Touch Football and Tired Camels

Puzzle Corner
by
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Here at Rockefeller University some of us play a little touch football on weekends. During the last few weeks, the New York Jets have had their top two quarterbacks injured. As that happens to be my position, I am beginning to have dreams of glory; but unfortunately their practices conflict with the computer course I teach.

As you may remember, the first problem in March/April, 1973, was printed incorrectly and revised in June. So this month we will print solutions to six problems—the revised **M/A1** and the five problems given in June.

Often I receive many correct responses to a given problem, and I've been asked how I decide which solution to print. If an answer is hard to read it is rejected. A typed solution gains a few brownie points, as does one which does not abound in many hard-to-type and hard-to-print special symbols. After that it comes down to my mood and the luck of the draw. I try to give preference to those whose names are new to Puzzle Corner; no extra priority is given to letters received early in the month.

Problems

Here is a novel bridge problem from Charles E. Blair:

DEC1 Construct a hand on which North-South can make seven of any suit but cannot make seven no trump. You are allowed to make up the East and West hands but you must allow for best play by the defenders.

Here is a geometry problem from Norman I. Apollonio:

DEC2 A hexahedron (I call such things cubes—ed.) has three regular sections: an equilateral triangle, a square, a regular hexagon. Prove that it cannot have a fourth—namely, a regular pentagon.

A political problem from Frank Rubin's doctoral dissertation:

DEC3 A body of N legislators is divided up into various (possibly overlapping) committees. An executive committee is formed consisting of at most one representative of each committee (the same individual will represent every committee of which he is a member). The executive committee is always chosen to be as large as possible without violating the rule that only one member from any other committee may be on the executive committee. How large is the executive committee?

On another topic, here's a question from Hal Varian about the possibility of stable marriages:

DEC4 Each of n men has a preference ordering of n women; each of the n women has a preference ordering of the n men. A marriage is an assignment of the n men to the n women. A marriage is called unstable if some man prefers another woman to his present mate and the preferred women also prefers this man to her present mate. Can you always find stable marriages?

Jeff Dodson writes:

DEC5 I can generate an infinite number of "magic squares" using

0	2	1
2	1	0
1	0	2

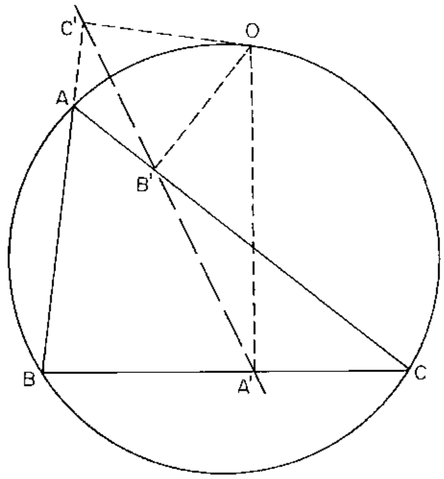
as well as its three rotations, and linear multiples. These all have rows, columns, and diagonals which add up to a multiple of three. Can you find a three-by-three array of integers all of whose rows, columns, and diagonals add up to the same number without this number being a multiple of three?

Speed Department

This is from H. W. Hardy:

SD1 The sum of Mary's and Ann's ages is 44. Mary is twice as old as Ann was when Mary was half as old as Ann will be when Ann is three times as old as Mary was when Mary was three times as old as Ann was. How old is Ann?

Our last problem this month is from Henri Hodara:

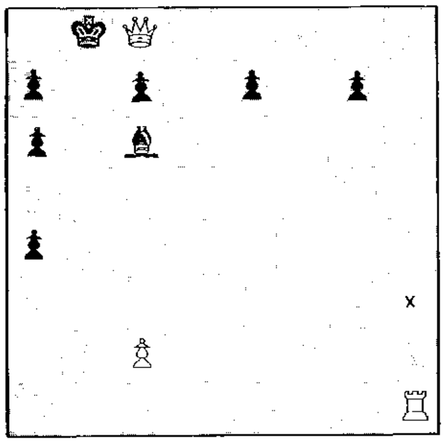


SD2 Inscribe a scalene triangle ABC in a circle. Pick any point O on its circumference. Draw from O the perpendicular to the three sides of the triangle and call the intersections, A', B', and C'. Show that these three points lie on a straight line.

