When Did Who Pour the Beer?

There have been several responses concerning our first permanent problem. This problem, originally numbered O/N 3, is now officially numbered PERM 1. A spin-off of this problem is now presented as MAY 2. As you may recall, PERM 1 asks you to construct expressions yielding all the integers using only the digits 1, 9, 7, and 3 together with various operators. By far the most controversial operator is the square brackets denoting greatest integer. As a result of this controversy I will print revised solutions if someone can generate an integer without the square brackets (see “Better Late Than Never,” below).

Eric Jamin has answered a query of Phelps Meaker. Mr. Jamin points out that PERM 2 as suggested in Dudney’s Amusement in Mathematics (Dover, 1958).

MAY 1. The following bridge problem is from Michael Kay:

\[
\begin{align*}
\text{K} & 3 2 \\
\text{J} & 5 2 \\
\text{Q} & 7 2 \\
\text{A} & Q J 10 5 \\
\text{K} & 6 4 \\
\text{A} & 9 7 \\
\text{A} & \\
\end{align*}
\]

Bidding:

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<tr>
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<th>N</th>
<th>E</th>
<th>S</th>
<th>W</th>
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<td>P</td>
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<td>4C</td>
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<td>4D</td>
<td>P</td>
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<tr>
<td>6S</td>
<td>P</td>
<td>P</td>
<td>P</td>
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</tbody>
</table>

North-South, playing the precision club system, arrive at an optimistic contract of six spades. The opening lead is ♥A, followed by ♥Q. East follows to the second heart as you win with the king. How can South make the contract?

MAY 2. Prove that PERM 1 can be solved for all integers. Bruce Appleby has sharpened this natural conjecture to the following: Let n be any integer greater than 2. Prove that the set of numbers

\[
\left\lfloor \sqrt{\sqrt{\cdots \sqrt{n}}} \right\rfloor
\]

is dense on the interval [1, ∞).

MAY 3. Hervé Thiriez is interested in square matrices composed of just zeros and ones. He feels that any such matrix of size n by n can have determinant no greater than \(F_n\) (F for Fibonacci) where \(F_n\) is defined by \(F_1 = F_2 = 1\) and for \(n \geq 3\)

\[
F_n = F_{n-1} + F_{n-2}
\]

There are at least three \(F_n = F_{n-1} + F_{n-2}\). There is an example to show that \(F_n\) is actually achieved. Is M. Thiriez correct?

MAY 4. Neil Cohen wants to know how many different possible bridge auctions (legal sequence of bids) there are.

Jim Cassidy wants to know about some Hanky Panky At The Picnic:

MAY 5. There was this picnic attended by Belinda the wife, Henry her husband, Joe their son, Mimi their daughter, and Pete, Belinda’s brother. At some time during the picnic one of the members poured a can of beer over the head of another member. At that time:

1. A man and a woman were at the table.
2. The victim and the guilty one were at the beach.
3. One of the children was in swimming.
4. Belinda and her husband were not together.
5. The victim’s twin was not the guilty one.
6. The guilty one was younger than his victim.

Who did it?

Speed Department

Frank Rubin has submitted the following:

SD 1 Many problems present scores on a target such as 17, 23, 32, 40, 49, and ask how a shooter totals 100 points. I want you to devise a best such problem in the following senses (i.e. there are two problems here):

1. The target whose score total is maximum and on which some number from 1 to 100 cannot be achieved.
2. The target whose score total is a maximum on which a score of 100 cannot be achieved.

Note that in each part all scores must be distinct integers in the range 1 to 100.

Winthrop M. Leeds has sent the following quickie which he attributes to Sam Lloyd.

SD 2 On a checkboard, place the White king on Q63, the White queen on KN4, and a white bishop on KN1. The problem is to place the Black king on a particular square where it is not in check and yet where White on the move can Mate in one.

Solutions

The following solutions are to problems published in the January issue.

O/N 1 as revised in January: Given a game consisting of all legal moves terminating with the board as shown—what piece (black or white) is at x?

