How R. R. Rowe Found Our Goof

Puzzle Corner:
Allan J. Gottlieb

A week ago Alice and I attended an N.C.A.A. semi-final hockey match. As a devoted pro fan I was surprised at the excitement of a college game. The Cornell fans were impressive but were somewhat overwhelmed by the Wisconsin supporters. As our friend Lou D'Angelo said, "It's tough to compete with the Big Ten in tribal rituals." Cornell scored the first four goals, and each time all their fans shouted "Sieve!" at the goalie. We started to feel sorry for him. But then Wisconsin scored and the place went wild. When the Wisconsin crowd shouted "Sieve!" it sounded like the chorus of a Greek tragedy. I don't know how the goalie could avoid feeling guilty.

My congratulations to Dave and Sue Lapin and everyone else at Wisconsin on the national championship awarded their team and the Gottlieb championship awarded their fans.

Problems

We begin with a bridge problem from Paul Berger:

MAY1 With the following hands, South holds a contract for five diamonds. West's lead is ♠4. Do you want to play offense or defense?

	♠ K, 7	
	♥ K, 5, 4, 2	
	♦ 5, 4, 2	
	♣ 10, 9, 4, 3	
♠ J, 9, 6, 4, 3, 2		♠ Q, 10, 8
♥ Q, 8, 7		♥ A, J, 9, 6, 3
♦ 3		♦ K, 8
♣ K, 6, 2		♣ J, 8, 5
	♠ A, 5	
	♥ 10	
	♦ A, Q, J, 10, 9, 7, 6	
	♣ A, Q, 7	

Here is an interesting contradiction, which Arthur Flerser calls "a demonstration that two equals four:"

MAY2 Given that

$$x^{x^{x^{\cdot^{\cdot^{\cdot}}}}} = 2, \text{ find } x.$$

Recall that in multiple exponentiation, the evaluation starts at the top and proceeds downward. Thus the substitution

$u = x^{x^{x^{\cdot^{\cdot^{\cdot}}}}}$ yields

$x^u = x^2 = 2$, so that $x = \sqrt{2}$.

Now suppose we try solving the equation

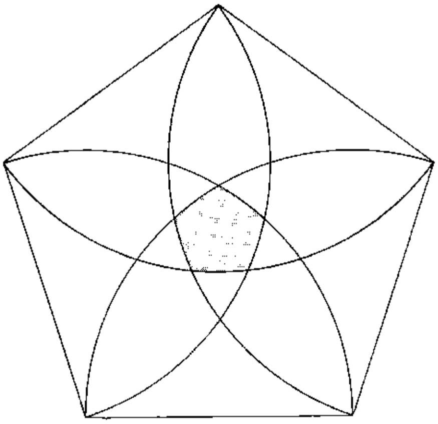
$$x^{x^{x^{\cdot^{\cdot^{\cdot}}}}} = 4.$$

The same reasoning as before leads to the conclusion that $x = \sqrt[4]{4}$.

But the square root of 2 and the fourth root of 4 are precisely the same quantity, both being approximately 1.414. So we are led to the conclusion that 1.414 . . . exponentiated upon itself an infinite number of times, yields both 2 and 4. So 2 = 4. Q.E.D.! What is wrong?

This geometry problem is from Professor Lee Casperson:

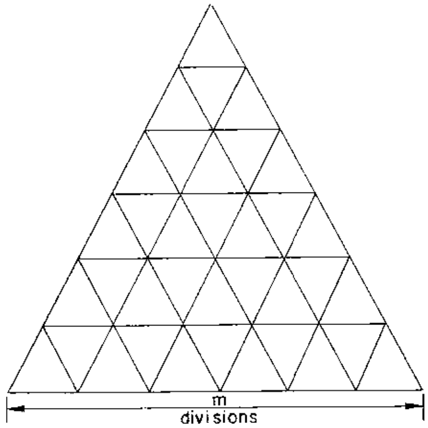
MAY3 Find the exact area of the shaded space in the pentagon with unit sides:



A number theoretic problem from Frank Rubin:

MAY4 Find the quadratic equation with integer coefficients ≤ 10 whose root is the nearest possible approximation to π. (Computer specialists may want to change quadratic to quintic and change 10 to 100.)

