zees. Drs. Deets and Harlow have learned what abnormal emotional responses result when a monkey is raised in an abnormal environment and that during one period of his life—from 3 to 18 months of age—isolation irreparably damages his responses of and to fear and aggression.

I question if what they have learned could not be learned by the natural and kindly method of Dr. Goodall. I question, even if information of value is so obtained, whether we have the right to condemn another animal—and especially one whose emotions are so evident and so strong—to the years and lives of cruelty that they report.

Ever Tried a Cross Number Puzzle?

Puzzle Corner:
Allan J. Gottlieb

I have been nearly deluged by first-time responses—people who read the column but have never written me before—in response to our offer in October/November. I called the Editor to ask him how many gifts he was prepared to send. He said that he expected I would receive perhaps six or eight letters; I told him I already had five times that many and they were still coming. He was delighted, but I thought he sounded a little worried, too . . . Every first-time respondent was to receive a gift; was he worried about a few dozen extra Christmas presents?

From now on, if the proposer of a "speed" problem submits a solution with the problem, that solution will appear at the end of the column in which the problem is published.

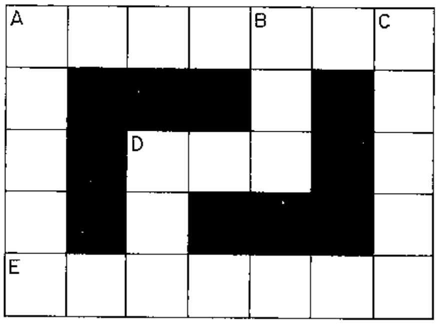
Problems

This month we begin with a chess problem from Alan La Vergne, which he credits to his M.I.T. classmate Robert Wolf:

JA1 A game of chess has just concluded, leaving (after Black's last move) White's king at his K1 and Black's king at his KR5 (White's KR4). Black, out of whimsy, asks if he can have his last move back. White, never one to give something away for nothing, says all right, if *he* can have his last move back too. Black agrees and takes back his last move. Then White does the same and makes another move, whereupon Black moves and gives checkmate. Problem: find the moves.

Here is a novel problem from Harry Nelson, a cross-number puzzle. He explains that it is quite like a crossword puzzle, with clues across and down, but the clues are about numbers and the blanks are to be filled with digits. Each square is to contain a single decimal digit; for example, A across is a seven-digit number.

JA2 Each of the letters in the clues (R, S, T, W, X, Y, and Z) stands for a decimal integer which may have many digits. The problem is to find numbers satisfying the equations in the clues and properly filling the blanks.



Across:
A = R! + S
D = X − Y − Z − Z
E = W^W/T

Down:
A = T^S
B = X + X + X + X
C = X + X − Y
D = Y

This problem is from James C. Wilcox:

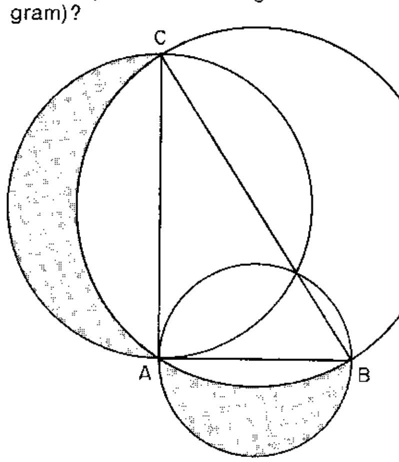
JA3 Four observers N, E, S and W depart the center of an unaccelerated two-dimensional cartesian coordinate system at the same time with equal, constant speeds. N and S travel in the direction of the positive and negative y axes while E and W travel in the direction of the positive and negative x axes, respectively. N directs a ray of light toward E who reflects it with a mirror to S who reflects it to W who reflects it back to N. Each observer measures the angle between the directions of propagation of the received and transmitted rays with a theodolite. They all find the same angle. What is it?