The following solution is from Theodore Mita:

The piece on White’s KR4 is White’s black-square bishop.
1. Black’s king is in check, so White’s last move was to take Black’s piece on Q8 with his pawn, promoting it to a rook.
2. The piece taken by this pawn was not a rook or a queen, since White’s king is on the eighth rank and White would have been in check prior to Black’s last move.
3. Black’s black-square bishop must never have moved (pawns on Black’s K2 and KN2) and there are two Black knights on the board; so the piece taken by this pawn was a promoted Black pawn. This accounts for all eight black pawns.
4. The piece, X, could not be Black’s queen or a Black rook again due to the placement of the White king. Also, Black’s dark-square bishop and his two knights are accounted for, as are all his pawns (see 2). Thus the piece was White.
5. The minimum number of White pieces taken by Black’s pieces (including the KRP which promoted) is 5. White has 10 pieces showing on the board, so 5 is the maximum number of pieces Black can have taken in order for a piece to be on White’s KR4.
6. All five pieces taken by Black’s pawns were on White squares, so the only possible piece left is White’s black-square bishop.

Responses were also received from Eric Jamin, Harry Zarembo, Peter Silverberg,
The following is a composite of the solutions submitted by Frank Rubin, Kevin O'Brien, and the proposer, Andrew Fink:

\[ \begin{align*}
A & \quad B \\
1 & P-K3 \\
2 & P-K4 \\
3 & P-K4 \\
4 & P-K3 \\
5 & P-K3 \\
C & \quad D \\
1 & B-KN3 \\
2 & B-K3 \\
3 & B-K3 \\
E & \quad F \\
1 & P-QN3 \\
2 & P-QB3 \\
3 & B-R3 \\
4 & B-R6 \\
5 & B-R5 \\
6 & B-R6 \\
7 & B-BN3 \\
8 & B-R5 \\
JAN 2 Barry Basset makes berry baskets with six pieces cut from a rectangular strip of wood material, using the accompanying pattern for his no-waste dissection. Three of the pieces are isosceles trapezoids and another is made from two halves; the sixth piece is the bottom of the basket, which when finished looks like the isometric figure. If the area of the strip is one square foot, what is the level-full capacity of the biggest berry basket Barry Basset can make?

Very interesting responses to this one. Every response agreed to four places, but there most differed. Both Robert Pogoff and Dr. Winslow Hartford arrived at a high-order polynomial which they used some numerical method to solve (or circumvent). Jack Parsons fell into a trap clearly set by the proposer and assumed that the horizontal section is a square. As Mr. Rowe points out, this leads to a solution which is approximately 0.009 882 9 ft. too small. However, Mr. Parsons was able to give an exact irrational answer, whereas Mr. Rowe felt that anyone who fell into the trap could only get an approximate solution (of the type found by Mr. Pogoff and Dr. Hartford). My personal favorite was submitted by Harry Zarembea. However, it used the Lagrange multiplier and every time I print such an "advanced" solution I have to tell me that this supposed to be a friendly column and not a mathematics journal. The following solution (essentially an elementary version of Mr. Zarembea’s) is from Eric Jamin, and a similar solution was received from the proposer.

Nothing in the stated problem imposes, as the drawing would make believe, the bottom piece to be a square. So let it rather be a rectangle of dimensions (2a \times (2b)\). It is quite obvious that, with sides being isosceles trapezoids, the top of the basket will be a rectangle of dimensions (2a + 2y) \times (2b + 2y). Thus we have the cutting pattern

\[ \begin{align*}
2a + 2y & \quad 2a + 2y \\
2b & \quad 2b \\
\alpha - y & \quad 2 \alpha - y \\
\alpha - y & \quad 2 \alpha - y
\end{align*} \]

with constant area 2a(4a + 6b + 4y) = A. Considering one fourth of the basket, for the volume V/4 = \( a \times b \times h \) (parallelepiped)

\[ + \frac{1}{2} bh^2 \]

\[ + \frac{1}{3} \alpha^2 h \]

\[ + \frac{1}{3} \alpha^2 h \]

with the altitude given by

\[ h = \frac{1}{4} \sqrt{a^2 - x^2} \]