In May, 1972, we published a problem about a tile layer and his mosaic surface with a triangle patterned from small equilateral triangles. He claimed that it required 10,000 individual units to form



the large triangle, and that he could retire comfortably if he had as many dollars as there were triangles of all sizes within the triangular pattern. Now the contractor is back again with a new problem, from L. R. Steffens:

MAY5 A tile contractor has laid two floors each composed of 10,000 square pieces—one floor 100 x 100 and the second 80 x 125. What is the total number of squares formed each containing only whole tiles?

Speed Department

Gilbert Shen offers:

SD1 Prove that $(\log_a b)(\log_c d) = (\log_a d)(\log_c b)$.

N. Judell has another proof that 0 = 1:

SD2 Given $\int(1/x)dx \equiv \int(1/x)dx$. Integration by parts gives

$\int(1/x)dx = x/x - \int x\, d(1/x)$. Restricting $x \geqq 1$ gives

$\int(1/x)dx = 1 - \int x(-1/x^2)dx$, because $(1/x) = x^{-½}dx$. Then

$\int(1/x)dx = 1 + \int(1/x)dx$. Q.E.D. $0 = 1$.

Solutions

JA1 A game of chess has just concluded, leaving (after Black's last move) White's king at his K1 and Black's king at his KR5 (White's KR4). Black, out of whimsy, asks if he can have his last move back. White, never one to give something away for nothing, says all right, if *he* can have his last move back too. Black agrees and takes back his last move. Then White does the same and makes another move, whereupon Black moves and gives checkmate. Problem: find the moves.

Peter Groot finds that the position before the two sets of "last" moves was
White: K at K1, R at R1
Black: K at KN6, Q at KR5.
The original "last" moves were R x Q and K x R. The second pair were 0-0, Q-R7 (mate).

Also solved by Harry Nelson, Norman Neff, and the proposer, Alan La Vergne.

JA2 Each of the letters in the clues (R, S, T, W, X, Y, and Z) stands for a decimal integer which may have many digits. The problem is to find numbers satisfying the equations in the clues and properly filling the blanks.

A				B		C
		D				
E						

Across:
A = R! + S
D = X − Y − Z − Z
E = W^W/T

Down:
A = T^S
B = X + X + X + X
C = X + X − Y
D = Y

R. Robinson Rowe found our typographical error and was thus able to solve the problem. His solution is shown below, with the values of the clues and digits in the diagram array.

R = 10	X = 224
S = 15	Y = 108
T = 2	Z = 5
W = 8	

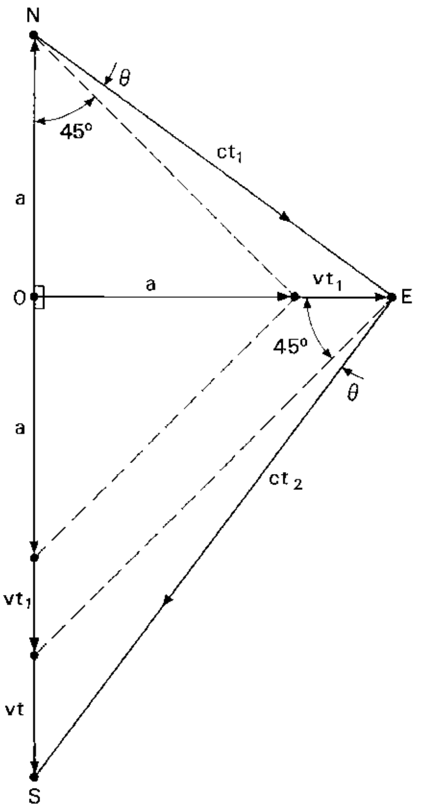
A 3	6	2	8	B 8	1	C 5
2				9		0
7		D 1	0	6		0
6		0				6
E 8	3	8	8	6	0	8

The typographical error was a plus instead of a times sign, so that C = X · X − Y. Mr. Rowe attaches a lengthy "random resume of my speculations" which led him to the typographical error and the solution, but space unfortunately does not permit its publication; readers who wish a copy may obtain one from the Editors at Room E19-430, M.I.T., Cambridge, Mass., 02139.

Also solved by Peter Groot and the proposer, Harry Nelson. The Editors join in apologizing for the typographical error which so challenged Mr. Rowe—but which may well have discouraged a good many others who found the problem intriguing.

JA3 Four observers N, E, S and W depart the center of an unaccelerated two-dimensional cartesian coordinate system at the same time with equal, constant speeds. N and S travel in the direction of the positive and negative y axes while E and W travel in the direction of the positive and negative x axes, respectively. N directs a ray of light toward E who reflects it with a mirror to S who reflects it to W who reflects it back to N. Each observer measures the angle between the directions of propagation of the received and transmitted rays with a theodolite. They all find the same angle. What is it?

