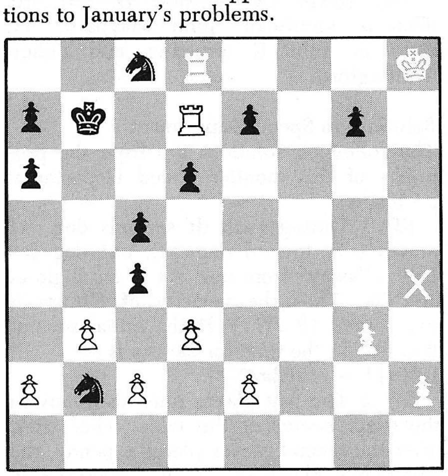
Barry Basset's Berry Basket

Puzzle Corner
by
Allan J. Gottlieb

I've just been introduced to a "war game" called RISK which I enjoyed very much and hope to play fairly often. I seem to remember some organization (perhaps just an informal collection of devotees) involved in playing and popularizing strategic board games. If any of you are "listening," I would appreciate hearing of other such games which are considered good. Especially interesting would be a game which would terminate after an hour or two with four players. The game to which I allude above did not meet this criterion. After nearly four hours of play, Alice and I were eliminated at 1 a.m., leaving our hosts to fight it out. Fortunately, they adjourned the game at that point.

Problem **O/N 1** is in error. The white knight on KR8 should be a white king. The corrected board is shown, and the question is—given a game consisting of all legal moves terminating with the board as shown—what piece (black or white) is at x? A solution will appear with the solutions to January's problems.



Problems
Our chess problem this month is from Andrew Fink.

JAN 1 Set the chess men in their normal starting positions. White and black are to work together to checkmate (selfmate) white, with the restriction that black is not allowed to make any captures. What is the minimum number of moves if black is to move (a) only pawns, (b) only one knight, (c) only one pawn once and the queen, (d) only one pawn once and a bishop, (e) only one pawn once and a rook, and (f) only one pawn once and the king?

The following "biggest berry basket" problem is from R. Robinson Rowe:

JAN 2 Barry Basset makes berry baskets with six pieces cut from a rectangular strip of stock material, using the accompanying pattern for his no-waste dissection. Three of the pieces are isosceles trapezoids and another is made from two halves; the sixth piece is the bottom of the basket, which when finished looks like the isometric figure. If the area of the strip is one square foot, what is the level-full capacity of the biggest berry basket Barry Basset can make?

The following word problem is from Karl Kadzielski:

JAN 3 Fill a 3-by-3 square with nine letters so that eight three-letter words are formed; the three rows are to be read left to right, the three columns are to be read top to bottom, and the two principal diagonals may be read in either order.

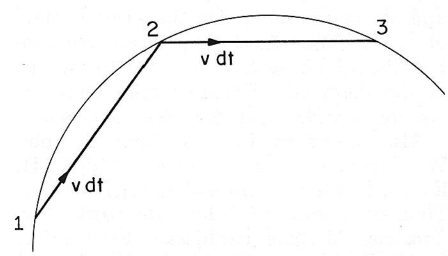
JAN 4 Given a unit circle and a point on it, William H. Collins would like to know how large a circle, centered at that point, would divide the area enclosed by the unit circle into two equal pieces.

The next is from Raymond P. Kremen:

JAN 5 A scoutmaster arrives with his troop at the local bus depot en route to a nearby camping site. Upon walking up to the information agent's booth and submitting a certified check for $52.93 for full payment of the fares of the entire troop (all of the scouts and his own fare, all fares being equal), the scoutmaster is told by the agent that due to the newly enacted "exact-fare" policy, each of the members of the troop will have to pay his own individual fare, giving the driver the exact amount. The agent then proceeds to cash the check, thus exchanging the total-payment check for individual exact-fare amounts which are distributed to members of the troop. As each scout boards the bus, he pays to the driver his individual fare with five coins—the exact same five coins that each other member of the troop utilizes in paying his individual fare. The driver counts the fares collected from the troop and finds that the troop did, in fact, wind up paying the originally calculated amount of the check. How many total nickels did the driver receive?

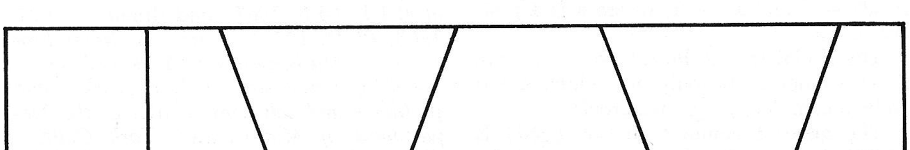
Speed Department
After you have solved **J/A2** (the solution is given on the next page), Donald E. Savage feels it should be easy to generalize to N dogs:

SD1 Each of N dogs located at the N vertices of a regular N-gon simultaneously spots the dog at the next clockwise vertex and runs toward him. All the dogs run at exactly the same speed and thus finally meet in the center of the polygon. How far will each dog travel?