Solutions

The following correctly states the first problem published in March/April, 1973:

M/A1 Given a chess game interrupted at the point shown by the board below, what piece must have been at X?



The answer was supposed to be a black knight. Unfortunately, Avi Ornstein can show that the piece cannot be white and cannot be black:

1. To be in the present condition, the, move before the interruption had to be White's KNP taking a piece at QB8, becoming a queen, and achieving a double check.
2. In moving Black's QNP to QR3, a white piece was taken.
3. In moving Black's QP to QB3 to QN4 to QR5, three white pieces were taken.
4. All of these captures were on white squares, so White's QB was captured elsewhere.
5. This accounts for all of White's pieces which are missing, so the piece knocked off of White's KR3 was not white.
6. White's KNP captured six black pieces by moving to KB3 to K4 to Q5 to QB6 to QN7 to QB8, where it became a queen.
7. Black's KB was captured elsewhere. This can be shown by two points: it could not have been on any of the white squares traversed by White's KNP; and it never left its original position, since both Black's KP and KNP have not moved.
8. Black's KRP was also taken elsewhere (he could not have moved to White's KR3 without having captured two addi-

tional white pieces).

9. This accounts for all of Black's pieces which are missing, so the piece knocked off White's KR3 is not black.

10. With 5. and 9. we therefore are back at the original difficulty. There were two errors, not one, in the puzzle as it originally appeared.

Thirty-one other readers have also responded—a list which is simply too long for publication here. Sorry—it doesn't often happen, but paper is scarce this year.

The following solutions are to problems published in the June, 1973, issue:

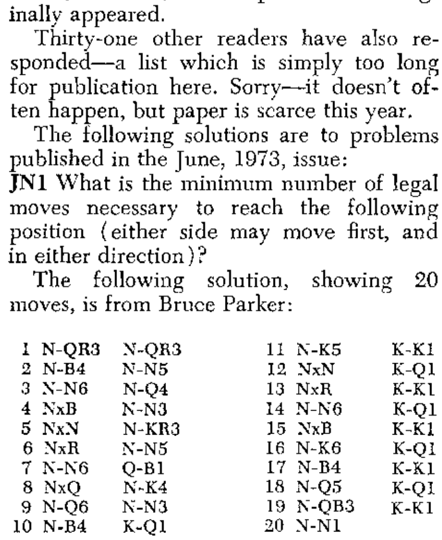
JN1 What is the minimum number of legal moves necessary to reach the following position (either side may move first, and in either direction)?

The following solution, showing 20 moves, is from Bruce Parker:

1	N-QR3	N-QR3	11	N-K5	K-K1
2	N-B4	N-N5	12	NxN	K-Q1
3	N-N6	N-Q4	13	NxR	K-K1
4	NxB	N-N3	14	N-N6	K-Q1
5	NxN	N-KR3	15	NxB	K-K1
6	NxR	N-N5	16	N-K6	K-Q1
7	N-N6	Q-B1	17	N-B4	K-K1
8	NxQ	N-K4	18	N-Q5	K-Q1
9	N-Q6	N-N3	19	N-QB3	K-K1
10	N-B4	K-Q1	20	N-N1	

Also solved by Edward Gurshuny, James Rumbaugh, Lars Sjodahl, and William Stenzler.

JN2 What is the shortest *curve* bisecting any given triangle?



Most people gave elementary solutions which assumed that the necessary curve was a circle having its center at a vertex. While this is in fact correct and seems "intuitively obvious," it should I believe be proved. The following solution from James Rumbaugh assumes only that the curve is differentiable. Unfortunately, it is very advanced—another case of mathematicians using high-powered machinery to prove "obvious" results:

Assertion: The shortest curve bisecting a triangle is a circle centered at the vertex V with the smallest angle. The radius of this circle is $(K/A)^{1/2}$ and the length of the relevant arc is $(KA)^{1/2}$ where K is the area of the triangle and A is the measure of the smallest angle (in radians, of course). Proof by calculus of variations (I warned you—ed.). Use polar coordinates about V and let A be the measure of the angle (it will be clear later that the correct vertex is the one with the smallest angle). We want to minimize

$$ds = \int_0^A (r^2 + r'^2)^{1/2}\, d\theta$$

with the constraint that

$$\int_0^A (r^2\, d\theta)/2 = K/2.$$

Let t be positive and try to minimize

$$\int H\, d\theta$$

where $H = (r^2 + r'^2)^{1/2} + t\, r^2/2$.