The following is by Harry Zaremba:
In the figure, let

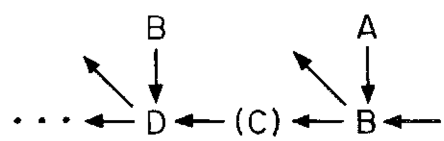


a = distance each observer has travelled before light is emitted from N,
c = velocity of light,
v = common velocity of the observers,
t_1 = time for light to travel from N to E, and
t_2 = time for light to travel from E to S.
From triangle NEO,
$(ct_1)^2 = a^2 + (a + vt_1)^2$, or
$t_1/a = (v \pm \sqrt{2c^2 - v^2})/(c^2 - v^2)$.
From triangle SEO,
$(ct_2)^2 = (a + vt_1)^2 + (a + vt_1 + vt_2)^2$, or
$t_2/(a + vt_1) = (v \pm \sqrt{2c^2 - v^2})/(c^2 - v^2) = t_1/a$.
Also from the figure,
$\tan(45 + \theta) = (a + vt_1)/a = 1 + vt_1/a$; and
$\tan(45 + \phi) = (a + vt_1 + vt_2)/(a + vt_1) = 1 + vt_2/(a + vt_1)$.
Since $t_1/a = t_2/(a + vt_1)$, then
$\tan(45 + \theta) = \tan(45 + \phi)$.
Therefore, angle $(45 + \phi)$ is the complement of triangle NEO, and the angle NES between the received and transmitted rays at E is 90°. The same procedure can be applied at points S, W, and N as illustrated above with similar results. Hence, the angle between received and transmitted light rays at each observer will be 90°. The result is the same irrespective of the magnitude of the observers' common velocity.

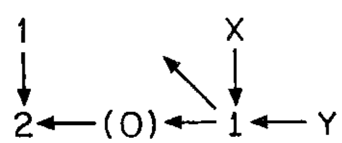
Also solved by J. Bledsoe, J. Fidel-Holtz, Peter Groot, Winslow Hartford, Woodrow C. Johnson, Hans Rasmussen, and R. Robinson Rowe.

JA4 What number ending with the digit 2 is such that when the last digit becomes the first, the resulting number is exactly twice the original?

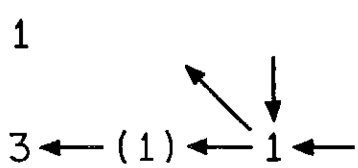
Here is a solution and a free generalization from Kenneth Hules:
Using the following technique we can construct the required number starting from the units place:



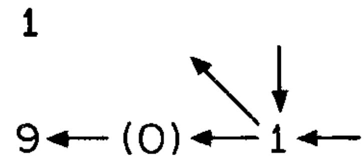
where
A = previous sequential digit established
B = units digit of operation (A · multiplier) + previous carry; in this problem the multiplier is 2.
C = carry digit of operation (A · multiplier) + previous carry.
D = units digit of operation (B · multiplier) + C.
From the problem statement we know the first digit (i.e., units) is 2 and the concluding configuration will look like:



Thus we construct:
1 0 5 2 6 3 1 5 7 8 9 4 7 3 6 8 4 2
2 1 0 5 2 6 3 1 5 7 8 9 4 7 3 6 8 4
to find the required answer:
105263157894736842.
Using this technique we can change the problem statement to any combination of last digit (to become new first digit) and multiplier. No guarantee is given that **all** combinations possess solutions. For examples:
Last digit of original number = 3, multiplier = 2. The terminal array is



and the construct is
N = 1 5 7 8 9 4 7 3 6 8 4 2 1 0 5 2 6 3
2N = 3 1 5 7 8 9 4 7 3 6 8 4 2 1 0 5 2 6.
Last digit of original number = 9, multiplier = 9. The terminal array is



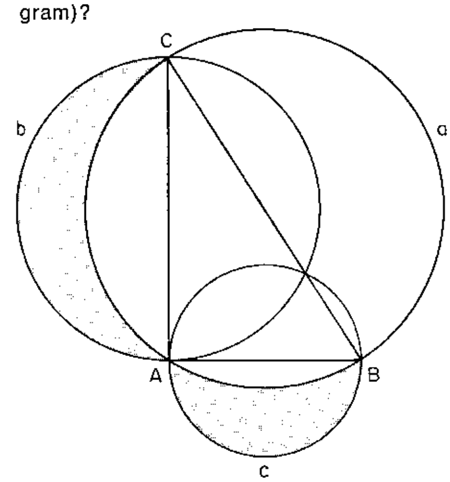
and the construct is the long number given in the box at the top of this page.

