solutions for minimal number of operators, but with no preference given to digit sequence, & the equivalent expression in base 10.

| | Base 19 | Base 10 | | Base 19 | Base 10 |
|---|---|---|---|---|---|
| 1. | (3-2)**45 | (3-2)**81 | 51. | 34-(2*5) | 61-(2*5) |
| 2. | 5+3-4-2 | same | 52. | ((2**3)+5)*4 | same |
| 3. | (2*5)-4-3 | same | 53. | (53/2)+4 | (98/2)+4 |
| 4. | 4-2-3+5 | same | 54. | (45/3)*2 | (81/3)*2 |
| 5. | (2+3-4)*5 | same | 55. | 25+(4*3) | 43+(4*3) |
| 6. | 2+3+5-4 | same | 56. | 53-24 | 98-42 |
| 7. | (3*5)-(2*4) | same | 57. | 24+(3*5) | 42+(3*5) |
| 8. | 2+4+5-3 | same | 58. | 54-23 | 99-41 |
| 9. | 45/(3**2) | 81/(3**2) | 59. | 32*(5-4) | 59*(5-4) |
| 10. | (25-3)/4 | (43-3)/4 | 60. | 35+2-4 | 62+2-4 |
| 11. | 54/(3**2) | 99/(3**2) | 61. | 23+(4*5) | 41+(4*5) |
| 12. | (5*4)-(2**3) | same | 62. | 2+(3*4*5) | same |
| 13. | (25-4)/3 | (43-4)/3 | 63. | 42-(3*5) | 78-(3*5) |
| 14. | 2+3+4+5 | same | 64. | 34+5-2 | 61+5-2 |
| 15. | (35-2)/4 | (62-2)/4 | 65. | ((3**2)+4)*5 | same |
| 16. | 42-35 | 78-62 | 66. | (54/3)*2 | (99/3)*2 |
| 17. | (2**3)+4+5 | same | 67. | (4**3)+5-2 | same |
| 18. | 34-25 | 61-43 | 68. | 34+5+2 | 61+5+2 |
| 19. | (24/3)+5 | (42/3)+5 | 69. | 43-(5*2) | 79-(5*2) |
| 20. | 35-24 | 62-42 | 70. | 35+(2*4) | 62+(2*4) |
| 21. | (42/3)-5 | (78/3)-5 | 71. | 34+(2*5) | 61+(2*5) |
| 22. | 45-32 | 81-59 | 72. | 45-(3**2) | 81-(3**2) |
| 23. | (2+4)*3-5 | same | 73. | 45-(2**3) | 81-(2**3) |
| 24. | (53-2)/4 | (98-2)/4 | 74. | (3**4)-2-5 | same |
| 25. | (53+2)/4 | (98+2)/4 | 75. | 45-(2*3) | 81-(2*3) |
| 26. | ((2*3)*5)-4 | same | 76. | 45-2-3 | 81-2-3 |
| 27. | (35/2)-4 | (62/2)-4 | 77. | ((2**4)*5)-3 | same |
| 28. | (34-5)/2 | (61-5)/2 | 78. | (2**4)+35 | (2**4)+62 |
| 29. | 34-(2**5) | 61-(2**5) | 79. | 32+(4*5) | 59+(4*5) |
| 30. | (3*4*5)/2 | same | 80. | 45+2-3 | 81+2-3 |
| 31. | 35/(4-2) | 62/(4-2) | 81. | 45*(3-2) | 81*(3-2) |
| 32. | 23-4-5 | 41-4-5 | 82. | 45-2+3 | 81-2+3 |
| 33. | (34+5)/2 | (61+5)/2 | 83. | (5**3)-24 | (5**3)-42 |
| 34. | 24-5-3 | 42-5-3 | 84. | 24*(5-3) | 42*(5-3) |
| 35. | (35/2)+4 | (62/2)+4 | 85. | 52-(3*4) | 97-(3*4) |
| 36. | 43-25 | 79-43 | 86. | 2+3+45 | 2+3+81 |
| 37. | (43-5)/2 | (79-5)/2 | 87. | (2*3)+45 | (2*3)+81 |
| 38. | (3**4)-25 | (3**4)-43 | 88. | ((5**2)-3)*4 | same |
| 39. | 32-(4*5) | 59-(4*5) | 89. | (2**3)+45 | (2**3)+81 |
| 40. | 45-23 | 81-41 | 90. | 45+(3**2) | 81+(3**2) |
| 41. | 23*(5-4) | 41*(5-4) | 91. | 54-(2**3) | 99-(2**3) |
| 42. | 23-4+5 | 41-4+5 | 92. | 53-2-4 | 98-2-4 |
| 43. | 25*(4-3) | 43*(4-3) | 93. | 54-(3*2) | 99-(3*2) |
| 44. | 24+5-3 | 42+5-3 | 94. | 54-3-2 | 99-3-2 |
| 45. | (53/2)-4 | (98/2)-4 | 95. | ((2**4)+3)*5 | same |
| 46. | 35-(2**4) | 62-(2**4) | 96. | 53+2-4 | 98+2-4 |
| 47. | 43-(2**5) | 79-(2**5) | 97. | 52*(4-3) | 97*(4-3) |
| 48. | (54-3)/2 | (99-3)/2 | 98. | 54-3+2 | 99-3+2 |
| 49. | 53/(4-2) | 98/(4-2) | 99. | 54*(3-2) | 99*(3-2) |
| 50. | 23+4+5 | 41+4+5 | 100. | 54-2+3 | 99-2+3 |

lovelihead, manlihood, millihenry, seemlihead, selihoth, superlikelihood, and unlikelihood.

**OCT SD 2** David Kaufman and Joseph Friedman note that $a = 3(\sqrt{2} - 1)$. This is the same as $3/(1 + \sqrt{2})$.

Responses have been received as indicated:

**J/J 1** Peter Sorant and (Ms.) Ronnie Rybstein.

**A/S 3** James Landau, Naomi Markovitz, Frank Rubin, and Harry Hazard.

**A/S 4** P. Jung.