We have the solution to the Euler equation:

$$H - r'(\partial H/\partial r') = C$$

and boundary conditions

$$\partial H/\partial r'|_0 = \partial H/\partial r'|_A = 0.$$

Plugging in gives

$$r^2 + (1 - t)r'^2 - C(r^2 + r'^2)^{1/2} = 0$$

and $r'(0) = 0$ and $r'(A) = 0$. So let r be the constant C (hence $r' = 0$), and we get a solution. Since we require $r^2A/2 = K/2$, we get that $rl = (K/A)^{1/2}$ and thus the length $= rA = (KA)^{1/2}$. As K is constant we should choose A to be the smallest angle.

Also solved by Winslow Hartford, R. Robinson Rowe, and Harry Zaremba.

JN3 If you take a two-digit number, A_1A_2, and subtract the number obtained when you reverse the digits, A_2A_1, you obtain a positive multiple of 5. Divide this number by 5 and you obtain one of the factors of both the original number and the reversed number. Find the original number.

The following solution is from Joseph Evans IV:

The two-digit number A_1A_2 may be represented as $10A_1 + A_2$. Similarly, A_2A_1 may be represented as $10A_2 + A_1$. The problem statement then becomes: $10A_1 + A_2 - 10A_2 - A_1$ is evenly divisible by 5, is positive, and is 5 times a factor of both the original and the reversed number; find A_1 and A_2. Or: $9(A_1 - A_2) > 0$ ("positive multiple"). And, since the difference of digits between 0 and 9 cannot exceed 9 nor be < 0, and 5 is the only multiple of 5 in that range, $A_1 - A_2 = 5$ ("multiple of 5"). Also, $[9(A_1 - A_2)]/5 = 9$ (9 is the common factor). The multiple of 9 the difference of whose digits is 5 is 72, and since $A_1 > A_2$, $A_1A_2 = 72$ and $A_2A_1 = 27$.

Also solved by Arthur Anderson, Bob Baird, Sam Bent, Joseph Evans IV, Mrs. Leonard Fenocketti, Edward Gershuny, David Glazer, Jean Goodwin, Winslow Hartford, John L. Joseph, Paul Karvellas, Mrs. Martin S. Lindenberg, Roger Milkman, Avi Ornstein, Bruce Parker, John Prussing, R. Robinson Rowe, James Rumbaugh, Joel Shwimer, Lars Sjodahl, William Stenzler, Paul G. N. de Vegvar, Neal Wellmer, D. Zalkin, and Harry Zaremba.

JN4 Two football teams of equal strength compete each year for a cup. The first team to win the game three years in a row keeps the cup. Assume each year's game is independent of the previous year's and each team has probability $p = 1/2$ of winning. What is the probability that a given team wins the cup at the nth trial?

An outstanding variety of methods was employed for this problem. After giving his solution, James Rumbaugh stated that a longer method was to use difference equations. This was prophetic, as H. M. Wilensky used these equations and some theory of Markov chains to get an unbelievably hairy method. I present a digested solution based primarily on those of Rumbaugh, Harry Zaremba, and Winslow Hartford: The number of possible n-strings of victories and losses for team is clearly 2^n. By inspection, there are exactly 1, 1, and 2 n-strings ending in a victory for A when n is 3, 4, and 5, respectively. They are WWW, LWWW, LLWWW, and WLWWW. Consider the Fibonacci sequence 1, 1, 2, 3, 5, 8, 11, etc., satisfying $F_{n+2} = F_{n+1} + F_n$. I claim that the number of n-strings ending in a victory for A (call this number S_n) is just F_{n-2}. By inspection this checks out for $n = 3$, $n = 4$, and $n = 5$. So all I must show is the recursion relation $S_{n+2} = S_{n+1} + S_n$. The answer will then be $(F_{n-2})/2^n$. Using standard results on the Fibonacci sequence we can see that the probability approaches $1/2$ as expected. Proof of recursion relation (n > 3): call an n-string legal if it contains neither LLL or WWW:

Let a_n = number of legal n-strings ending in LWW,
b_n = number of legal n-strings ending in LW,
c_n = number of legal n-strings ending in WL, and
d_n = number of legal n-strings ending in WLL.

