Crossing a 1,000-Mile Desert In a 10-m.p.g. Car

Puzzle Corner
by
Allan J. Gottlieb

Hello again.

During a weak moment, your editor has been convinced to play rugby. When I was at M.I.T. our intramural football games were often played adjacent to the rugby field. Whenever our ball strayed onto their field, we would wait until the "maniacs" went to the other end of their field before retrieving our ball. Surprisingly enough, I was only moderately hurt (bruised ribs) during our first game, and after missing game two I am ready for the third game next week. This may explain any columns for the remainder of this year which seem disjointed.

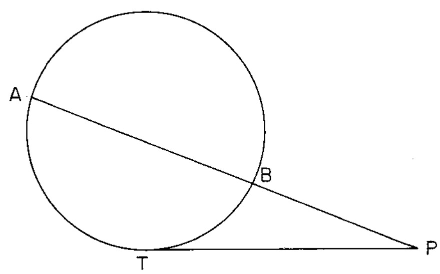
Problems

DEC 1 We start this month with a bridge problem from Michael Kay. With the following hands, South has a seven-hearts-doubled contract and West leads the ♠9. Can South make his contract?

	North	
	♠ A Q 10 7 ♥ 9 7 6 5 4 3 ♦ 2 ♣ Q 7	
West ♠ 9 8 5 4 ♥ J ♦ K Q 4 3 ♣ K J 9 8		**East** ♠ K J 6 3 ♥ 10 ♦ 9 7 6 5 ♣ 5 4 3 2
	♠ 2 ♥ A K Q 8 2 ♦ A J 10 8 ♣ A 10 6	

DEC 2 In January, 1974, Donald E. Savage sent us the following published as a "speed" problem: "Each of N dogs located at the N vertices of a regular N-gon simultaneously sees the dog at the next clockwise vertex and runs toward him. All the dogs run at exactly the same speed and thus finally meet in the center of the polygon; how far will each dog travel?" Now Mr. Savage is back with a similar, but harder, problem: Four hovering hummingbirds are equidistant from each other when the first sees the second, the second sees the third, the third sees the fourth, and the fourth sees the first. Simultaneously each begins flying toward the one it sees, always pointing directly at it; they all fly at the same speed. How far will each travel before there is a collision?

DEC 3 The following geometry problem is from Joseph Horton: In the diagram, PT is tangent to the circle and we are given that AB = PT. Prove that (AP)(BP) = $(AB)^2$.



DEC 4 Lawrence Smith wants you to prove or disprove the following: Given that $A^2 + B^2 = C^2$, that A, B, and C are integers, and that C is not prime; then, for some factor F, of C, there exist integers G and H such that $G^2 + H^2 = F^2$.

DEC 5 William Wagner has a question apropos our energy situation; he says it was brought to him by one of his students whose father picked it up at Xerox Research in Menlo Park, Calif. Apparently no one there has solved it completely (minimally) yet. The problem: To determine what strategy minimizes the amount of gas consumed (= miles travelled) traversing a 1,000-mile flat desert by car. The car gets 10 m.p.g.; the tank holds 25 gal., and gas may be carried in no other way; gas may be left at any point along the way (essentially, the tank holds 250 *miles*, and miles may be left anywhere along the way); there is an unlimited supply of gas at mile 0; but, except for what you leave, gas is not available anywhere along the way.

Speed Department

DEC SD 1 A matrix quickly from W. Arendt: Given that the sum of the elements of X is 1 and that

$$X^tBX = 3;\ B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix},$$

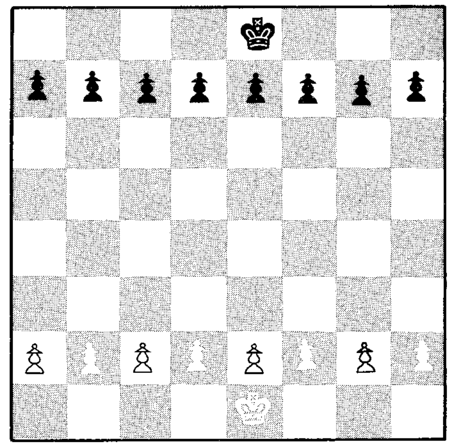
find X (X^t means X transpose).

DEC SD 2 The following is from R. Robinson Rowe: A fence around a circular corral is as long in rods as there are acres contained. What is the radius of the corral?

