A Grifter Meets a Yokel in Las Vegas

Puzzle Corner:
Allan J. Gottlieb

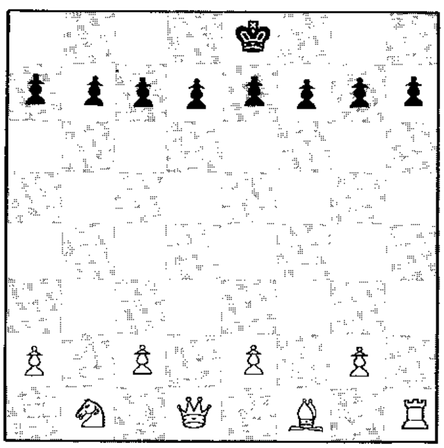
Spring has come again to New England. This was a mild winter, but warm weather and flowers are still appreciated. Yet this has been an "indoors" afternoon, and I am exhausted: I watched "my" Knicks beat the Celtics in double overtime. Alice was really surprised to see that after the game I was sweating and wanted to lie down. Who says television keeps people from being involved?

The first problem in the March/April issue, **M/A1**, is printed with an error: the white rook at KR1 should be removed. As originally printed there is no solution. Please remove the rook and try again; a solution to the revised problem will be given in the October/November, 1973, issue—along with the solutions to this month's new problems. Here they are:

Problems

Harry Nelson asks:

JN1 What is the minimum number of legal moves necessary to reach the following position (either side may move first, and in either direction)?



From R. E. Crandel:

JN2 What is the shortest *curve* bisecting any given triangle?

A number problem from Les Servi:

JN3 If you take a two-digit number, A_1A_2, and subtract the number obtained when you reverse the digits, A_2A_1, you obtain a positive multiple of 5. Divide this number by 5 and you obtain one of the factors of both the original number, A_1A_2, and the reversed number, A_2A_1. Find the original number.

John Bobbitt submits the following; he's a student at Purdue, and the problem comes from a probability course there:

JN4 Two football teams of equal strength compete each year for a cup. The first team to win the game three years in a row keeps the cup. Assume each year's game is independent of the previous year's, and each team has probability $p = 1/2$ of winning. What is the probability that a given team wins the cup at the nth trial?

Here's a banana problem, from a reader whose signature is simply not legible (but he's a graduate—S.M.—from M.I.T.'s Sloan School of Management, 1971):

JN5 A camel must carry 3,000 bananas from A to B, which are separated by 1,000 miles. The camel can carry no more than 1,000 bananas on its back, and it eats one banana a mile. How can it arrive at B with the most possible bananas?

Speed Department

Doug Hoylman asks:

SD1 What is the minimum number of pitches possible in a half-inning of a (completed) baseball game?

Here is a modified craps problem from Jack Parsons:

SD2 Carnival. A yokel met a grifter who said, "Here are three ordinary, honest dice. We'll take turns rolling them. Regardless of who's rolling, if one of your points comes up, you win. Now, you're a beginner, so I'm going to give you a break: I'll give you ten winning points and keep only six for myself. And I'll give you odds of three to two." This seemed like a more-than-fair game to the yokel, and they played for a long time. Sometimes the yokel won but more often he lost; in fact, when he studied his score sheet he found that, on the average, he had lost about 19 cents of each dollar he bet. What were the winning points the grifter gave the yokel?

Solutions

The following are solutions to problems published in the February issue:

FEB1 A scrambled sequence of letters is formed by choosing a key word (containing no repeated letters) and writing the remainder of the alphabet in order after it. The letters of this sequence are written as capitals on the 26 cards ♠A through ♥2. The letters of another sequence (based on a different key word) are written as lower case on the 26 cards ♦A through ♣2. On this basis a bridge game, with a contract of four spades held by South, might be written as shown in the box at the bottom of this page. What are the key words?

