Puzzle Corner

Allan Gottlieb is associate professor of mathematics and coordinator for computer mathematics at York College of the City University of New York; he studied mathematics at M.I.T. and Brandeis. Send problems, solutions, and comments to him at the Department of Mathematics, York College, Jamaica, N.Y., 11451.

100 Miles of Desert At 20 Miles a Day

Allan J. Gottlieb

As the volume number of Technology Review advances by one digit, here's a word of welcome back to all Puzzle Corner veterans and a warm greeting to those of you who I am meeting for the first time. For the benefit of the latter I would like to review the ground rules.

Each issue we publish five regular problems and two "speed" problems; three issues later the solutions to the regular problems appear. We also occasionally republish older problems to which solutions were never received. This month, for example, we are printing the solutions to problems published in the March/April, 1979, issue; and among the problems you will find an "NS" problem first published in May, 1977. Challenges to published solutions and acknowledgement of late responses appear in the "Better Late Than Never" department. The "speed" problems are not to be taken too seriously. Often whimsical, their solutions are usually given in the same issue as the problem is posed, and discussion of them rarely appears in the "Better Late Than Never" department.

Some interesting comments have been received from our veterans. First of all I want to send a great thank you to Judith Longyear. Dr. Longyear asked why the *Review* is using a circa 1967 photo of me. The truth is that the picture was taken two years ago expressly for "Puzzle Corner." If I appear younger than my years, perhaps it's because you have not seen the hundreds of shots we decided *not* to use.

Theodore Engle is still interested in factor champions and would like to correspond with Albert Mullin or anyone else interested in the subject. If you are interested in factor champions (e.g., smallest number with a million factors) please write to me for Mr. Engle's address.

Mary Lindenberg reports that she and her husband just returned from his 40th M.I.T. reunion where a classmate, upon seeing her name tag, asked if she was *the* Mary Lindenberg who contributes to "Puzzle Corner."

Finally, a personal note. Having completed my sixth year at York College, I am on sabbatical leave for 1979-80. I will be spending the year at the Courant Institute for the Mathematical Sciences in New York City. However, please continue to send all correspondence to me at the York College address given above.

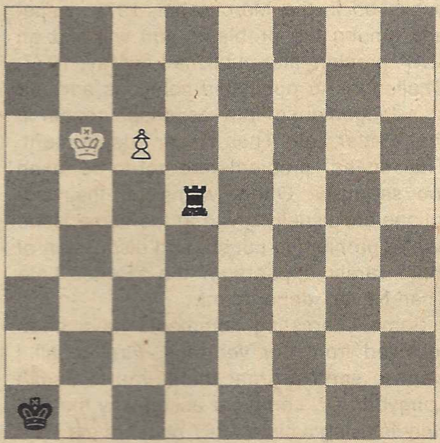
Problems

NS 16 (nee **1977 MAY 4**) We begin this month with a past problem that was **N**ever (completely) **S**olved. A palindrome is a number that reads the same left to right and right to left — e.g., 18781 or 372273. Take an arbitrary number and add it to its mirror image. If sum is not a palindrome add it to its mirror image. Keep going. Will a palindrome necessarily result? For example:

79 + 97 = 176;
176 + 671 = 847;
847 + 748 = 1595;
1595 + 5951 = 7546;
7546 + 6457 = 14003;
14003 + 30041 = 44044;

So 79 does yield a palindrome. From what I gather 196 is a particularly interesting one to try. I have heard claims that 196 never yields a palindrome but have not seen a proof.

A/S 1 We begin this month's regular problems with a simple-looking chess problem from Steve Grant which contains a few surprises:



White to play and win.

A/S 2 Charlie Bahne needs your help to cross a 100-mile-wide desert. His expedition can travel 20 miles in a day. However, there are no supplies in the desert, and they can only carry two days' provisions at a time (today's plus one extra day's). They can, of course, go out into the desert, deposit a day's supplies, and return to their base. Supplies are sealed in one-day packages; they cannot be broken up. What is the least number of days it will take to cross the desert? You might also try two variants: Is there a general solution for deserts of different widths? Would the answer be different if the provision against fractional days' supplies were eliminated?

A/S 3 William Hornick sent us some examples of problems he and his wife call "the first five." They simply involve the listing of the first letter of the five words of a common, ordered series. For example, OTTFF is One, Two, Three, Four, Five. That's an easy one. I offer you five more difficult First Fives; you have the aid of the clue that they all have to do with numbers.