Solutions

The following are solutions to problems published in the June issue.

JUN 1 What is the minimum number of moves needed to reach the position shown at the top of the next column:



Ted Mita, Eric Jamin, and the proposer, Frank Rubin, were able to solve this in 18 moves, but the best is the following in 17 moves from Richard I. Hess:

1	N-KB3	N-KB3
2	N-K5	N-K5
3	N-N6	N-N6
4	NxR	NxR
5	N-N6	N-N6
6	NxB	NxB
7	KxN	KxN
8	N-R3	N-R3
9	N-B4	N-B4
10	N-N6	N-N6
11	NxR	NxR
12	N-N6	N-N6
13	NxB	NxB
14	N-Q6	N-Q6
15	Q-K1	Q-K1
16	NxQ	NxQ
17	KxN	KxN

JUN 2 Consider a regular pentagon whose sides have unit length. Draw the five diagonals of the pentagon thus creating a five-pointed star (a pentagram) which in turn encloses a smaller regular pentagon. If this process is repeated indefinitely, show that the sum of the perimeters of the infinity of regular pentagons so created, including that of the original one, is 5ϕ units where ϕ is the so-called golden mean $[= \frac{1}{2}(1 + \sqrt{5})]$ and that the sum of the perimeters of the infinity of pentagrams so created is ten units.

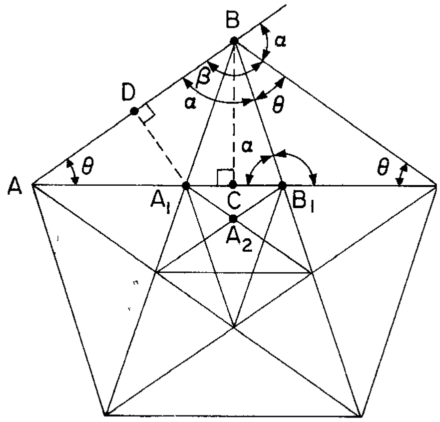
The solution shown at the top of the column, next page, was submitted by Harry Zaremba:

In the figure, $\alpha = 360°/5 = 72°$, $\beta = 180 - \alpha = 108°$, $\theta = (180 - \beta)/2 = 36°$, and AB = a = 1. By construction, AD = DB = a/2, and since triangle ABB_1 is isosceles, AB_1 = a = 1. From triangles ADA_1 and ABC, $AA_1 = a/(2\cos\theta)$, and $AC = a\cos\theta$.

Also, $A_1B_1 = 2(AC - AA_1)$

$$= 2[a\cos\theta - a/(2\cos\theta)] = aw, \quad (1)$$

where $w = (2\cos^2\theta - 1)/\cos\theta$. By relation (1), $A_1B_1 = w \cdot AB$. Thus, any two



successive similar pentagons and pentagrams will be related by the factor w, and the sums of the perimeters of the infinity of their number will form geometric progressions. If P_1 and P_2 are the sums of perimeters of the pentagons and pentagrams, respectively, then:

$$P_1 = 5a(1 + w + w^2 + \ldots)$$

$$= 5a/(1 - w), \text{ and} \qquad (2)$$

$$P_2 = 10a/(2\cos\theta)(1 + w + w^2 + \ldots)$$

$$= 5a/[\cos\theta(1 - w)]. \qquad (3)$$

Also from the figure,

$$A_1B_1 = AB_1 - AA_1 = a - a/(2\cos\theta), \text{ or}$$

$$A_1B_1 = (2\cos\theta - 1)a/(2\cos\theta). \qquad (4)$$

Equating (4) and (2) and solving for cos θ, $\cos\theta = (1 + \sqrt{5})/4$. Hence, $w = (\sqrt{5} - 1)/(\sqrt{5} + 1)$, which when substituted with cos θ into (2) and (3) gives

$P_1 = 5(\sqrt{5} + 1)/2 = 5\phi$, and

$P_2 = 10$ $(a = 1)$.

Also solved by Richard Hess, Eric Jamin, John E. Prussing, R. Robinson Rowe, Frank Rubin, Joseph Haubrich, and the proposer, William Thompson.

JUN 3 Enter in the open squares all the missing numbers from 1 to 256 (no duplications), so that the total of all the numbers in each horizontal row, the total of all the numbers in each vertical row, and the total of the numbers in each of the two diagonal rows (corner to corner) will equal 2,056 in each of the three categories.