Following Rumbaugh we note that $a_n = d_n$, $b_n = c_n$, $a_{n+1} = b_n$, and $b_{n+1} = d_n + c_n = a_n + b_n$. Thus $b_{n+2} = b_{n+1} + b_n$. Since $b_2 = 1$ and $b_3 = 2$, $b_n = F_n$. Each n-string in S_n is just an element of b_{n-2} with WW stuck on the end. So $S_n = b_{n-2} = F_{n-2}$ as desired.

Also solved by Arthur Anderson, Bob Baird, Edward Gershuny, Jack Parsons, R. Robinson Rowe, and Lars Sjodahl.

JN5 A camel must carry 3,000 bananas from A to B, which are separated by 1,000 miles. The camel can carry no more than 1,000 bananas on its back, and it eats one banana a mile. How can it arrive at B with the most possible bananas?

For the camel's health I have selected the solution from Ray A. Brinker, M.D.:

1. At first it takes five bananas eaten per mile gained because five one-way trips are required.
2. Where is the first stopping point? This is probably where only three one-way trips are subsequently required; therefore, 200 miles out. After five trips, there are 2,000 bananas and one tired camel at point 200 miles.
3. For the second trip it is likely that the best distance is where 1,000 bananas are left (and so only one trip to point B). This is 333 miles from the last stop and 533 miles from the start. At this point we have 1,001 bananas and one *very* tired camel.
4. There are now 467 miles left to go—using 467 bananas for camel food leaves 533 bananas to sell at point B (plus one banana wasted in the desert).

Dr. Brinker explains that "the above solution is rambling because, among other reasons, I am an applied biologist, not a scientist."

Also solved by Arthur Anderson, Bob Baird, Bruce Cuthbertson, Edward Gershuny, Otto Hadler, Karl Kadzielski, Erich Kranz, Mrs. Martin Lindenberg, Roger Milkman, Avi Ornstein, Bruce Parker, R. Robinson Rowe, James Rumbaugh, Joel Shwimer, Lars Sjodahl, Benjamin Whang, Norman Wickstrand, and Harry Zaremba.

Better Late Than Never

In connection with **SD1** in June, James Rumbaugh, Sam Bent, Roy Schweiker, and Benjamin Whang have pointed out that a half inning of baseball can occur with *no* pitches thrown. For example, batters can be declared out for not respecting rules concerning the batter's box; the umpire can call a "ball" if he decides the pitcher is delaying the game; and pitchers wetting their fingers while on the rubber cause the batter to take first. Surely now we are at the theoretical minimum.

JA4 John E. Prussing has furnished solutions for all bases from 3 to 10. His answers are as follows:

Base	Solution	Decimal equivalent
3	1,012	32
4	102	18
5	102,342	3,472
6	1,031,345,242	10,993,850
7	103,524,563,142	2,129,428,800
8	1,042,	546
9	10,467,842	5,064,320
10	105,263,157,894,736,842	same

DEC4 (1972) Lars Sjodahl agrees with the proposer that the published solution is in error. He pinpoints the mistake to be the assumption that all solutions of the algebraic equation are solutions to the problem.

FEB3 Harold Chelemer points out that if we work base-14 there is a unique solution, namely

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             7 3 4 3
1 5 3 8 3 7)9 Y 4 9 9 5 3 V 7
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where we have denoted the fourteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, V, X, and Y.

Solutions have been received from the following as indicated:
B. Rouben—**MAY2** and **MAY3**
Paul G. N. deVegvar—**MAY4**

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