How Four Dogs Meet In a Field, etc.

Puzzle Corner:
Allan J. Gottlieb

Next year Alice and I will be in New York; I'll be in the Mathematics Department of York College, a new campus in the City University of New York system, while Alice will be a graduate student at Rockefeller University.

Here's an indication of how long I've been out of "the city" (old-time provincial New Yorker talking). While crossing the street today I noticed that another pedestrian was nearly run over right in front of me. I gasped. A third pedestrian turned and said, "You must be from out of town. That wasn't even close—he missed him by two inches!" Two inches seemed close to me, but no one else was alarmed.

Help! A critical shortage of "speed" problems!

Even before proceeding to new problems, let me give Harry Nelson a chance for rebuttal on M1, originally published in May, 1972, on which we published Hallock G. Campbell's remarks in May, 1973 (p. 00):

"I concur that the laws of chess require 'only a three-time repetition of the same position' but not that it is 'quite independent of the previous moves.'"

"The first 17 moves by black and white which I sent in (plus the 18th white move) were needed to establish that the possibility of both black and white castling on either side has not been precluded. Thus with black's 27th move (KR—R2), the situation is changed; now black may no longer castle on the King's side. Similarly at moves 36W, 44B, and 53W. Thus, even after move 61W, the same position has not occurred three times, 'with the possible moves of all the men unchanged.'"

Problems

A bridge problem from Winslow H. Hartford:

J/A1 South has won a contract of six spades. West's opening lead is A3, taken by A A. East returns 4. How can South make the contract?

J/A2 Each of four dogs, located at the four corners of a square field, simultaneously spoils the dog in the corner to his right and runs towards that dog, always pointing directly toward him. All the dogs run at exactly the same speed and thus finally meet in the center of the field. How far did each dog travel?

Our third offering is from Gordon J. McKinnon, Jr.:

J/A3 A circular table is pushed into the corner of a rectangular room. A coin resting on the edge of the table is 9" from one wall and 8" from the other. What is the diameter of the table?

Jerry Blum wonders:

J/A4 If you have just been given the dice at a crap table, what are the odds against your winning at least once?

Richard T. Bumby wants you to:

J/A5 Find the greatest common divisor of a - 1 and a + 1, where a is a positive integer.

Speed Department

A flagpole problem from H. W. Hardy:

SD1 Three flagpoles of 60 ft., 80 ft., and 100 ft. height are erected at the three corners of a triangular field, 100 ft. on each side. A ladder is placed at a point in the field so that it can be leaned to the exact top of each of the three poles. How long is the ladder, and where is it located?

A baseball puzzle from Norman Brenner:

SD2 The Yankees shut out the Orioles in Yankee Stadium by scoring n runs in the ninth inning, for n. What was the final score?

Solutions

The following are solutions to problems published in the March/April issue of the Review—except recall that M/A1 was modified in June, so we start this month with M/A2:

M/A2 If (1) F is continuous from 0, (2) for all t > 0, the sequence F(t), F(2t), F(3t), . . . → 0; then F(x) → 0 as x → ∞ [x ∈ 0, ∞).

Jason L. Bitsky, a member of the Department of Mathematics at Princeton, writes, "I offered this problem to a group of hackers at tea. It was quite obvious to everybody that Baire's Category Theorem was called for. The only hitch was how to apply it. The key point (marked after the fashion of chess analyses with a double explanation point) was brought up by Bob Israel. After that the solution was almost trivial." Here is the complete proof:

Let ε > 0 and define for each positive integer k the set A_k(ε) = {t > 0: f(nt) ≤ ε}. It is easy to see that each A_k(ε) is closed in (0, ∞) as follows: the function f_n from (0, ∞) into (0, ∞) defined by f_n(t) = 1/(1 + t^2) is continuous (because f is continuous), and so A_k(ε) = f_n(0, ε) is the inverse image of a closed set under a continuous function. Now let A(ε) = ∩_k=1 A_k(ε) = {t > 0: f(t) ≤ ε} for all m = 0, n = 0. Each A(ε) is closed (the intersection of closed sets). We claim that

lim A(ε) = {t > 0: f(t) = 0}. By hypothesis, for each t > 0, there exists a positive integer n such that f(nt) < ε whenever m ≥ n. This precisely the assertion that t ∈ A(ε). Since (0, ∞) is locally compact Hausdorff space, the Baire Category Theorem implies that some A_k(ε) must have non-empty interior; say A_k(ε) contains an open interval (α, β), β > α. Choose a positive integer M ≥ N so that (M + k) - 1 < M and M + k > N. The inequality * implies that the intervals (M + k - 1, M) and (M + k, M + k + 1, M + k + 1, M + k + 1) overlap. In fact, we have that for all k ≥ 0 the intervals ((M + k - 1, M + k) and (M + k + 1, M + k + 1)) overlap. Thus U ((M + k - 1, M + k), M + k + 1) overlaps. Now, if t > Ma, then t lies in some interval ((M + k), (M + k + 1), k ≥ Ma). Hence, f(t) ≤ f(M + k)/H(k + 1), since H(1) = 1/(M + k + 1). Thus, f(t) ≤ e - k, for all t > Ma, and this elementary theorem in analysis asserts that every open set on the real line is a countable union of disjoint open intervals (a_n, b_n). Thus, the proposition above tells us that no matter how we select a_1, a_2, . . . , a_k, b_1, b_2, . . . , a_k, there is always a t > 0 such that infinitely many points of the sequence {b_t = a_t} lies in U = U {α, β}, a non-obvious result.