Walter Penny, who submitted this problem, deserves extra credit: many favorable comments were received on it. Here is John Ribb's solution:

The two key words are "VEXATIOUS" and "copyright." The hands would have been:

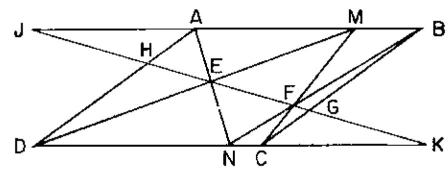
West	North	East
	♠ 6 5	
	♥ 9 4 2	
	♦ A K J 8 2	
	♣ A Q 5	
♠ Q J 10 8		♠ 4 2
♥ A K 8 6		♥ Q J 10 7 3
♦ 10 5		♦ 9 7 6
♣ 7 4 2		♣ 10 9 8 6
	♠ A K 9 7 3	
	♥ 5	
	♦ Q 4 3	
	♣ K J 7 3	

Also solved by Michael Auerbach, James Dotson, Winslow Hartford, Doug Hoylman, Harry Nelson, Charles Rivers, Jr., R. Robinson Rowe, Shou-ling Wang, Benjamin Whang, Mary Youngquist, and the proposer.

FEB2 Given a paralellogram ABCD. Choose two random points M and N on the parallel sides AB and CD. Draw MC, MD, NA, and NB intersecting at E and F. Prove that the line EF divides the parallelogram into two equal areas.

The following purely geometric proof is from Michael Sutherland. A shorter solution would be possible using Cartesian geometry:

The problem is to prove that area ABGH equals area DCHG.



Triangles AEJ and NEK are similar, and triangles AEM and DEN are similar in the same ratio, since AE and EN are common sides. Thus

$$JA/NK = AM/DN. \quad (1)$$

Similarly, triangles JFM and CFK are similar in the same ratio as triangles BFM and CFN. Thus,

$$JM/CK = MB/NC. \quad (2)$$

Substituting $NK = NC + CK$ and $JM = JA + AM$, equation (2) yields

$JA = CK \cdot MB/NC - AM$.

Using this in equation (1) and solving for CK,

$$CK = [NC \cdot AM(DN + NC)]/[- MB \cdot DN + AM \cdot NC]. \quad (3)$$

Now solving (2) for CK yields

$CK = (NC/MB)(JA + AM)$.

Using this in equation (1) and solving for JA,

$$JA = [NC \cdot AM(AM + MB)]/[MB \cdot DN - AM \cdot NC] \quad (4)$$

Since $AM + MB = DN + NC$, $JA = - CK$, or

$/JA/ = /CK/$ from (3) and (4). This implies that triangle JAH = triangle CJK, and that AH = CG. Therefore DH = BG, and figure ABGH is identical to figure DCGH, and the areas are thus equal.

Also solved by Mrs. Martin Lindenberg, R. Robinson Rowe, Gilbert Shen, and the proposer, John T. Rule.

FEB3 Fill in the dashes with digits to make it a correct long division:

```
                     7 - 4 -
- 5 - - 3 -)9 - - - - - - - -
             - - - 1 - -
               - - - - X -
               - 1 - - - 7
                 - - 2 - - -
                 - - - - - 0
                   4 - - - - -
                   - - - - - -
                             0
```

A very nice solution is from Raymond Dyba:

The problem is solvable in base-14 number system. There are two solutions differing in the value of X; in both cases the quotient is 7343. The two divisors are 150437 (with X = 0) and 150637 (with X = 8). In the complete solutions shown below, the additional digits required for base-14 are given by y for 10 in the decimal system, z for 11, w for 12, and v for 13.

```
                  7 3 4 3
1 5 0 4 3 7)9 z 8 w y 7 5 y 7
            9 7 2 1 y 7
              4 6 z 0 0 5
              4 1 0 w y 7
                5 y 2 3 w y
                5 6 1 3 0 0
                  4 1 0 w y 7
                  4 1 0 w y 7
```

```
                  7 3 4 3
1 5 0 6 3 7)9 z 9 v 4 1 z y 7
            9 7 3 1 y 7
              4 6 z 7 8 z
              4 1 1 4 y 7
                5 y 2 w 4 y
                5 6 1 z 0 0
                  4 1 1 4 y 7
                  4 1 1 4 y 7
```