1. U T H T T
2. F S T F F
3. S D T Q Q
4. O F N S T
5. O S T S O

A/S 4 Cyrus Berstein has a question concerning surface-to-air missiles (SAMS): A SAM tracks its quarry through a heat sensor in the nose. Assume that there is a large square marked off in the sky. A plane enters the square at the southeast corner at 600 m.p.h. and proceeds on a straight course to the northwest corner. At the same moment that the plane enters the square, a SAM enters the southwest corner of the square going at 1200 m.p.h. Instead of "leading" the plane to intercept it more quickly, the SAM points at the plane at all times, thus traveling in an arc as it alters direction to follow the plane. The plane is following a diagonal line. How far up this diagonal line will the plane go before it is intercepted by the SAM?

Those of you who recall the hummingbird problem from a while ago should not have too much trouble with this one.

A/S 5 Emmet Duffy has a geometry problem for which he desires a non-calculus solution: Given an acute-angled triangle, find the inscribed triangle having minimal perimeter.

Speed Department

A/S SD 1 Greg Jackson wants you to punctuate the following sentence: Jack where John had had had had had had had had had a more pleasant sound to it.

A/S SD 2 John Prussing wants to know how long it will take him to parlay $100 into $1 if he agrees to play a game of coin tossing in which he must bet half his current bankroll on each toss. If the coin comes up heads he wins, tails he loses. All winning bets are paid at fair odds 1:1. He begins with $100 and must resign if his bankroll becomes less than $1. During a certain game he won as many tosses as he lost but was forced to resign. After how many tosses did he resign? (Assume bets of arbitrary fractions of dollars are allowed).

Solutions

M/A 1 Evaluate this four-story gabled HOUSE on its GROUNDS so generous that there is both a front yard and a back yard.

```
      U
    O U S
  H O U S E
  H O U S E
  H O U S E
  H O U S E
-------------
G R O U N D S
```

Thomas Petterson, like nearly everyone else, assumed a decimal point is permitted. His solution was: HOUSE = 79804, GROUNDS = 329816.0. Here is how it works:

S = 0

Since D and N cannot be zero, E cannot be 0, 1, 2, or 3.

Similarly, since G and R cannot be 0, H cannot be 0 or 1.
Now, 6 * U = XU, where X is some other digit, but U cannot equal zero. So

U = 2, X = 1; or
U = 4, X = 2; or
U = 6, X = 3; or
U = 8, X = 4.

And O is such that 5 * O + X = YO and O cannot equal U. Then if

U = 2, O cannot equal anything; or
U = 4, O can equal 2 or 7; or
U = 6, O cannot equal anything; or
U = 8, O can equal 4 or 9;
so OU = 24, 48, 74, or 98.

For each of these four possibilities, there are only a few possibilities for H and E. If OU = 24, then since ND = 4 * E we have E = 0, 1, 2, 3, 4, 5, 6, 7, and 8; so E = 9. But then, since GR = 4 * H + 1 (when OU = 24), and H = E, H = 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9; so H cannot equal any digit and OU cannot equal 24. By exhaustive search of the possible choices of H and E for the remaining three choices of OU (only 18 total), the unique solution is found: OU = 98, H = 7, and E = 4.

Also solved by Winslow Hartford, Harry Hazard, Richard Hess, Winthrop Leeds, Avi Ornstein, Frank Rubin, John Trifiletti, Harry Zaremba, and the proposer, the late R. Robinson Rowe.

M/A 2 South, on lead, to make the remaining six tricks with hearts as trump:

♠—
♥8 7
♦J 5
♣A 2

♠Q 7	♠J 6
♥—	♥—
♦Q 7	♦10 8 6
♣J 3	♣8

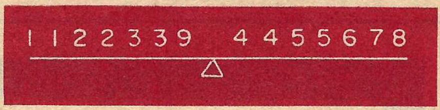
♠10 9
♥—
♦A K 9
♣10

Jerome Shipman had little trouble with this one:
The declarer has five top tricks: his ♦A and ♦K, and dummy's two trumps and ♣A. His problem is to promote an additional trick from among his ♦9, one of his two spades, and dummy's ♣2. He does this in the following way (assuming best play on the part of East and West): *Trick 1:* Declarer leads the ♦A, dropping dummy's jack under it, while West and East follow with the ♦7 and ♦6, respectively. *Trick 2:* Declarer leads the ♠10, trumping with the ♥8, while West and East follow with the ♠7 and ♠6, respectively. *Trick 3:* Dummy leads the ♥7, East discards the ♣8 and South the ♣10. West cannot discard a club, for then Dummy's ♣2 would be promoted to a winner; he cannot discard the ♦Q, for then declarer could finesse through East's ♦10. Therefore, West must discard the ♠Q. *Trick 4:* Dummy leads the ♣A. If East discards a diamond, South discards the ♠9 and the ♦K and ♦9 (reached through Dummy's ♦5) are the fifth and sixth tricks. If East discards the ♠J, South discards the ♦9, and the ♦K and ♠9 are the fifth and sixth tricks.