The solution shown at the right is from Mrs. Leonard Fenocketti; the numbers in grey are those originally given, those in black are Mrs. Fenocketti's:

Also solved by Richard Hess, Frank Rubin, Eric Jamin, Dick Boyd, Harry Zaremba, William Wong, and the proposer, Mark Yellon.

JUN 4 A man is sitting in a wooden rowboat in his swimming pool. Both the pool and the boat have gauges on them to measure the water level. How does each gauge reading change as the water level changes, when:

1. The man sits in the boat?
2. The man loads the boat with bricks?
3. The man drops all the bricks overboard into the pool?
4. The boat develops a fast leak and swamps?

This is similar to a "basic smartness test" which I was unofficially given while an employee of Grumman Aircraft. I don't remember how I did, but I was rehired for eight summers. Joseph Haubrich would surely qualify; here is his solution:

When the man sits in the boat, it displaces more water and thus both gauges go up. The same thing happens when the man loads the boat with bricks, assuming the boat will hold the bricks. When the man dumps the bricks overboard, the boat, freed of the weight, moves up, so its water gauge shows falling water; the bricks sink, and the water in the pool lowers because the bricks do not displace their full weight any more. As the boat sinks, the gauge on the boat shows water rising; the gauge on the pool shows no change, because people and wooden boats are still slightly buoyant, displacing the same amount of water as they did previous to the sinking.

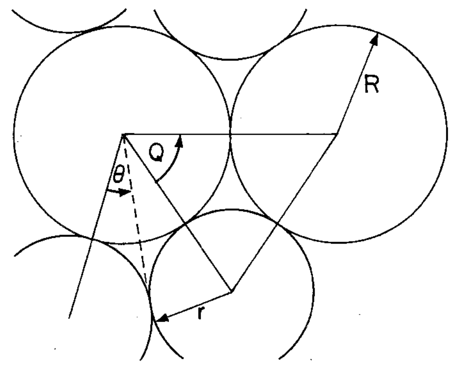
Also solved by Frank Rubin, Richard Hess, Capt. E. Jarman, R. Robinson Rowe, Bruce Fleischer, Harry Zaremba, Ted Mita, and the proposer, Roy Schweiker.

JUN 5 Two large coins and six small coins are placed on a table, each just touching its neighbors. What are the relative diameters of the coins?

The following is from Horace and Alfred Sklar:

First we must define "touching" as follows: two touching circles have a common tangent and hence the line joining their centers goes through the point of contact. Defining θ and φ as in the drawing at the top of the next column, we have:

1. $\cos\varphi = R/(R + r) = 1/(1 + \alpha)$;
$\alpha = r/R$, $0 < \varphi < 90°$
2. $\sin\theta = r/(R + r) = \alpha/(1 + \alpha)$;
$0 < \theta < 45°$



These two equations have three unknowns. The third equation is obtained by counting all the angles:

3. $10\,\theta - 2\varphi = 2\pi$

Solving:

$\cos\varphi + \sin\theta = 1$

$\sin\theta + \cos(\pi - 5\theta) = 1$

$\sin\theta - \cos 5\theta = 1$, $0 < \theta < 45°$

A numerical solution using the Newton approach is now done:

$\theta_{k+1} = \theta_k - f(\theta)/f'(\theta)$

$f'(\theta) = \cos\theta + 5\sin 5\theta$

$f(\theta) = \sin\theta - \cos 5\theta - 1$

$\theta_{k+1} = \theta_k - (\sin\theta - \cos 5\theta - 1)/(\cos\theta + 5\sin 5\theta)$

Let $\theta_0 = 25° = .436\text{---}$ rad.

$\theta_1 = \theta_0 - f(.436\text{---})/f'(.436\text{---})$

$= \theta + .00076$

$\theta_1 = .437093059$

$\theta_2 = .437093916$ (error is less than 10^{-10})