In order to describe the process of solution concisely, label the unknown digits of the divisor d_i and of the quotient q_i, as shown below. Note also that the first digit of the bottom row must be 4 because the bottom row must be exactly equal to the one just above it. We are also given that $d_6 \neq 0$, and from the requirements of the division process we can see that $q_3 \neq 0$ or 1, $d_1 \neq 0$, and $q_1 \neq 0$.

```
                          7 q3 4 q1
d6 5 d4 d3 3 d1)9 - - - - - - - -
                - - - 1 - -
                  - - - - X -
                  - 1 - - - - 7
                      - - 2 - - -
                      - - - - - 0
                          4 - - - - -
                          4 - - - - -
                                    0
```

The problem is apparently impossible to solve because

☐ The product of the smallest possible divisor and the smallest possible quotient is larger than the largest possible dividend; and

☐ d_1 must meet two apparently inconsistent conditions:

4 x d_1 = a number ending in 0, and

q_3 x d_1 = a number ending in 7.

The only way to eliminate these inconsistencies is to assume that the numbers are written to a base other than 10. One can show that any base greater than 11 will eliminate the first inconsistency, and from simple considerations of the products of two digits one can show that only in base 14 can d_1 satisfy the two conditions given above, and then only for d_1 = 7. One must adopt some convenient symbols for the four digits greater than 9. Then from the requirement that the product of the smallest possible divisor and smallest possible quotient cannot be larger than the largest possible dividend, one can show successively that $d_6 = 1$, $q_3 = 3$, and $d_4 \leq 4$. One can then show from the requirement that

q_1 (1 5 d_4 d_3 3 7) = 4 – – – – – –

that $q_1 = 3$; therefore the quotient is 7343. And one can show from the requirement that

7 (1 4 d_4 d_3 3 7) = – – – 1 – –

that d_3 must be an even number. Having with the help of the preceding steps established that

3 (1 4 d_4 d_3 3 7) = 4 1 – – y 7

(where y = decimal 10),

4 (1 4 d_4 d_3 3 7) = 5 – – – 0 0, and

7 (1 4 d_4 d_3 3 7) = 9 – – 1 y 7,

one can substitute into the appropriate spaces of the original problem. Then making use of the only given digit, 2, that has not yet been used, and the requirement that $d_4 \leq 4$, one can avoid an inconsistency only if $d_4 = 0$ and $d_3 = 4$ or 6. One can then complete the entire calculation for both possible values of d_3 separately by working upward to the dividend, and in the process one finds that X = 0 or 8.

Also solved by Christopher Brooks, Richard Dreselly, Raymond Dyba, R. E. Efimba, Winslow Hartford, Fred Heutiuk, Ramchandran Jaikumar, Harry Nelson, Gilbert Shen, and Michael Sutherland.

FEB4 Who drinks water? And who owns the zebra?

1. There are five houses, each of a different color and each inhabited by men of different nationalities, with different pets, drinks, and cigarettes.
2. The Englishman lives in the red house.
3. The Spaniard owns the dog.
4. Coffee is drunk in the green house.
5. The Ukranian drinks tea.
6. The green house is immediately to the right (*your* right) of the ivory house.
7. The Old Gold smoker owns snails.
8. Kools are smoked in the yellow house.
9. Milk is drunk in the middle house.
10. The Norwegian lives in the first house on the left.
11. The man who smokes Chesterfields lives in the house next to the man with the fox.
12. Kools are smoked in the house next to the house where the horse is kept.
13. The Lucky Strike smoker drinks orange juice.
14. The Japanese smokes Parliaments.
15. The Norwegian lives next to the blue house.