Setting y = 2x and A = 12k, we have

\[ b = \frac{k - 5x}{3x^2} - 3x^3, \]

\[ V/4 = \left( a^2 - x^2 \right)^{1/4} (ak - 2a^3/3 - \alpha x + kx^3/2), \]

or, letting z = x/a,

\[ V/4 = \left( 1 - z^2 \right)^{1/4} (ak - 2a^3/3 - \alpha z^2 + a k z) \]

To maximize, we set the partials of V with respect to a and z equal to zero. The first gives

\[ (1 - z^2)^{1/4} (k - 2a^2 - 3a z + k z) = 0. \]

Excluding the case 1 - z^2 = 0, giving V = 0, we have

\[ z = \left( k - 2a^2 \right) / \left( 3a z - k \right) = 0. \]

Since z must be positive we get

\[ \left( k - 2a^2 \right) / a \geq 0 \]

\[ \left( k - 2a^2 \right) / a \geq 0 \]

Next set the second partial mentioned above equal to zero and multiply both sides of the resulting equation by \( \left( 1 + (4 a^2 x^2)^{1/2} \right) / \left( 4 a^2 \right) \) to get

\[ -2a^2 (k - 2a^2) - \left( k - 2a^2 / 3 \right) \]

\[ + (k - a^2) = 0. \]

Substitute back z = x/a and perform some algebra to get 15(a^2k^3) - 25(a^2/k) + 8 \geq 0

\[ which \ gives \ a^2/k = \left( 25 \pm (145)^{1/2} \right) / 30. \]

The inequality given above forces us to take the minus sign making A/3 a/3

\[ V = 27776 \]

\[ 1046 - 38 (145)^{1/4} \]

But we are given that A = 1 and computing the square roots to three places we get V is approximately 109.

JAN 3 Fill a 3-by-3 square with nine letters so that eight three-letter words are formed; the three rows are to be read left to right, the three columns are to be read top to bottom, and the two principal diagonals may be read in either order.

The following was submitted by Ronnie Appel:

\[ P \ E \ T \]

\[ E \ R \ R \]

\[ A \ N Y \]

Also solved by: George Ropes, Mary Youngquist, William Steel, C. R. Lakin, W. J. Arendt, Avi Orseston, Eric Jamin, Gerald Blum, Frank Blum, R. Robinson Rowe, and the proposer, Karl Kadzielski.

JAN 4 Given a unit circle and a point on it, how large a rectangle, centered at that point, would divide the area enclosed by the unit circle into two equal pieces?

For this problem everyone had to use some numerical method, so no exact solutions were found. The following nearly organized solution from R. Robinson Rowe is typical of the techniques employed:

[Diagram of a circle with a point on it and a rectangle inscribed within the circle.]

In the figure, the area of the unit circle is \( \pi \) and the lune PAMB must have its half. The area of the lune equals the sum of the sectors OAkB and PAMB less the overlapped quadrilateral PAOB. Let the desired radius PA = FB = x = 2a and the angle POB = \( \phi \).

Sector OAkB has an angle 2\( \phi \) and radius 1.

Sector PAMB has an angle pi - \( \phi \) and radius x.

Quad PAOB = 4 rt triangles with base x and hypotenuse 1.

Where

\[ \phi + \frac{1}{2} \pi (x - \phi) - x / 1 - x^2 = \frac{1}{2} \pi \]

(1)

But sin \( \frac{1}{2} \pi \) = u = \( \frac{1}{2} x \), so that (1) can be reduced to either

\[ \pi (x^2 - 1) / 2 \]

\[ x \sqrt{4 - 2 x^2} \sin^{-1} \frac{1}{2} x \]

(2)

or

\[ \frac{1}{2} \pi (1 - 2 \cos \phi) + 4 \cos \phi - \sin \phi = 0 \]

(3)

Neither of these can be solved explicitly. With tables, (3) might be easier, but with a HP-45 computer, (2) worked best.

