Help a Farmer Build a Bridge

Puzzle Corner
by
Allan J. Gottlieb

As we start another year, let me welcome back our regular readers and say hello to newcomers.

For the latter, here are the ground rules: Each month we publish five problems and several "speed problems," selected from those suggested by readers. The first selection each month will be either a bridge or a chess problem. We ask readers to send us their solutions to each problem, and three issues later we select for publication one of the answers — if any — to each problem; we also publish the names of other readers who submitted correct answers. Answers received too late or additional comments of special interest are published as space permits under "Better Late Than Never." And I cannot respond to readers' queries except through the column itself.

Here goes.

Problems

O/N 1 We begin this issue with a bridge problem from Kenneth Barbour (both bridge and chess problems are in short supply). At a recent bridge party North and South had the following hands:

♠ K
♥ A Q 9 8 7 6 2
♦ 9 8 4
♣ J 10

♠ A Q J
♥ J 10 5 3
♦ A K Q 5
♣ K 6

I sat as South and bid six diamonds only to discover six hearts was beginners' play. However, six diamonds is a good exercise and was made. By playing all probabilities to the fullest, seven diamonds could be made; the problem is to show how, given the fact that neither East nor West bid. West's lead was ♠2.

O/N 2 Roger Lustic has a problem concerning magic cubes: Is it possible to construct a 3 × 3 × 3 magic cube in which the elements are not all identical? (A magic cube of size n × n × n is a cubic array of numbers in which any set of n colinear numbers totals a constant magic sum S.)

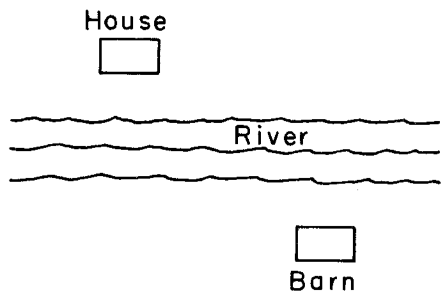
O/N 3 This problem, from James Edmonds, seems very hard to me. Perhaps the physicists in my audience (and others, too?) will show me up. There are two parts:

A. Complex numbers of the form $c = a[e_0] + b[ie_1]$ have *one* kind of "complex conjugation," $c^* \equiv a[e_0] - b[ie_1]$. Here $[e_0][e_0] \equiv [e_0]$, $[e_0][ie_1] \equiv [ie_1][e_0] \equiv [ie_1]$, and $[ie_1][ie_1] \equiv -[e_0]$. This conjugation has the property $(cc')^* = c'^*c^* = c^*c'^*$. Note that $\equiv$ is used to represent "equals by definition." Complex quaternions (or "hypercomplex" numbers)

$$q = a[e_0] + b[ie_0] + c[e_1] + d[ie_1] + e[e_2] + f[ie_2] + g[e_3] + h[ie_3] \equiv \sum_{n=0}^{3} a[e_n] + b[ie_n]$$

have *two* kinds of "complex conjugation" with the property $(qq')^* = q'^*q^*$. The hypercomplex number basis elements have a simple multiplication table: $[e_0] \leftrightarrow 1$ (i.e., $[e_0]$ acts like 1), $[ie_1] \leftrightarrow i$, $[e_1][e_2] = [ie_3] = -[e_2][e_1]$ and cyclic 1, 2, 3, $[e_1][e_1] = [e_2][e_2] = [e_3][e_3] = [e_0]$, $[ie_n] \leftrightarrow i[e_n]$ for multiplication. Find the conjugations q^* and $q\#$ (not so bad) and prove them unique.