There were many solutions to this problem, including one by architect Toby Hanks, who sent in a model which I cannot figure out how to reproduce. The following less exotic solution is from Mary Lindenberg, who writes that she and her husband found the February *Review* upon returning from a cruise aboard M.S. *Starward,* a Norwegian vessel out of Miami; the headline on "Puzzle Corner" reminded them of their exciting adventure, and Mrs. Lindenberg started working on the problems even before unpacking. Her answer:

The Norwegian drinks the water ("I found this, too, to be true on our cruise!") and the Japanese owns the zebra. The analysis is diagrammed in the box above.

	Norwegian	*Ukranian*	*Englishman*	*Spaniard*	*Japanese*
Color of house	Yellow	Blue	Red	Ivory	Green
Pet	Fox	Horse	Snails	Dog	**Zebra**
Drink	**Water**	Tea	Milk	Orange juice	Coffee
Cigarette	Kool	Chesterfield	Old Gold	Lucky Strike	Parliament

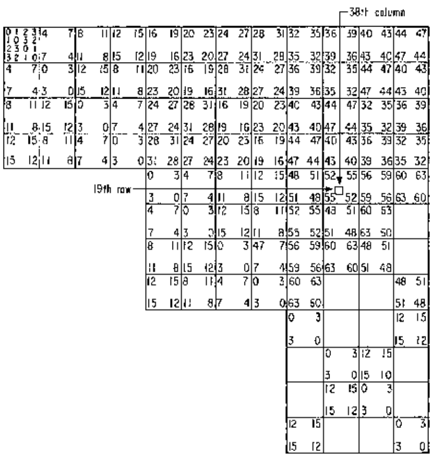
Many readers solved this one: Michael Auerbach, James Bledsoe, Mark Bucciarelli, James Dotson, Carl Estes II, Bruce Fauman, Joann Fray, Marty Geer, David Geisler, Anne Goetting, Jean Goodwin, Winslow Hartford, Denis Hayes, Ronald Jablonski, Ramchandran Jaikumar, Michael McNutt, Dianne Maar, Jerry Miller, Russell Nahigian, Avi Ornstein, Charles River, Jr., R. Robinson Rowe, Mitchell Serota, Gilbert Shen, Deborah and David Smart, Michael Sutherland, Benjamin Whang, George Wynne, Harry Zaremba, and the proposer, Bruce Orr.

FEB5 In each square of an infinite checker board, put the smallest (no negatives) integer not already occurring to the left in that row or above in that column. What number is in the 19th row and the 38th column?

```
0 1 2 3 4 5 6 . .
1 0 3 2 5 4 . .
2 3 0 1 6 . .
3 2 1 0 7 . .
4 . .
. .
.
```

We publish two solutions. The first is a mixed solution picture from Harry Zaremba:

The number which is in the 19th row and 38th column of the infinite matrix is 55. A portion of the matrix is shown below:



The matrix is symmetrical about the principal diagonal whose elements are zero.

Two very different methods are suggested by Gilbert Shen:

Suppose we have an n x n array in the upper left corner of the checkerboard containing only integers 0 through $n - 1$. Then each row and column will contain all integers 0 through $n - 1$. Now consider the 2n x 2n array in the upper left corner. Divide this square into four equal quadrants. (The upper left quadrant is our original n x n array.) It is clear that the upper right quadrant will have the identical structure as the upper left quadrant, if the correspondence is made that $0 \rightarrow n$; $1 \rightarrow n + 1$; . . . $n - 1 \rightarrow 2n$. The same is true for the lower left quadrant. Also, the lower right quadrant is identical to the upper left quadrant since the numbers 0 through $n - 1$ do not occur to the left or above it. This implies that diagonally opposite quadrants are identical to each other and that the 2n x 2n array contains only integers 0 through $2n - 1$. Since the assumption is true for $n = 1$ and $n = 2$, by induction it is true for all $n = 2^s$, where s is a non-negative integer. We are to find the value of the element at row m and column p (hereafter called "the element"). We proceed by boxing it in with successively smaller boxes of sides $n = 2^s$. At each step, if the element falls in a lower quadrant, we equate it to the corresponding element in the diagonally opposite quadrant, which then becomes "the element." We shift the corner of the checkerboard to the upper left corner of the quadrant containing the element before subdividing further. Eventually, when we reach $n = 1$, the element will be in the top row (which we observe to be the integers in their normal order). The procedure is equivalent to the following prescription:

□ (1) Write $m - 1$ in binary.