Since PM was greater than PO, I tried x = 1.1 and 1.2 Interpolating the results, I next tried x = 1.155 and 1.159. A second interpolation led to this result:

\[ x = 1.1587 \]

\[ 2478 \]

\[ \phi = 1.3235 \phi \]

\[ 96923 = 70^8 \times 42\] which checked (1) with 1.3235 \( 96923 + 1.2793 \times 42780 - 0.9444 \times 43378 = 1.5707 \]

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JAN 5 A scoutmaster arrives with his troop at the local bus depot en route to a nearby camping site. Upon walking up to the information agent's booth and submitting a certified check for $525.93 for full payment of the fares of the entire troop (all of the scouts and his own fare, all fares being equal), the scoutmaster is told by the agent that due to the newly enacted "exact-fare" policy, each of the members of the troop will have to pay his own individual fare, giving the driver the exact amount. The agent then proceeds to cash the check, thus exchanging the total-payment check for individual exact-fare amounts which are distributed to members of the troop. As each scout boards the bus, he pays to the driver his individual fare with five coins—the exact same five coins that each other member of the troop utilizes in paying his individual fare. The driver counts the fares collected from the troop and finds that the trip had, in fact, wind up paying the originally calculated amount of the check. How many total nickels did the driver receive?

The following is from Steve Sussman: The prime factors of 52393 are 79 and 67. Therefore 72 scouts paid 67¢ each or 67 scouts paid 79¢ each, A fare of 79¢ in 5 coins requires 4 pennies and a 75¢ piece which is not in our repertoire. It follows that the fare is 67¢ made up of a half dollar, a dime, a nickel, and two pennies. The driver received 79 nickels.


Better Late Than Never
There are many contributions to PERM 1, which began life as O/N3 (see above). The first solutions were published in this column in February; now there are more: Col. E. W. Kelley has made the following corrections:

93 0 = (9 - 3) x + 9 + 1 86 = 79 + 31 + 1
97 = 1 + (3 + 7) x 31 98 = 79 + 31 - 1
99 = 97 + 3 - 1 103 = 91 + 31

Thanks to Richard Haberman, our list can be extended to 130 without the greatest integer.

109 = (31 + 1) x 7 + 3
116 = (9 - 3) x + 9 + 1
117 = 31 + 7 - 1
118 = 31 + 7 - 9
122 = 31 + 7 + 9
123 = 31 + 7 + 9
125 = 31 + 7 + 9
Y. Iwasa has extended our solution to 150 without greatest integer except for 135 and 149:

131 = 137 + 3
137 = 137 + 3
133 = 137 + 3
135 = 137 + 3
140 = 137 + 9
141 = (9 - 3) x + 1 + 3 - 7
142 = 7 x + 9
143 = 137 + 9
144 = (9 - 3) x + 1 + 7 - 9
145 = 7 x + 9
147 = 137 + 9
148 = 31 + 7 + 9
149 = 137 + 9
150 = 137 + 9

The grandest of the grand is Bruce Applebee's extension up to 250. Of course he uses greatest integer. Our next goal will seem to be 135 and 149 without greatest integer.

151 = (9 - 3) x + 1
152 = (9 - 3) x + 1 + 3
153 = (9 - 3) x + 1 + 7
154 = (9 - 3) x + 1 + 9
155 = (9 - 3) x + 1 + 9
156 = 9 x + 1 + 9
157 = 9 x + 1 + 9
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promised in March to examine possibilities for new research and development efforts on both food and materials through its new Science Policy Office, the still-untitled successor to the discarded White House Office of Science and Technology. This assignment would seem to be one of N.S.P.F.'s first tests as an originator of federal science policy.

Victor Cohn reports on major science-oriented affairs for the Washington Post.

Books
Life's Goals Achieved; What Can Follow?