B. In physics, there is a 16-element extension of this number system which also seems to describe reality in a fundamental way. We can write it $E = a_n[e_n] + b_n[ie_n] + c[f_n] + d[if_n]$, where again sum on n = 0, 1, 2, 3. Let $E \equiv e + f$ for short. Then $E^+ \equiv e^* + f^\#$ and $E^\wedge \equiv e^\# + f^*$ and $E^\vee \equiv e^\# - f^*$, and all have the property $(EE')^{conj} = E'^{conj} E^{conj}$. The question is open, so far as I know, as to whether any other conjugations can be defined with this property. The multiplication table is as follows: $e \leftrightarrow f$ and $e \leftrightarrow f$; that is, f is like e with respect to $()^*$ and $()^\#$; e and f are like (+) and (−) in multiplication, $ef \rightarrow f$, $ee \rightarrow e$, $ff \rightarrow e$, $fe \rightarrow f$; if f is the *left* multiplier only, then fe is like ee given before ($[f_1][e_2] = [if_3]$); if f is the *right* multiplier then $()^{\#*}$ is taken on the *left* before the regular multiplication is done ($[e_1][f_2] \rightarrow [e_1]^{\#*}[f_2] = -[e_1][f_2] \rightarrow -[if_3]$ and $[if_1][f_2] \rightarrow [if_1]^{\#*}[f_2] = +[if_1][f_2] \rightarrow -[e_3]$). I invented this number system (which is isomorphic to Eddington's E-numbers and the Dirac-Clifford algebra) while trying to extend the complex quaternion system to fit mass into the relativistic quantum wave equation (heavy stuff — ed.). Anyone interested in this material should read the proposer's article in the *American Journal of Physics* (Vol. 42) or/and contact him at San Diego State College.



O/N 4 In a somewhat lighter vein, we have the following agriculture problem from Ray Brinker: A farm is cut by a river flowing from east to west, the house on one side, the barn on the other. The farmer wants to build a north-south bridge from his house to his barn. Where should he locate the bridge to minimize his walk from house to barn?

O/N 5 This problem from Robert Baird sounds hard (a proof — in number theory, no less!), but it's not so bad: Prove that any set of n integers (not necessarily distinct) contains a nonempty subset whose sum is divisible by n.

Speed Department

O/N SD 1 Here is a speed problem from R. Robinson Rowe which even I could solve quickly: Maine, Iowa, Utah, and Idaho do not recognize hereditary barrenness as legal justification in a suit by a husband for a divorce. Why?

O/N SD 2 Ted Mita knows that February 29, 1972, was a Tuesday. When will February 29 next be a Tuesday?

Solutions

The following solutions are to problems published in the May, 1975, issue of *Technology Review*.

MAY 1 South dealt the following hand with neither side vulnerable:

♠ A J 5 3 2
♥ 3 2
♦ 10 8
♣ A K 8 5

West		East
♠ Q 10 9 6		♠ 8 7
♥ K J 9 7 5		♥ 10 4
♦ K 7 3		♦ 6 4
♣ 9		♣ Q J 10 7 4 3 2

♠ K 4
♥ A Q 8 6
♦ A Q J 9 5 2
♣ 6

North-South use Blackwood, and the bidding was:

S	*W*	*N*	*E*
1D	1H	1S	P
3D	P	4D	P
4NT	P	5H	P
6D	P	P	P

Use the bidding (and not the East-West hands) to guide you to South's winning play.

The following is from William Butler: Since West bid one heart, it is apparent that South assumes West has most of the outstanding high cards, including the ♠Q and the ♥K. Also, since West bid hearts he might possibly be short in some other suit. Therefore, efforts should be made to keep East out of the lead (where practical) to prevent a possible club lead and ruff or a heart lead through South's ♥A and ♥Q. In the play of the hand South must not lead hearts until a squeeze position develops (or other problems are solved), and he cannot finesse the ♠J until after the ♠K has been played. If West leads a heart, South's problem is immediately solved and 12 tricks are produced with a simple spade finesse. If West leads a diamond or a club, South wins in the dummy and immediately leads diamonds (overtake the second diamond in his own hand). After West takes the ♦K, South winds any return, runs diamonds, and takes the ♠K until the following position develops:

♠ A J 5
♥ 3
♣ K

♠ Q 10 9
♥ K J

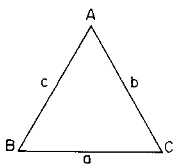
♠ 4
♥ A Q 8 6

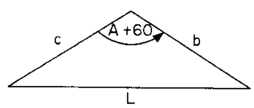
South now leads the ♠4 and finesses dummy's ♠J. South leads ♣K from dummy and discards a heart. West must unguard either spades or hearts. Note that if West could lead a second club after winning the ♦K, then the squeeze would develop when South leads the last diamond. If West's initial lead is a spade, South must win in his own hand and not take the finesse immediately. He must then pull trumps without using clubs as an entry to take a diamond finesse, by first leading a low trump from his hand and using the ♦A on the second diamond trick. (Any other combination leads to re-entry problems.) When West has the lead with the ♦K, the situation is similar to that described above as long as he returns something other than a spade. If he returns a spade (his second spade lead), South finesses the ♠J and returns to his hand by ruffing a third spade. South wins the remaining tricks by running diamonds, taking the ♥A, and using the carefully preserved club entry to take the clubs and remaining spades.

Also solved by William Delehanty, Emmet J. Duffy, Winslow H. Hartford, Richard I. Hess, R. Robinson Rowe, and the proposer, Michael Kay.

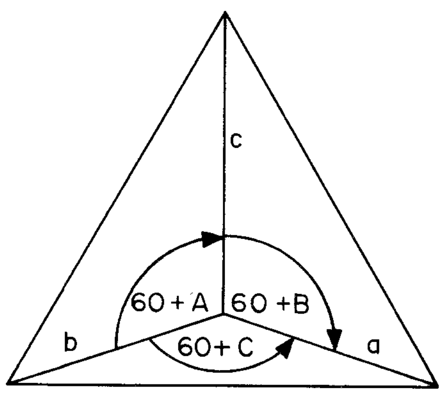
MAY 2 Given three lengths a, b, and c ($a < b + c$, $b < a + c$, and $c < a + b$), find the side of an equilateral triangle, inside which a point joins the three vertices with distances a, b, and c.

Robert Kimble feels that the point in question should be required to lie inside the circumscribed circle, not the equilateral triangle. His solution is to construct a triangle with sides a, b, and c and call its area X. Now he constructs a triangle with sides b and c and included angle ($A + 60$) and claims that the side opposite the angle ($A + 60$) is the desired length. Proof:





Proof: Let L be this desired length. Then $L = b^2 + c^2 - 2bc \cos (A + 60) = b^2 + c^2 - 2bc(\frac{1}{2} \cos (A - \sqrt{3})/\sin A) = b^2 + c^2 - bc \cos A + \sqrt{3}bc \sin A = b^2 + c^2 - bc(b^2 + c^2 - a^2)/2bc + 2\sqrt{3}X = \frac{1}{2}(a^2 + b^2 + c^2) + 2\sqrt{3}X$. Thus the length, 1, is clearly independent of which pair of sides was chosen. The completed diagram would then be the following:

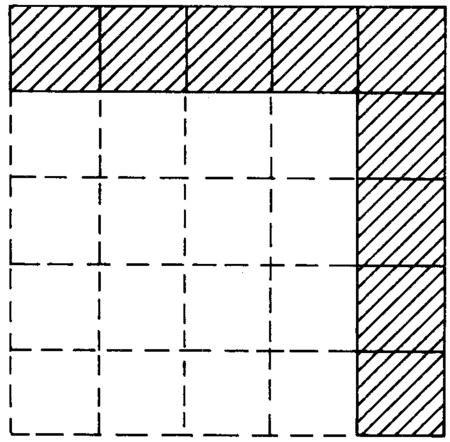


Also solved by Gerald Blum, William Butler, Bob Lutton, Robert Pogoff, R. Robinson Rowe, Frank Rubin, Hal Vose, and the proposer, Eric Jamin.

MAY 3 Show that for any integer $A > 2$, there exist integers B and C such that $A^2 + B^2 = C^2$.