□ (2) Write $p - 1$ in binary.

(This decomposes the element into boxes of different values of s. Each binary place corresponds to a different s and a step in our reduction procedure.)

□ (3) Combine the two binary numbers digit-by-digit with an exclusive Boolean "OR" (i.e., 1.0R.1 = 0). A "1" or "0" in $p - 1$ corresponds to the element being in a right or left quadrant, while a "1" or "0" in $m - 1$ corresponds to a lower or upper quadrant. The result of the Boolean operation indicates whether or not the origin should be shifted by 2^s.

□ (4) Convert the result back to a decimal number, which is the required answer. For $m = 19$ and $p = 38$: (1) $18_{10} = 1\,0\,0\,1\,0_2$; (2) $37_{10} = 1\,0\,0\,1\,0\,1_2$; (3) and (4) $1\,1\,0\,1\,1\,1_2 = 55_{10}$.

Also solved by James Bledsoe, Ramchandran Jaikuma, Dianne Maar, Harry Nelson, Charles Rivers, Jr., R. Robinson Rowe, and Michael Sutherland.

Better Late Than Never

Concerning **O/N3,** Mike Rolle writes as follows:

The geodetic-net puzzle printed recently does not have only the solution printed in February. Your published solution assumes that only a regular polyhedron will satisfy the conditions given in the problem. However, there are in fact an infinity of solutions. If you think about the problem you can find some of them.

I fear that, as usual, Mike is right. For all it's worth, I fell into the same trap and would conjecture that the proposer intended to imply a regular polyhedron.

Speed Department Solutions

The proposers' solutions to the Speed Department problems above are:

SD1 One. It would occur in the bottom half of ninth inning, score tied, if the first pitch results in a home run.

SD2 The lowest point with three dice is 3; the highest, 18. These are the least probable. The more probable points are grouped around the average of the distribution, which is 10½. So the grifter kept for himself 8, 9, 10, 11, 12, and 13 and gave 3, 4, 5, 6, 7, 14, 15, 16, 17, and 18 to the yokel. This is confirmed by calculating the yokel's expectation:

Point	*Ways*
8, 13	21
9, 12	25
10, 11	27

There are 146 ways to lose and 216 − 146 = 70 ways to win. The probability of winning is 70/216 = 0.325. If the odds are 3 to 2, the expectation is 0.325 x $2.50 = $.81.

Allan J. Gottlieb, whose undergraduate degree in mathematics was given by M.I.T. in 1967, teaches at York College, Jamaica, N.Y. Send solutions and new problems to him at the Department of Mathematics, York College, 150-14 Jamaica Ave., Jamaica, N.Y. 11432.

(Columns continued from p. 7.)

Farming the Oceans: Lagging Technology

Carle O. Hodge

People always have perceived the sea as a limitless cornucopia, a belief as fallacious as it is venerable. Nonetheless, because of the chronic spectre of famine, it is important to ponder the possibility of extracting far more food from the waters. The fundamental obstacle is that man at sea, unlike terrestrial man, remains a hunter and a gatherer rather than a cultivator—and his techniques and tools have changed little over the centuries. Too, neither the real extent of marine life nor the intricacies of marine food chains is fully comprehended.