Also solved by Hugh Barrie and the proposer, Mike Rollin.

M/A3 Every day for a week (of seven days) a class of 15 school girls went for a walk. They walked in five rows of three each. Each day, each girl had two new "row" mates. How did they do this?

R. Robinson Rowo submitted the following solution and comment: this is the classic problem T. P. Kirkman proposed on page 48 of the Lady's and Gentleman's Diary in 1850. It has 0,000 solutions, but my favorite is that shown in the box at the top of the next page. The best analysis of the general problem, with the number of girls = G = 3(2d + 1) arranged differently for d = 0,3 = 1852-232 at pp. 193-223 of W. W. R. Ball's Mathematical Recreations of the Nineteenth Century, 10th edition, 1937. Henry E. Dudeney in his Amusement in Mathematics says, "There are no fewer than 15,567,552,000"
1. Construct BC = a, angle B giving a line l.
2. Construct from C as a center the distance (b - h) toward B on line a. This is point P1.
3. Construct a line parallel to a at a distance x above it, and the circle arc of radius (b - h) + x. The intersection is point P2.
4. Construct more points P to form a curve; the intersection of this curve with line l will have x = h, and AC = (b - h) + x = b. Note that there are two solutions for b > h, one solution if b = h, and no solution if b < h.

Also solved by Mary Lindenberg, R. Robinson Rowe, and David B. Smith.

Better Late than Never
Concerning DE1 John Schwartz, the proposer, writes:
I take exception to Mr. Cutter's statement in his solution that it is very hard not to find a way to make six spades. The point is that the hand was one actually played at the bridge table, and it should be played with the defensive hands unseen. The problem is made trivial if all the hands are visible, in which case it is indeed easy to make the contract.

DE2 Harvey Piazzo was going through back issues and found himself personally interested in Frank G. Smith's construction. He writes. I checked it, and it works for n = 3, n = 4, and n = 6—and perhaps for some other values of n, but it certainly cannot work for all n. Assuming the construction is correct, the diagram below shows what happens. The equation for collinearity is: (sin 2π/n)/√3 = (cos 2π/n - 1 + 4/n)/(1 - 4/n); it is not satisfied in general. If Mr. Smith's construction were correct he would be able to trisect many angles. Even 20° = 60°/3 cannot be constructed with ruler and compass because it requires the solution of a cubic.

Richard Lips, the proposer, feels that the solution given to DE4 must be wrong, as he knows the numerical solution and the one obtained do not agree. Comments?

Solutions to the following problems have also come from the following readers:
DE3 Mary J. Youngquist
DE4 Peter Groot
JN3 Thomas Kaufman
JA1 Roger Sinnott
JA2 Robert Baird and Michael Rolle
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AEC, as has been devised for the use of nuclear explosives in civilian application, and the project which is now the center of Commission interest: the breeder reactor. The book makes a substantial attempt to present an unbiased approach to the radiation safety controversies of Ernest J. Sternglass and of John W. Golman and Arthur R. Talmibin in their book, Population Control Through Nuclear Pollution.

H. Peter Metzger, a biochemist who is now a newspaper writer on technical issues, takes a broader aim in a more serious book. His purpose is to show how the Joint Committee and the AEC changed from healthy adversariness into pals: how the Committee was transformed from a critic into an apologist, from an attacker of the AEC into its defender, while the A.E.C. itself was reduced to a fanatically defensive protectionist clique of (tenured?) bureaucrats who have been drawing job security from... the Manhattan Project... and whose best efforts since then have been divided between widely inappropriate technological adventures and the justification of these past mistakes. His story shows that the weaknesses of the AEC and the supporters are grave indeed and how as a consequence the U.S. nuclear power program is in such much trouble. He invokes most of the known or suspected errors of the A.E.C. in supporting his views, drawn from the U.S. nuclear weapons acquisition and testing programs, the effects of radioactive fallout (the AEC's "body in the morgue" approach), uranium mine radon, a far wider spectrum of waste storage irregularities than Mr. Lewis' documents, and a whole panopoly of derelict "atomic gadgets." The latter includes the plutonium-heated "long johns" for deep sea divers produced by the AEC for the Navy, one set of which contains a kilo-