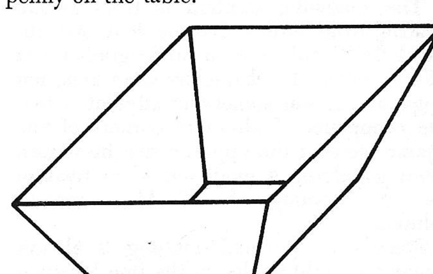


The following is from John T. Rule:

SD2 Each of two persons has an infinite number of pennies. Each takes turn placing one penny at a time on a rectangular table. The pennies may touch each other and may hang over the edge of the table, but they may not overlap. Establish a system so that either the first or second player must inevitably put the last possible penny on the table.



Cutting pattern



Isometric of berry basket

	♠ 6 5 ♥ A K 10 8 2 ♦ 10 4 ♣ K J 7 3	
♠ 3 ♥ Q 7 5 4 ♦ Q 9 7 3 ♣ Q 10 8 5		♠ A 4 ♥ J 9 6 ♦ J 8 5 ♣ A 9 6 4 2
	♠ K Q J 10 9 8 7 2 ♥ 3 ♦ A K 6 2 ♣ —	

Solutions

The following are solutions to problems published in July/August, 1973:

J/A1 South has won a contract of six spades with the hands shown at the top of this page. West's opening lead is ♠3, taken by ♠A. East returns ♠4. How can South make the contract?

The following is from Ed Gershuny:

South drops a high spade under the ♠A and, by playing his ♠2, wins the second trick in dummy. Dummy leads ♣K, East must cover (or South would pitch a small diamond and the other one would be discarded under the second high heart), and South trumps. South then runs the rest of his trumps, coming down to four diamonds and a heart. Dummy must retain ♥A, ♥K, ♥10, a diamond, and ♣J. West must retain ♣Q and either three hearts or three diamonds (his fifth card doesn't matter), while East protects the other red suit. If West has saved hearts, declarer plays two rounds of diamonds. On the second diamond: if East discards the ♣Q, dummy discards the ♥10, winning the last three tricks with two high hearts and the ♣J; if East discards one of his hearts, dummy discards the club and wins the last three tricks with ♥A, ♥K, and ♥10. If West has saved diamonds, declarer will lead a heart and play two rounds. On the second round of hearts, West is also squeezed. He cannot discard his ♣Q, so he must throw one of the diamonds. Declarer then wins the last three tricks with ♦A, ♦K, and ♦6.

Also solved by John G. Connine, John W. Dawson, Charles Estes, Ralph H. Evans, Stanley A. Horowitz, Michael Kay, Thomas Mauther, John Maynard, Avi Ornstein, Michael Padlijosky, Fred Price, Smith D. Turner, David W. Ulrich, and the proposer, Winslow H. Hartford.

J/A2 Each of four dogs, located at the four corners of a square field, simultaneously spots the dog in the corner to his right and runs towards that dog, always pointing directly toward him. All the dogs run at exactly the same speed and thus finally meet in the center of the field. How far did each dog travel?

The following solution is from Lydall Morrill, who writes that he first encountered this problem as an undergraduate at M.I.T., when the characters were ants, not dogs; and it was sometimes stipulated that the occupants of alternate corners of the square were of the opposite sex, he writes, "thus supplying a modicum of motivation for the ensuing chase." Mr. Morrill's solution:

Since each pursued ant/dog is always moving at right angles to the line between him and his pursuer, his own motion contributes nothing to diminish the distance between them. Thus only the pursuer's motion accounts for the steady shortening of that distance. The pursuer always moves directly toward the pursued and must traverse the distance that separated them at the start—namely, the side of the original square field. The rotating motion of the square formed by the four ants/dogs of course causes their trajectories to be spiral in shape, but this is charmingly irrelevant to the distance each pursuer has to travel.

Note that by considerations of symmetry the four dogs must remain on the vertices of a square (of decreasing side length, of course).—Ed.

Also solved by 20 readers, for whose names there simply isn't space this month.

J/A3 A circular table is pushed into the corner of a rectangular room. A coin resting on the edge of the table is 9" from one wall and 8" from the other. What is the diameter of the table?

The following solution is from Phelps Meaker; he complains that "most of your puzzles and problems are way above my waning capabilities." But when he turned the page upside down while studying **J/A3**, "the x and y coordinates fairly shouted for attention":

$(R - x)^2 + (R - y)^2 = R^2$
$(R - 8)^2 + (R - 9)^2 = R^2$
$R^2 - 34R + 145 = 0$; $R = 29$ or 5;
$D = 58''$.

Mr. Meaker continues: "I became curious about the root 5, a positive whole number. Then it came to me that a smaller circle passing through the same point might be crowded into the corner, and a radius of 5 might fit. So I wrote a new equation:

$(8 - R)^2 + (9 - R)^2 = R^2$.