A little number problem from H. W. Hardy:

JA4 What number ending with the digit 2 is such that when the last digit becomes the first, the resulting number is exactly twice the original?

Finally, a geometry problem from Mary Lindenberg:

JA5 Given right triangle ABC with coplanar circles constructed on each of its three sides. The center of each circle is on the midpoint of a side of the triangle, and the length of a radius of a circle is equal to half the length of the side of the triangle. If the area of triangle ABC is 12 ft.², what is the combined area of the smaller circular regions which are not intersected by the largest circular region (the shaded regions in the diagram)?



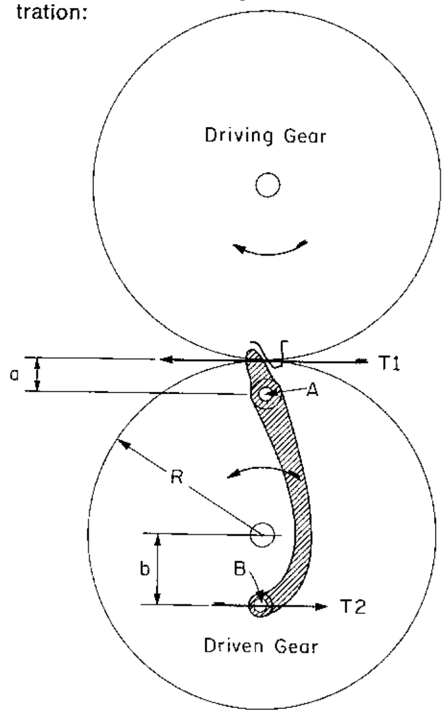
Speed Department

A paradox from Norman Brenner:

SD1 Define α to be the complex angle for which $\tan \alpha = i$. Then, for any θ, $\tan(\alpha + \theta) = (\tan \alpha + \tan \theta)/(1 - \tan \alpha \tan \theta) = (i + \tan \theta)/(1 - i \tan \theta) = i = \tan \alpha$. How it this possible?

Here is one from T. Davidson; who writes that someone tried to sell his firm the issued patent for a gear set to produce increased torque in the driven gear at the same r.p.m. as the driving gear. (He does not have the patent number nor the inventor's name.):

SD2 Given the arrangement in the illustration:



Both gears are the same diameter. The driving gear is conventional with teeth cut in the rim. The driven gear is a disc with teeth formed by levers supported by pins A and B. As both gears have the same diameter, input and output speeds will be equal. The driven gear will have the same tangential load (T1) as a conventional gear, but output torque will be increased by reaction of the lever on pin B producing an additional output torque (b · T2).

Solutions

The following are solutions to problems published in "Puzzle Corner" for July/August, 1972:

Jy1 Given these hands:

	♠ K Q J 10 ♥ A K x x ♦ x x ♣ A x x	
♠ 9 8 ♥ Q x x x ♦ J x x x ♣ x x x		♠ A 7 6 ♥ J x x ♦ Q x x ♣ J x x x
	♠ 5 4 3 2 ♥ x x ♦ A K x x ♣ K Q x	

West leads ♠ 9; East takes the first trick with the ♠ A and returns the ♠ 7. Can South make his contract of six spades?

George J. Todd proposes that South can indeed make six spades; here are the 13 tricks he proposes:

1 Lost to East.
2 Taken in dummy.

3 Third round of trumps taken in dummy.
4, 5, 6 Taken by ♣ A, ♣ K and ♣ Q in any order. By now West has had to discard a red card; for the sake of argument, let's say he chose to discard a heart.
7, 8 ♥ A, ♥ K taken in dummy.
9 Small heart ruffed in closed hand.
10, 11 ♦ A, ♦ K taken in closed hand.
12 Small diamond ruffed in dummy.
13 Fourth heart in dummy is now good for twelfth trick.
If West had discarded a diamond on tricks 3 or 6, merely interchange hearts and diamonds in tricks 7 through 13.