Neil Hopkins sent us both graphical and analytical solutions. First the pictures:

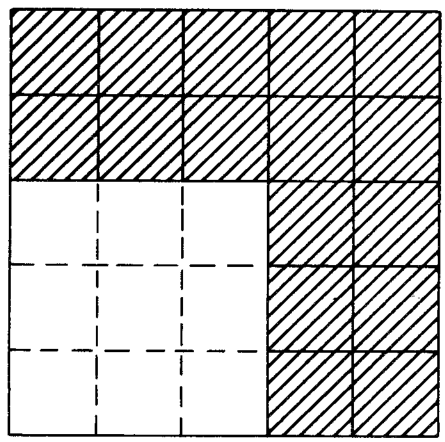
A is odd. Arrange in equal vertical and horizontal legs, single lines, the total value of A^2. The following example is for $A = 3$:



From this drawing, A^2 encloses on two sides B^2, and the total is C^2. From this it follows that the values of B and C are:

$B = (A^2 - 1)/2$, and $C = (A^2 + 1)/2$.

A is even. Arrange the value of A^2 in a double vertical and horizontal row. The following example is for $A = 4$:



From this drawing, $B = (A^2 - 4)/4$, and $C = (A^2 + 4)/4$.

Analytically, Mr. Hopkins also separates the two cases: A is odd, $A^2 = A^2 \cdot 1 = [(A^2 + 1)/2 + (A^2 - 1)/2] \cdot [(A^2 + 1)/2 - (A^2 - 1)/2] = [(A^2 + 1)/2]^2 - [(A^2 - 1)/2]^2$.

A is even, $A = A^2/2 \cdot 2 = [(A^2 + 4)/4 + (A^2 - 4)/4] \cdot [(A^2 + 4)/4 - (A^2 - 4)/4] = [(A^2 + 4)/4]^2 - [(A^2 - 4)/4]^2$.

Also solved by Gerald Blum, William Butler, Emmet J. Duffy, Kenneth Friedman, George Grover, Winslow Hartford, P. V. Heftler, Richard I. Hess, Ken Kiesel, Judith Q. Longyear, Bob Lutton, Thomas O. Mahon, Jr., James H. Michelman, John D. Mill, William J. Moody, Avi Ornstein, R. Robinson Rowe, Frank Rubin, Norman M. Wickerstrand, and the proposer, Winthrop Leeds.

MAY 4 Starting at zero-zero latitude and longitude at 12:00 M on Sunday, Aaron Ott flew his plane at a constant 225 knots loxodromically North 60° West. Where was he at 12:00 M on Monday?

The proposer, R. Robinson Rowe, set two traps and no one else avoided both pitfalls. The first trap is that you are to use local time, the second is that there are *two* solutions. His solution follows:

Let λ = west longitude, θ = north latitude, φ = colatitude = 90° − θ, A = rhumb angle between path and meridian, V = velocity, S = length of path in degrees, and T = elapsed time in hours. From the general differential equations of the loxodrome, $d\lambda = \pm \tan A \cdot d\phi / \sin \phi$ and $dS = \sec A \, d\phi$, the boundary conditions derive the particular relations:

$$\lambda = 180\sqrt{3}/\pi[\ln \cot (45° - ½\theta)] \quad (1)$$

$$S = 2e \quad (2)$$

$$T = 4S/15 \quad (3)$$

(If this is not clear, note that A = 60°, tan A = $\sqrt{3}$, sec A = 2, and that V = 225 knots is equivalent to 3.75° per hour.)

Note first that when Aaron Ott had flown 24 hours, he would be in a different time zone where it would not yet be 12:00 M. From a rough approximation or graphical solution, it appears that he will meet that time condition in the ninth or tenth time zone west of the Greenwich meridian.

Try the ninth time zone (Yukon). Then T = 24 + 9 = 33. From (3), S = 123.75. Then from (2) θ = 61.875, and from (1) λ = 137.38163. This longitude would use Yukon Time (135th Meridian), checking the assumption.