N = 1 0 1 1 2 3 5 9 5 5 0 5 6 1 7 9 7 7 5 2 8 0 8 9 8 8 7 6 4 0 4 4 9 4 3 8 2 0 2 2 4 7 1 9
9N = 9 1 0 1 1 2 3 5 9 5 5 0 5 6 1 7 9 7 7 5 2 8 0 8 9 8 8 7 6 4 0 4 4 9 4 3 8 2 0 2 2 4 7 1

This technique may be used to construct and solve the class of problems wherein the number sought when multiplied by a second number (less than 10) yields a third number whose first digit(s) bear some given relation to the last digit of the number sought (i.e., not necessarily the same digit, as in **JA4**). The trick seems to be to identify the terminal array so you know when to stop.

Also solved by F. F. Assmann, J. Bledsoe, Philip Bobko, C. Brooks, J. Fidelholty, Peter Groot, Winslow Hartford, Greg Jackson, Woodrow Johnson, Thomas Jones, Jonathan McCray, J. D. Miller, Terry Montlick, E. A. Nordstrom, Hans Rasmussen, R. Robinson Rowe, B. Rouben, Sheri Schneider, Jay Schwartz, Robert Shooshan, Larry Wischhoefer, Harry Zaremba, and the proposer, H. W. Hardy.

JA5 Given right triangle ABC with coplanar circles constructed on each of its three sides. The center of each circle is on the midpoint of a side of the triangle, and the length of a radius of a circle is equal to half the length of the side of the triangle. If the area of triangle ABC is 12 ft.², what is the combined area of the smaller circular regions which are not intersected by the largest circular region (the shaded regions in the **dia**gram)?



The following is from Larry Wischhoefer:
We know by geometry that point A lies on the large circle. By Pythagoras, $AB^2 + AC^2 = BC^2$, and thus the area of two circles
$A_b + A_c = A_a$.

Areas of the respective half-circles are also equal:
$\frac{1}{2} A_b + \frac{1}{2} A_c = \frac{1}{2} A_a$.
We also see that
$\frac{1}{2} A_a = A_{triangle} + A_{shaded} = 12 + A_{shaded}$, and
$\frac{1}{2} A_b + \frac{1}{2} A_c = A_{shaded} + A_{of\ interest}$.
Combining the last two expressions,
$A_{shaded} + A_{of\ interest} = 12 + A_{shaded}$, or
$A_{of\ interest} = 12$.

Also solved by Jordan Backler, J. Bledsoe, Earl Creekmore, J. Fidelholtz, Raymond Gaillard, S. Glazer, Peter Groot, Winslow Hartford, John P. Hoche, Woodrow C. Johnson, Anastasios Jsiatis, J. D. Leber, Jonathan McCray, E. A. Nordstrom, E. R. Pejack, Robert Pogoff, John E. Prussing, Hans Rasmussen, Ben Rouben, R. Robinson Rowe, W. H. Stephenson, Roger A. Whitman, Harry Zaremba, anonymous, and the proposer, Mary Lindenberg.

Better Late Than Never
The following names should have appeared last month as having submitted solutions to problems published in December:
DE1 Bill Friedmann
DE2 Harold Rice
DE4 Harry Nelson, Steven Alexander, and Thomas Weiss.
Additional solutions have come from the following readers to the problems indicated:
JY3 Frank Rubin
O/N-1 Frank Rubin, Les Servi
O/N-4 Frank Rubin, Ron Moore
O/N-5 Frank Rubin

Hallock G. Cambell has the following comment on the solution given for M1 as printed in *Technology Review* for October/November, which he says "is quite wrong:"
Puzzler Harry Nelson mistakenly believes that a positional draw in chess requires repetition of three *moves*. No, it requires only a three-time repetition of the same *position,* quite independent of the preceding moves. Game M1 becomes a draw at move 29a if either player calls attention to this third repetition (with black to move) of the diagrammed position, steps 25a, 27a, and 29a.

Speed Department Solution
SD1 Change the variable: $b = d^k$ for some k (the variable b is replaced by k).
$(\log_a b)(\log_c d) = \log_a d^k \log_c d$
$= k \log_a d \log_c d = \log_a d \log_c d^k$
$= \log_a d \log_c b$.