Most commercial fishing takes place along continental shelves and where upwelling occurs. Therefore, these delimited areas tend to become impoverished. And because they generally lie near populous shorelines, our effluent society imperils them. Less than one per cent of the world's sea food comes from the deeper, open oceans, which are relatively barren and inaccessible.

From the statistics, one might almost suspect that the fisheries are developed deliberately to deplete the oceans. In three decades the world catch has nearly tripled. Some of the ecological consequences already are easy to assess, for the oceanic fauna, like all resources, are finite. Although seafood still contributes only a tenth of the animal protein in the global diet, tuna has been over-fished almost everywhere, as have cod and ocean perch in the North Atlantic. The western Pacific has been all but rid of bottom-fish, the Bering Sea of flatfish and the North Atlantic of hake, to name a few. A large sardine industry prospered along the California coast during the 1930s. Owing to intensified pressure from fishermen, coupled perhaps with minor oceanic changes, that pursuit has lapsed, and a less profitable but similar species, the anchovy, has filled the emptied ecological niche of the sardine. In another two decades, virtually no substantial stocks of commercial fish will remain underexploited. A side effect is worth noting: laboratory experiments suggest that when over-utilization becomes severe, the food-web efficiency of the remaining population decreases.

While legislation and international agreements could (but probably will not) curb excesses, the effects of the waste from our industrial civilization are considerably more complex. These effects are not necessarily all negative. Pollution perhaps is the greatest menace to mariculture. Yet, for more than a half century, the sewage of Berlin and Munich has enriched high-yielding carp ponds. Flue gases from a power plant in Dorset, England, are washed with seawater, which, laden with carbon dioxide, is put to use in diatom culture, and the diatoms then are fed to bivalves and shrimp.

Chemical pollutants are another matter. Oysters and other filter-feeding animals may concentrate poisons to levels far higher than those found in the surrounding medium. Anatomically, nature provides fish with scant protection. Mainly because of their gills, they come into intimate contact with whatever may be suspended in the surrounding water; in concentrations of less than one part per billion the pesticide Endrin eliminates many fish. Industrial contamination combined with channel dredging and other marine developments may be blamed for the fact that there has been no consequential natural set of the American oyster in Long Island Sound for more than a decade.

Domesticating Fish

None of these difficulties are likely to be resolved by conventional fishing methods. Trawling techniques certainly will improve. Fish stocks will be hunted and herded with sonar, attracted with mercury vapor lights and concentrated with electrical fields. Still, if seafood production is to grow, there must be other new approaches that are both skillfull and ecologically sound.

Biologists generally agree that to further exploit marine productivity will require a technology not unlike that of modern agriculture. To do this, one at the very least would have to limit predation, control diseases and interspecies competition, and perhaps exercise some control over the growing environment. Obviously, none of this is likely to occur soon in the open ocean.

On or near land, rudimentary fish farming is not new. A Chinese scholar wrote the first treatise on the subject in 475 B.C. At about the same time, juvenile shrimp were captured in the Far East on tidal exchanges and "pastured" in lagoons. The Japanese now raise these crustaceans from the egg to market.

Even these somewhat elementary methods can enlarge yields enormously. In Asia, carp sometimes are stocked and cultivated. Large-scale feeding, under otherwise comparable conditions, multiplies more than 5,000-fold the fresh-weight yield of the same fish.

These numbers can become prodigious when compared with terrestrial output. A ton or more of fish or 100 tons of shellfish can be harvested from the same space that a few hundred pounds of beef cattle would require; a pig farmer needs a man-year to produce 25 tons of live pigs, the same time it takes an oyster grower to raise 40 to 60 tons of oysters, not counting the shells. This is a matter of ecological efficiency, of maximum conversion of solar energy into animal protein.

Thus, the potential appears promising, even with the rather artless state of present-day aquaculture. When fishermen of the Japanese inland sea were beset by diminishing catches, their government developed an industry in culturing a jack, the yellowtail. The men who now grow the yellowtail make more money than they did netting for it. The yellowtail is a high-cost food, in common with shrimp and