$\theta_2 = 25.04363665°$

$.423308386 = \alpha/(1 + \alpha)$

256	243	242	253	48	35	34	45	32	19	18	29	208	195	194	205
245	250	251	248	37	42	43	40	21	26	27	24	197	202	203	200
249	246	247	252	41	38	39	44	25	22	23	28	201	198	199	204
244	255	254	241	36	47	46	33	20	31	30	17	196	207	206	193
80	67	66	77	160	147	146	157	176	163	162	173	128	115	114	125
69	74	75	72	149	154	155	152	165	170	171	168	117	122	123	120
73	70	71	76	153	150	151	156	169	166	167	172	121	118	119	124
68	79	78	65	148	159	158	145	164	175	174	161	116	127	126	113
144	131	130	141	96	83	82	93	112	99	98	109	192	179	178	189
133	138	139	136	85	90	91	88	101	106	107	104	181	186	187	184
137	134	135	140	89	86	87	92	105	102	103	108	185	182	183	188
132	143	142	129	84	95	94	81	100	111	110	97	180	191	190	177
64	51	50	61	240	227	226	237	224	211	210	221	16	3	2	13
53	58	59	56	229	234	235	232	213	218	219	216	5	10	11	8
57	54	55	60	233	230	231	236	217	214	215	220	9	6	7	12
52	63	62	49	228	239	238	225	212	223	222	209	4	15	14	1

$\alpha = .73402903 = r/R$
$R/r = 1.362343939$.

Also solved by John E. Prussing, Winthrop Leeds, Carl Muckenhoupt, Winslow Hartford, R. Murphy, Meredith Schoppee, R. Robinson Rowe, Richard Hess, Joseph Haubrich, Harry Zaremba, James Friend, John Bobbitt, Ken Kivenko, Ralph Runels, Jeffrey L. Kenton, Eric Jamin, J. J. Williams, Arthur W. Anderson, and Ruth Fox.

M/A 4 (as revised in June) The God of Truth and the God of Falsehood are obvious; then there is also the God of Malice, who gives random answers to any question. You are to ask three questions and determine from the answers who is who.

The following is a slightly modified version of a solution submitted by Homer Schaaf; the idea of this solution is to use the first question to find one god who is *not* the God of Malice. Initially there are six possibilities:

	G^1	G^2	G^3
1	T	M	F
2	T	F	M
3	M	T	F
4	M	F	T
5	F	T	M
6	F	M	T

Let the first question be addressed to G^1: "The God of Truth and the God of Falsehood are opposites; the God of Malice is not the opposite of any god. If I asked you if G^2 were your opposite, would you say yes?" An answer of "yes" eliminates permutations 1 and 6 (so that G^2 is not the God of Malice), and an answer of "no" eliminates 2 and 5 (G^3 is not the God of Malice). Now ask the god known *not* to be Malice if 1 plus 1 is 2; if he says "yes" he is Truth, and if he says "no" he is Falsehood. You know, then, whether or not to believe him when you ask him who is the remaining God.

Responses were also received from John Joseph, R. Robinson Rowe, Eric Jamin, Ted Mita, Neil Hopkins, and Carl Faflick.

Better Late Than Never

PERM 1 As mentioned previously, only solutions without the greatest integer function will be printed. I have also omitted solutions using a decimal point. Recall that $x^{1/2} = \sqrt{x}$ is legal. This month's contributions come from Greg Girolami, Eric Jamin, Alfred Aburto, Harry Zaremba, Frank Rubin, Woodrow Johnson, and an anonymous doctor from Bridgeport, Conn. This month I present solutions from 257 to 300—a series in which there are still many gaps. For 1 to 256, see several issues in last year's volume. Solutions to numbers omitted from the following have already been printed.

$257 = 3!!/\sqrt{9} + 17$
$258 = \sqrt{9}![7(3!) + 1]$
$259 = 37(1 + \sqrt{9}!)$
260 =
261 =
$262 = 7[\sqrt{9}! + (3! - 1)!]$
263 =
264 =
265 =
$266 = 3(91) - 7$
267 =
268 =
269 =

$270 = 9^{1/2}3!!/(7 + 1)$
$271 =$
$272 = 7(39) - 1$
$273 = 7(39)1$
$274 = 7(39) + 1$
$275 =$
$276 = 17(9) - 3$
$277 =$
$278 =$
$279 = 9[7 + (3 + 1)!]$
$280 = 91(3) + 7$
$281 = 7!/(3!9^{1/2}) + 1$
$282 =$
$283 = (3! - 1)! + 7(9)$
$284 =$
$285 =$
$287 = (9^{1/2}!)^3 + 71$
$288 = (97 - 1)3$
$289 = 17^{(3!/9^{1/2})}$
$293 =$
$294 = (97 + 1)3$
$295 =$
$296 =$
$297 =$
$298 =$
$299 =$
$300 =$

One correction has been received:
$108 = 9(7 + 3! - 1)$

M/A 3 Comments were received from George Cain and R. Robinson Rowe.