Architecture by Theory, Computer

Book Review:
William W. Caudill

Third Generation: The Changing Meaning of Architecture
by Philip Drew
Praeger Publishers, New York, 1972, 176 pp., $25.00

The Architect and the Computer
by Boyd Auger
Praeger Publishers, New York, 1972, 135 pp., $13.50

Buy both. The value of both these books is their affluence of theory. Architects and their firms die without theory. And some get sick because of an overdose of theory. There must be a mixture of theory and practice. This applies also to schools. There are too many sick firms and too many sick schools that fail to find the balance needed for professional health. Theory needed? It's in the reading.

The Value of Theory
Philip Drew's *Third Generation* was pure inspiration to this hardnosed practitioner who has a great need for a strong dose of theory. The first chapter, "The Uncertain Future," is worth the price of the book. Drew doesn't preach gloom. He challenges. He states, "The first generation," listing Wright, Gropius, Mies van der Rohe, Le Corbusier, Nervi, Neutra, and Fuller, "scavenged science and technology for levers to extricate architecture from the iron grasp of the past and launch it into the new machine age. Their task was to drag architecture into the 20th century. The challenge facing the third generation," he writes, listing the superstars born in the inter-war period, 1918-38, including Rudolph, Van Eyck, Utzon, Roche, Otto, Venturi, Stirling, Chalk, Kirutake, Isozaki, Andrews, Kurokawa, Cook, Alexander, and Safdie, "is to see it safely through, and they now need to review those features of architectural ideology which were taken over from science and technology in the 1920s." That's the gist.

Drew introduces the ideas and resulting products of this selected group of "the third generation" with a high degree of thoroughness. Chapter 2, "Pattern Language" is a beautiful dissertation on architectural form. He points out the problem of modern buildings "expressed in a special form which makes it inaccessible to all but a small band of initiates. Unlike the pattern language of unselfconscious cultures which are shared by all members of the community, the language of modern architecture remained the exclusive property of architects." Bullseye! Most buildings are public domain. The public owns the views, if not the building. Architecture is too important to be personal expression. Architecture, however, is a personal experience—a birthright. In Chapter 3, Drew states that "the third generation reacted against the tyranny of a too-explicit functionalism" and pointed out that Corbu's Ronchamps initiated "a number of important third generation themes" bringing together "rational geometric and intuitive organic ideals in a dynamic synthesis." I take it to mean that the third-generation architects are trying to discover that there exists a symbiosis between functionalism and formalism. If so, I could not agree more.

Functionalism in the 1950s became a nasty word. Formalism followed—just as nasty. Since then we have matured professionally. Today I offer no apologies for the formalists—architects obsessed with form—playing on our CRS team. Nor do I offer apologies for the functionalists—architects, including programmers, who are obsessed with function. They are both needed to hold up their end of team action. We feel architecture is too important to be entrusted to one man. When a job comes into CRS we give the leadership to a project troika—a manager, who by nature is a functionalist; a designer, who is by nature a formalist; and a technologist, whose passion is to put things together with minimum means to obtain maximum effect.

Unquestionably the author of this stimulating book will receive arguments on how he arbitrarily drew the lines to separate first generation, second generation (which he says very little about but which includes such outstanding architects as Aalto, Kahn, Jacobsen, Breuer, P. Johnson, Saarinen, Tange, and Doxiadis), and the third generation. And there will always be at least one critic who will scream to high heaven because his favorite superstar was not included. I cried a little.

One might question the author for putting so many architectural theologians in the third generation. But why not? The book is about architectural ideology. We need architects who seriously delve into theory and dabble in practice. They contribute by stimulating the practitioners. They need not build. Our firm at one time hired two highly theoretical professors from Rice University for a specific project to spend time at a site and in the community writing "specifications for an architecture" solely to inspire our designers. They gave us the theory we didn't have. When I served as Director of the School of Architecture at Rice, I deliberately hired theoretical people who could think and write on theory but who had never designed a building. I also balanced the situation by mixing the faculty with top flight architects who were stronger in practice than in theory. Drew's broad choice of architects representing the third generation makes sense. There's a good mix of theologians and practitioners.

The Value of Proposal
Some may wonder why the inclusion of so many proposals for projects, discussed in detail. Not I. Proposals do influence. And they are more pure. And easier to understand than real projects with all the nuances of programmatic