Try the 6th time zone (Alaskan). The same procedure finds T = 34, S = 127.5, θ = 63.75 and λ = 144.49218. This longitude would use 150th Meridian (Alaskan) Time, also checking the assumption. Thus there are two solutions; twice on Monday Ott passes over spots where standard time is 12:00 M, viz:

1. At latitude 61° 52′ 30″, longitude 137° 22′ 54″, on the Nisling River north of Aishinik, YT.
2. At latitude 63° 45′, longitude 144° 29′ 32″, on the Tanana River near Dot Lake, Ak.

Also solved by William Butler, R. I. Hess, James Shearer, and Harry Zaremba.

MAY 5 A band of pirates was chased, and one was caught. A search of the pirate was made, and a description of the location of buried treasure was found; it read: "From the great tree are nine rock formations. Counting from left to right turn around at the ninth rock counting the eighth as ten then again turning around at the first rock, counting it as the seventeenth, the second rock the eighteenth, etc. When the number 1,000 is reached, the treasure is buried five paces north of this rock." One of the natives read this description and immediately figured out where the treasure was located without going through all the steps. What formula did he use? And near what rock was the treasure buried?

Edward Friedman found this to be almost too easy. His solution: Except for the first trip, eight piles are counted on each trip and 16 on each round trip (one must be added for the first trip). Since 1,000 is an odd multiple of 8, we must be one pile from the end on a "forward" trip. Therefore we must be at the eighth pile.

Also solved by Gerald Blum, William Butler, Kenneth Friedman, Ben Gunter, Winslow Hartford, R. I. Hess, Neil Hopkins, Winthrop Leeds, Judith Q. Longyear, Bob Lutton, John D. Mill, Avi Ornstein, R. Robinson Rowe, Frank Rubin, James Shearer, George Sinclair, Hal Vose, Anthony M. Weiner, and Harry Zaremba.

Better Late Than Never

JAN 1 The published solution is wrong; the final position is not mate! To explain how I could have failed to notice this obvious shortcoming, I should mention that the only chess award I have ever received is the Horatio Q. Patzer Award for the most rapid resignation at a Rockefeller University simultaneous event. The following solution is from Norman M. Wickerstrand and Jack O. Dunn:

1	N − QB7	ck	K − Q5
2	R − B4	ck	P × R
3	P − B3	ck	P × P
4	P × R	ck	P × P
5	R − K4	ck	P × R
6	B − K5	ck	P × B
7	N(Q6) − N5	ck	R × N (a)
8	Q − Q8	ck	Q − Q3 (b)
9	Q × Q	ck	B − Q4
10	Q × B	ck	P × Q
11	N × R	mate	

(a) If P × N, white's penultimate move is mate.
(b) If B − B4, skip the next move.

Responses were also received from Peter Bishop, Michael Laufer, Charles H. Pierce, Frank Rubin, and Jerome J. Taylor.

Solutions have also come from the following to the problems indicated:
DEC 2 Uri Reychav and Frank Rubin.
JAN 2 David Kaufman.
JAN 4 William J. Butler.
JAN 5 William J. Butler.
M/A 1 Gerald Blum and Frank Rubin.
M/A 3 Gerald Blum, Gregory C. Daley, Winslow Hartford, William T. Moody, Carl F. Muchenhoupt, R. Robinson Rowe, and William Swedush.

Proposers' Solutions to Speed Problems

O/N SD 1 In those states, as elsewhere, barrenness cannot be hereditary.

O/N SD 2 February 29, 2000. There is always a gap of 28 years unless some peculiar year like 1900 (multiple of 100 but not 400) intervenes.

Allan J. Gottlieb studied mathematics at M.I.T. (S.B. 1967) and Brandeis (A.M. 1968, Ph.D. 1973); he is now Assistant Professor of Mathematics at York College of C.U.N.Y. Send problems, solutions, and comments to him at the Department of Mathematics, York College, 150-14 Jamaica Ave., Jamaica, N.Y., 11432.