M/A 5 Winthrop Leeds prefers the following solution, since it uses common words:

P	R	A	M
L	E	V	Y
U	N	I	T
G	O	S	H

MAY 2 Another proof was submitted, this one from A. C. Williams.

Proposers' Solutions to Speed Problems

DEC SD 1 $\begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$

DEC SD 2 Each rod of fence bounds a one-acre sector, with an area of ½R. Since 160 square rods equal an acre, the radius must be 320 rods, which is just one mile.

Allan J. Gottlieb studied mathematics at M.I.T. (S.B. 1967) and Brandeis (A.M. 1968, Ph.D. 1973), and he is now Assistant Professor of Mathematics at York College of the City University of New York. Send problems, solutions, and comments to him at the Department of Mathematics, York College, 150-14 Jamaica Ave., Jamaica, N.Y., 11432.

Books

Management Is Not Enough

Management: Tasks—Responsibilities—Practices
Peter F. Drucker
New York: Harper and Row, 1974; 839 pp., $15.00

Reviewed by Leopold R. Michel and Richard B. Maffei

At the outset of this immensely comprehensive, pro-management work, Peter Drucker announces that "performing, responsible management is the alternative to tyranny and our only protection against it. . . . If our institutions do not perform in responsible autonomy, we will not have individualism and a society in which there is a chance for people to fulfill themselves."

Can we advance toward happier fulfillment simply by emerging from the "management boom" (the decades of the 1950s and 1960s) and pursuing now a straight path toward "management performance" through better knowledge of the tasks and required skills of the managers? This appears to be the central thrust of this book, and through it all there remains the unannounced but profusely illustrated implicit need to be moral and virtuous. To be sure, Mr. Drucker sidles away from any blatant statement, hewing closely to that pragmatic position that has always made his work directly appealing to businessmen and less acceptable to some scholars in the field. But he has clearly moved to weld ethics to practice in ways that are acceptable to practitioners of business in industrial institutions.

However, there remains the need to defend efficient practices and performance by management with a well defended philosophical posture. Drucker does not do this. Perhaps he regards it a non-problem; but to these reviewers it is the heart of the problem. Tyranny in the past has been made possible by excellence in management and planning; tyrants have been able to manage their uprisings. Externalities neglected by yesterday's and today's managers now cause national and international concern; in even the least productive bureaucracies there is much traditional "management." In his emphasis on people and planning, Drucker neglects both entity goals and aggregate system goals. But especially after the nation's moral trauma of Mr. Nixon's second term, we clearly need a statement of management philosophy which focuses on the moral and human issues—the need to provide every individual on spaceship earth with a viable and aesthetic set of economic purposes, and to provide each institution with a larger sense of values and purpose than can be measured by efficiency, income, and profit. Legitimacy and quality of life deserve deeper and extended interest.

There are guidelines in bodies of thought outside the field of management which need to be known and shown: cultural and philosophical anthropology, selected fragments of theology, and legal theory.

Though this book is a milestone on the way toward a richer definition of freedom and responsibility through viable organized practice, it is also pragmatic and question-begging. "We are moving from management boom to management performance," writes Mr. Drucker, proposing what he calls the leitmotiv of this book. A new opus—grand and enjoyable. But we are now seeking a differently orchestrated leitmotiv.

Richard B. Maffei is Associate Dean—Academic Affairs of the School of Management at Boston College. Leopold R. Michel, a management consultant, was Visiting Lecturer in the Boston College School of Management before his retirement one year ago. Both have studied at M.I.T., Dr. Maffei in the Sloan School of Management (where, he has been a member of the faculty) and Mr. Michel in the Department of Mechanical Engineering.

Inflation: No Simple Cure for Subtle Causes

The Earnings Conflict
Wilfred Brown
New York: John Wiley and Sons, 1973; 126 pp.

Reviewed by Daniel Quinn Mills

The problems of wage and price inflation continue to bedevil the British even more than they do the U.S. Wilfred Brown, an industrialist and writer on management topics, has proposed in *The Earnings Conflict* a plan to resolve the inflation problem in Britain. Brown has some 25 years of experience as managing director and chairman of a British manufacturing corporation and has held influential government posts in recent years, so his ideas deserve to be taken seriously.

Brown perceives inflation in the British economy as an economic problem with