Also solved by Edmund Chen, Jerry Grossman, Winslow Hartford, Richard Hess, Warren Himmelberger, George Holderness, Paul Horvitz, Thomas Mouthner, Scott Nason, John Rutherford, Stuart Schulman, John Trifiletti, Donald Trumpler, Shirley Wilson, and the proposer, Noland Poffenberger.

M/A 3 Eleven open bags, numbered 1 to 11, each contain two coins. Ten bags contain genuine coins all of the same weight. One bag contains false coins which differ slightly in weight from genuine coins. The two false coins weigh the same but a false coin is either 10 grains heavier or 10 grains lighter than a genuine coin. You are given a balance scale and 20-grain weight. Using the balance scale two times only, find a method to identify the false coins and tell whether they are heavy or light. Placing stated coins on stated sides of the scale, and, if required, placing the weight on the scale, shall be considered a use of the scale.

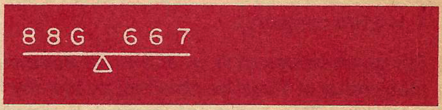
This one is a little subtle. The key is that adding the 20-grain weight does not count as a new weighing. The following solution is from Dean Lytle:
Place the coins as shown below, with coins from bags 10 and 11 held out:



If the scale balances, the false coins are in bag 10 or 11, and we proceed as follows:

The left hand side will be either heavy or light. Suppose it is heavy. Place the 20-grain weight on the right side. If the scales balance, coins from bag 10 are counterfeit-heavy. If the scales are heavy on the right, bag 11 is counterfeit-heavy. The same procedure applies if the left side is light, except the weight is put on the left side. If the first weighing does not balance, we proceed as follows:
□ Place the weight on the right (light) side. If balance is achieved, bags 1, 2, or 3 are heavy or bags 4 or 5 are light; go to the second step (below). If the right side becomes heavy, bag 9 is heavy or bag 6 or 7 is light; go to the third step (below).
□ Place the coins as follows, G denoting coins from bags 6 through 11.

If the left side is doubly heavy — i.e., it balances with the 20-grain weight added on the right), 1 = H. If the left is singly heavy, 2 = H. If the right side is doubly heavy, 4 = L. If the right side is singly heavy, 5 = L. If the two sides balance, 3 = H.
□ Place the coins as follows:



If the left side is doubly heavy, 6 = L. If the left side is singly heavy, 7 = L. If the right side is doubly heavy, 8 = L. If the two sides balance, 9 = H.

Also solved by Edward Gaillard, Raymond Gaillard, Michael Haney, Richard Hess, Winthrop Leeds, Maurie Libner, Scott Nason, Thomas Peterson, Frank Rubin, John Trifiletti, and the proposer, Emmet Duffy.

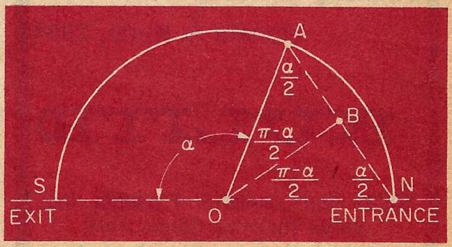
M/A 4 Find a collection of (ordinary) English words that contains the fewest possible total number of letters while including each of the 26 letters at least once.

Surprisingly, Frank Rubin and Winthrop Leeds found essentially the same solution; the only difference between them was to place the s at the end of a different noun. Their solution, with only 27 letters, is: BAWD FLOCKS JIG NYMPH QUARTZ VEX.

Also solved by Edward Gaillard, Raymond Gaillard, Steve Feldman, Winslow Hartford, Richard Hess, Terence Langendron, Stuart Schulman, William Stein, Ernest Thiele, John Trifiletti, Shirley Wilson, and the proposer, Hugh Thompson.

M/A 5 A unicyclist moving at velocity v enters the north end of a table rotating at an angular velocity w. He wishes to leave via an exit ramp at the south end of the table in the minimum possible time, t. Find t and the path required as a function of v, w, and r (the radius of the table).