Also solved by Burt S. Barrow, Winslow Hartford, Walter Hausz, Jim Kemphner, Bryan Lewis, Thomas Mauthner, John Meader, Dave Mohr, James Poynter, R. Robinson Rowe, Les Serve, Harry Terkanian, Smith D. Turner, and the proposer, Michael Kay.

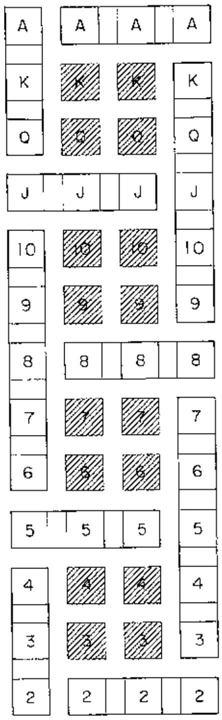
Jy2 Let p be a prime. Can p^2 divide $2^n - 1$ when p does not divide n?

This solution is from Tom Glennon:
Suppose p is a prime ∍ $p^2 \mid (2^n - 1)$; since $2^{\phi(p^2)} \equiv 1 \bmod p^2$, we have $p^2 \mid 2^{p(p-1)} - 1$. Hence $p^2 \mid \gcd(2^n - 1, 2^{p(p-1)} - 1)$. But $\gcd(2^n - 1, 2^{p(p-1)} - 1) = 2^{\gcd[n,p(p-1)]} - 1$, so that if $p \nmid m$, then $p^2 \mid 2^{\gcd[n,(p-1)]} - 1 \Rightarrow p^2 \mid 2^{(p-1)} - 1$. Thus the original problem is equivalent to finding those primes p ∍ $p^2 \mid 2^{(p-1)} - 1$. Of course it is always the case that $p \mid 2^{(p-1)} - 1, p \neq 2$.
The square of a prime p ∍ $p^2 \mid (2^{(p-1)} - 1)$ is called a Wieferich square after A. Wieferich, who proved the following theorem in 1909:
If $p^2 \nmid 2^{(p-1)} - 1$, p prime, then the equation
$a^p + b^p = c^p$ has no solution in integers not divisible by p.
It has not been proven that there are infinitely many Wieferich squares, nor has it been proven that there are infinitely many primes p ∍ $p^2 \nmid 2^{(p-1)} - 1$. In fact, there are only two primes $< 100,000$ whose squares are Wieferich squares; these are $p = 1,093$ and $p = 3,511$. As a side note, the Wieferich squares form a subclass of another rare class of numbers called the composite fermatians. These are the composite numbers N ∍ $N \mid 2^{(N-1)} - 1$. There are only 2,043 composite fermatians $< 10^8$.

Also solved by Gerald Blum, Raymond Gaillard, Judith Longyear, and R. Robinson Rowe.

Jy3 Arrange a full deck of cards in any mixture of groups of three or more by kind or by consecutive sequence of the same suit (example: four hearts, four spades, and four diamonds; or eight spades, nine spades, and 10 spades). What is the maximum number of cards that can be left out such that they cannot be formed into groups of sequences nor added to those previously made?

This solution is from Edward Gaillard:
The maximum number of excluded cards is 16, as the drawing shows:



In order *not* to be able to be added to another group of cards, each card excluded can have at most one card to the right or left which is not part of a previously picked *horizontal* group and at most one card above or below it which is not part of a previously picked *vertical* group. These excluded cards are the shaded cards. I solved this problem by laying out the cards as shown. Since I am only 11 years old, I asked my father to make the drawing for me.