Return to Earth
Edwin E. Aldrin, Jr., with Wayne Warga
New York: Random House, 1973; 338 pp., $7.95

Reviewed by James E. Oberg

In general outline "Buzz" Aldrin's career is well known to most Americans. A graduate of West Point, Colonel Aldrin was a fighter pilot in the Korean War and then was assigned fighter duty in Europe and elsewhere. After earning his Sc.D. (including a thesis in orbital rendezvous) at M.I.T., Aldrin became an astronaut in 1963. He was co-pilot on a Gemini flight in 1966, served on the back-up crew for the first flight around the moon (Apollo 8), and was on the crew of Apollo 11; as such, Aldrin became the second man on the moon on July 20, 1969.

Back from the moon, and following a series of world tours, tumultuous parades, and assorted ceremonial functions, Aldrin left N.A.S.A. late in 1969 to become Commandant of the Aerospace Test Pilot School at Edwards Air Force Base. He retired from the Air Force the following year amid growing rumors of mental problems, severe depressions, and psychiatric help.

In Return to Earth Aldrin has written two books in one, confirming in effect that he has in fact lived two lives in one. The book is a full account of the two important voyages of his life: to the moon and back, and into his own mind. The two half-books may not appeal to a single audience; most readers may be tempted to skip one or the other half. But that is not what the author intends, for Aldrin's purpose is to show that both halves are important for ordinary people as well as for people who walk on the moon.

The first half—Aldrin's astronaut training, his two space flights, the technological, engineering, and scientific problems which had to be faced and overcome to put a man on the moon—is all that anyone can ask for. One might suspect that it has all been told before, but many revealing aspects of space flight could not be described by a man in the pay of N.A.S.A., writing for a family magazine.

When Armstrong, Aldrin, and Collins return to Earth from the moon, Aldrin's most important journey was only beginning. The book in fact begins with Apollo 11 splashdown in the Pacific, in a sense to emphasize Aldrin's feeling that what came after was his real "return to Earth".

The reader may be tempted to assume that the pressures of parades and publicity might be enough to unhinge any man. But Aldrin makes it clear that it was not these pressures—which his colleagues and earlier space heroes like John Glenn had successfully passed—that broke on his depression, mental problems, and "dysfunction." When he finally sought psychiatric counseling—counseling too long delayed by friends' disability to believe that an American here might be anything but perfect, and by Aldrin's concern that a psychiatric record might ruin his career—Aldrin probed deep into his own mind to find the real reasons for his problems.

According to Aldrin, his trouble began with success. He had always been goal-oriented, accepting first his father's guidance, then the rigid codes of West Point, then the directions specified by the Air Force and N.A.S.A. That he truly loved his work was beside the point. Suddenly, in 1968, he was back from the moon a hero with the chance to write his own ticket.

But what could he do to match what he had already finished?

The poets, Aldrin tells us, wrote of "the melancholy of all things done." Worse than the burden of Aldrin's career was the thought that there had no more worlds to conquer. Philosophers have eternally speculated about what would happen to a man who suddenly achieved all his heart's desires; most agreed that it would not be a happy ending.

Perhaps we should fault Aldrin for his inability to form real goals in his life; but if we follow that reasoning we come very close to home, each of us having his own goals toward which he strives and—perhaps mercifully—only approaches but never reaches.

Here Aldrin makes a point for all of us. His mental problems would not have been so serious if at any time he had been able to talk to others at a man, not as an astronaut hero. Aldrin has found a new and meaningful goal: help people recognize their needs for psychiatric counseling, and help them find it soon enough.

The courage of astronauts flying small, fragile spaceships out to the moon and back is proverbial, although the astronauts themselves disclaim any glory or credit for it. They know their spacecraft, and are voyaging along routes well charted and laboriously practiced. Aldrin's point is that he has now journeyed on a truly dark and mysterious voyage of a kind which any of us may one day have to face. His courage in describing his own journey may help thousands, if they have in themselves courage enough to listen and learn.

Captain James E. Oberg (U.S.A.F.) is a member of the staff at the Department of Defense Computer Institute in Washington; he writes frequently on the Soviet and American space programs.