The quadratic was identical to the first, and I had a 10" circle in the corner! It would be nice to hear how Mr. McKinnon created this problem, with nice whole numbers."

Also solved by 35 other readers—a list which is simply too long to print.

J/A4 If you have just been given the dice at a crap table, what are the odds against your winning at least once?

The following is from Winslow H. Hartford:

If you throw a 7 or 11 you win; $P_w = 8/36$. If you throw a 2, 3, or 12, you lose; $P_l = 4/36$. Thus the following table:

Throw	4	5	6	8	9	10
P_{Throw}	3/36	4/36	5/36	5/36	4/36	3/36
P_{Win}	3/9	4/10	5/11	5/11	4/10	3/9

The total probability of a win is $8/36 + 1/36 + 16/360 + 25/396 + 25/396 + 16/360 + 1/36 = 1952/3960$. So the probability of a loss is $2008/3960$.

Also solved by Archie Gann, Jack Parsons, Henry Randall, Frank Rubin, John T. Rule, Smith D. Turner, George Wynne, Harry Zaremba, and the proposer, Jerry Blum.

J/A5 Find the greatest common divisor of $a^m - 1$ and $a^n - 1$, where a is a positive integer.

The following technical solution from S. D. Comer is the only one which seems to be a completely rigorous proof:

The greatest common divisor (gcd) is $a^{gcd(m,n)} - 1$. Proof: Let $z_n = e^{2\pi i/n}$. $F_n(x)$, the nth cyclotomic polynomial, is $\pi_{\substack{1 \leq h \leq n \\ (h,n)=1}} (x - z_n^h)$, where $(h, n) = gcd(h, n)$. $F_n(x)$ is irreducible in $Z[x]$ and its roots (in C) are the primitive nth roots of unity. (1) If k/n, then $(a^k - 1)/(a^n - 1)$. k/n implies $x^n - 1 = \pi_{d/n} F_d(x) =$

$$\pi_{d/k} F_d(x) \cdot \pi_{\substack{d/n \\ d/k}} F_d(x) = (x^k - 1) \cdot \pi_{\substack{d/n \\ d/k}} F_d(x).$$

Now let $x = a$.
(2) Therefore $a^{gcd(m,n)}$ divides both $a^m - 1$ and $a^n - 1$.
(3) $d \neq d'$ implies $F_d(x) \neq F_{d'}(x)$ because every root of $F_d(x)$ generates a cyclic group of order d and every root of $F_{d'}(x)$ generates a cyclic group of order d'.
(4) $F_d(x)$ divides $x^n - 1$ implies d/n. The unique factorization into irreducible factors of $x^n - 1 = \pi_{d'/n} F_{d'}(x)$ so $F_d(x)$ dixides $x^n - 1$ implies $F_d(x) = F_{d'}(x)$ for d'/n. Thus $d = d'$ divides n.
(5) Therefore if $F_d(x)$ divides $x^n - 1$ and $x^m - 1$, then d divides $gcd(m, n)$.
(6) Thus,
$s(x) = \pi_{\substack{d/n \\ d/(m,n)}} F_d(x)$ and $t(x) = \pi_{\substack{d/m \\ d/(m,n)}} F_d(x)$ are relatively prime.
(7) Thus there exists $q(x)$ and $r(x)$ in $Z[x]$ such that
$1 = q(x)s(x) + r(x)t(x)$.
Now multiply both sides by $a^{(m,n)} - 1$:
(8) $x^{(m,n)} - 1 = q(x)s(x)(x^{(m,n)} - 1) + r(x)t(x(x^{(m,n)} - 1) = q(x)(x^n - 1) + r(x)(x^m - 1)$.
Let $x = a$:
(9) $a^{(m,n)} - 1 = q(a)(a^n - 1) + r(a)(a^m - 1)$. So any factor of $a^n - 1$ and $a^m - 1$ also divides $a^{gcd(m,n)}$. From (2) and (9) we get the desired result.

Also solved by Winslow H. Hartford, Thomas Kauffman, John Maynard, Avi Ornstein, John E. Prussing, and Robert Wallingford.

Solutions to Speed Department

The following solutions are from the proposers of this month's Speed Department problems:

SD 1 During each dt seconds dog #1 moves v dt toward dog #2, but dog #2 moves "away" from dog #1 at an angle of 360°/N. Thus the velocity of closure is $v[1 - \cos(2\pi/N)]$. If the initial separation was L, the time for closure is $L/(v[1 - \cos(2\pi/N)]$.

SD 2 The first player puts his penny in the exact center of the table. Thereafter, after the second player places a penny, the first player places his on the opposite side of the table so that the center penny is the mid-point of the line connecting player #1's penny and the last penny of player #2. Player #1 must place the last penny.

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