Apparently it is not possible to give an *explicit* formula for T. Emmet Duffy submitted the following derivation of an *implicit* formula:
Let T = time in seconds to make the trip; let w = the angular velocity of the table in radians, R = the radius in feet, v = the velocity of cyclist in feet per second, and t = variable time in seconds. The cyclist should move in a straight line (on the table) to some point A on the periphery of the table so that his time of travel to point A is the same as the time it takes point A to travel to the exit.



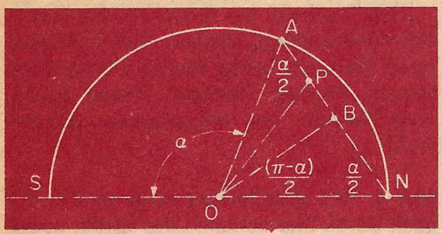
The cyclist's path will be NA. Draw the perpendicular bisector OB of path NA. Let angle AOS = α. Then angle AON = $\pi - \alpha$. As OA = ON = R, then angles OAN and ONA are equal and are $\alpha/2$. Path NA then is equal to 2R cos $\alpha/2$. Then

$(2R \cos \alpha/2)/v = \alpha/w$.

Since T = α/w, we get

$T = 2R/v \cos(wt/2)$.

For given values of R, v, and w, an approximate value of T can be obtained by standard methods. The curve of motion is easily found as a ρ,θ curve with the x-axis passing through N (entrance) and S (exit), with the origin 0 at the center of the table.



Let t be the parameter in seconds. Then

OP = ρ, where P is a point on the curve. Angle PON + wt = θ. PN = vt. BN = R cos (α/2). Then:

$PB = vt - R \cos(\alpha/2)$, as
$OB = R \sin(\alpha/2)$ and
$OP = [(PB)^2 + (OB)^2]^{\frac{1}{2}} = \rho$. Then
$\rho = ([vt - R \cos(\alpha/2)]^2 + [R \sin(\alpha/2)]^2)^{\frac{1}{2}}$
θ = Angle PON + wt = Angle POB + Angle BON + wt. Therefore,

$\theta = \arctan([vt - R\cos(\alpha/2)]/R \sin(\alpha/2)) + (\pi - \alpha)/2 + wt$.

We do have a small controversy, however. The proposer, S. Baranow, claims (without a complete derivation) that

$t = [2 \tan^{-1}(Rw/v)]/w$.

Since this explicit formula is incompatible with Mr. Duffy's implicit solution, we may hear more about this problem later.

Also solved by Winslow Hartford, Richard Hess, Gary Lewison, Dave Mohr, Frank Rubin, F. Steigman, and Harry Zaremba.

Better Late Than Never

NS 13 Although we still have not received an *exact* answer, two interesting replies were received. Frederic Vose found several trigonometric relations which give him a new approximation method and Felix Alexa submitted a beautifully done 1 × 2 foot drawing.

1978 M/A 5 Apparently we have trouble getting parenthesis correct. Irving Hopkins notes that

$\cos(18°) = [(5 + \sqrt{5}) / 8]^{\frac{1}{2}}$

J/J 1 Mark "the shark" Aquino has an alternate method of playing the hand which gives a 97.2 per cent probability of success.

D/J 3 Several readers were unhappy with the published solution. Since Dr. Rubin, the proposer, had previously written to say that the problem itself was incorrectly printed, I have asked him to send me another copy of the problem so that we can try again. Responses have been received from Jonathan Hardis, Greg Hunter, Thomas Mahon, Ron Moore, and James Voss.

Y1978 Harry Hazard notes that

3 = 1 − [(9 + 7)/8]
89 = 1 * (97 − 8)

1979 FEB 2 Allen Keith, Hugh Thompson, and Gary Lewison have responded.

FEB 3 Gary Lewison, William Stein, and Harry Hazard have responded.

M/A SD 1 Theodore Edison and Joseph Friedman note that the area is 144 square inches and that the formula is $(R^2 - 4^2)$.

Proposers' Solutions to Speed Problems

A/S SD 1 Jack, where John had had "had had," had had "had"; "had had" had a more pleasant sound to it.

A/S SD 2 If the number of wins and losses are equal, the number of tosses must be even and the order of the wins and losses is immaterial. Assuming an initial bankroll B_0, after 2n tosses the bankroll is

$B_{2n} = (3/4)^n B_0$.

The smallest value of n for which $(3/4)^n$ is less than 0.01 is 17. Thus, he resigned after 34 tosses.