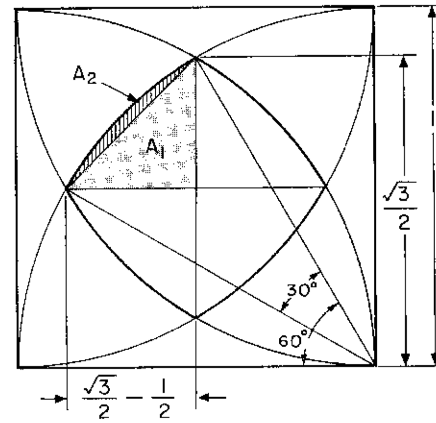
Also solved by Gerald Blum, David Mohr, and the proposer, David Merfeld.

Jy4 Seven thieves stole some gold bars. Unfortunately, when they started to divide the take it didn't come out even. But finally they figured out how to divide the bars: the first thief received one plus one-seventh of the remaining, the second man two plus two-sevenths, etc., the last man receiving seven plus seven-sevenths—i.e., all the remaining bars. In this way they didn't have to divide any bars. What is the smallest number of bars they could have stolen? And which man received the most?

This solution was submitted by Gerald Blum:
Let A be the first man, B the second, . . . and G the seventh. Using the fact that G takes all that's left at that point, we see $F = 6 + 6G$, $E = 5 + 5/2(F + G)$, etc., yielding after much algebra
$F = 6 + 6G$, $E = 20 + (35/2)G$, $D = 116/3 + (98/3)G$, $C = 103/2 + 343/8)G$, $B = 727/15 + (2401/60)G$, and $A = 5119/180 + (16807/720)G$. This implies $G \equiv 2 (\bmod 3)$, $G \equiv 4 (\bmod 8)$, $G/4 \equiv 8 (\bmod 15)$, $G/4 \equiv 23 (\bmod 180)$; the smallest satisfactory $G = 92$, giving $F = 558$, $E = 1630$, $D = 3044$, $C = 3996$, $B = 3730$, and $A = 2176$—a total of 15226. Clearly C, the third man, gets the most and G gets gypped!

Also solved by John Bobbitt, Charles Faulders, Winslow Hartford, Karl Kadzielski, Dave Mohr, J. Ralbrancha, R. Robinson Rowe, John Rule, Joel Schwimmer, George Todd, Richard Wobus, and Harry Zaremba.

Jy5 Given a unit square with unit radius arcs drawn centered at each corner, find the exact area of the four-sided (shaded) space bounded by the arcs below:



This solution is from M. Sacid Ozker, who uses A to designate the area required:
$A/4 = A_1 + A_2$
$A_1 = (1/2)(\sqrt{3}/2 - 1/2)^2 = (1/8)(4 - 2\sqrt{3})$
$A_2 = (1/2)(30° n/180°) - \sin 30°$
$A_2 = (1/2)(n/6 - 1/2) = (1/4)[(n/3) - 1]$
Therefore
$A/4 = 1 - \sqrt{3} + n/3 = 1 - 1.7321 + 1.0472 = 0.3151.$

Also solved by Robert Anthonyson, Gerald Blum, Charles Faulders, Arthur Flexser, Raymond Gaillard, Winslow Hartford, Walter Hausz, Joseph Horton, Andrew Kazdin, G. Harold Klein, Mary Lindenberg, Samuel Loebl, Dave Mohr, J. F. Parsons, Robert Pogoff, John Prussing, Claude Rabache, J. Ralbrancha, R. Robinson Rowe, John Rule, David Sales, Donald Savage, Les Servi, W. M. Stephenson, Jr., George Todd, Brian Witzke, Harry Zaremba, and the proposer, Charles Landau.

Speed Department Solutions

Solutions to the "speed" problems given above, as supplied by their proposers:
SD1 $\propto = -\infty$.
SD2 From the following table:
Load:
T2 $T1[a/(R - a + b)]$
TA $T1[(R + b)/(R - a + b)]$
Torque:
$T1[ab/(R - a + b)]$
$T1[(R + b)(R - a)/(R - a + b)]$
The last expression can be written
$T1(R^2 - Ra + Rb - ab)/(R - a + b)$
and the total torque becomes $T1 \cdot R$.

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