Thompson, and Willis Whitney, among others, both the humanity of the people and their quest for knowledge become fascinating and exciting.

The Answer Is Ham-on-Rye

Puzzle Corner:
Allan J. Gottlieb

Tonight I was able to listen to "Sports Huddle" for the first time in a year. As I mentioned before, Central California may have beautiful weather but it has awful radio programs—and sports coverage. So it's good to hear Eddy, Mack, and Jim after such a long absence.

I've just finished my first two weeks of full-time teaching. They were enjoyable—and, I feel, successful . . . and I wonder if my students feel the same way. It is hard to realize that my school days are over and I'm all grown up now; it seems like only yesterday when I went to my first class at M.I.T. I remember it well. The class was 8.01, the room was 26-100, and I was late.

Problems

This month we start with a bridge problem from John Schwartz:

DE-1 Here is a hand actually encountered at the bridge table:

North:
♠ K 10 5
♥ K 10 7
♦ A Q J 4
♣ J 8 3

West:
♠ 8 4
♥ Q 9 8 6 5 4
♦ 6
♣ K 10 5 4

East:
♠ 9 7 2
♥ A
♦ 10 9 8 7 5 3 2
♣ Q 9

South:
♠ A Q J 6 3
♥ J 3 2
♦ K
♣ A 7 6 2

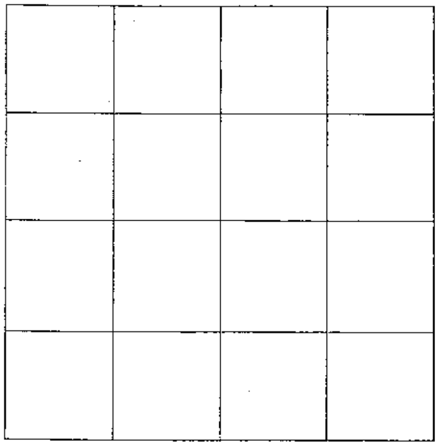
West leads ♦6. How is South to make six spades?

The following is from Frank G. Smith:

DE-2 Give an algebraic proof of the following geometrical construction method for inserting within a circle a figure of any number of sides:

1. Draw the diameter of any circle you may have chosen.
2. Divide this diameter into the number of units you wish to have inscribed in the circle.
3. At the extremities of this diameter, scribe two intersecting arcs with radius equal to the diameter.
4. From the intersection of these two arcs draw a line through the second division point from the circle, extending it to the circle.
5. From this intersection with the circle, draw a line to the (zero) point of the diameter on the circle. This line will then be the side of a figure inscribed in the circle, having the number of sides into which the diameter was divided.

A magic square from George L. Uman:



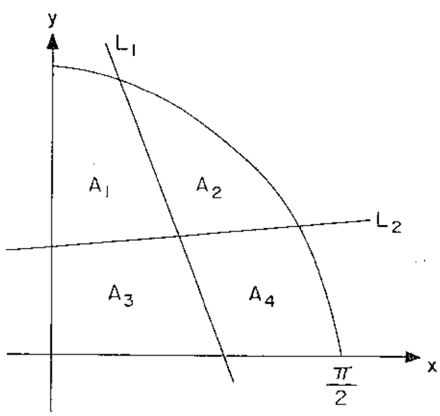
DE-3 In each of the 16 squares of the figure, place a *different* letter, selected so each row, column, and long diagonal will spell a *different* four-letter word when the letters are selected consecutively in one or the other of the only two possible directions, as we do with numbers. There will be a total of 10 different words, all of which must be defined in any one edition of Webster's dictionaries.

Richard Lipes submits the following problem, reporting that he has solved it but only by resorting to a computer to solve a transcendental equation involving x^2, sin x, and cos x; he asks if any "Puzzle Corner" reader can find a solution without a computer.

DE-4 The area under the curve

$y = \cos x,\ 0 \leq x \leq \pi/2$

and the line $y = 0$ is to be divided into four equal areas by a line parallel to the y-axis and another line. Give the equation of the two lines. (In the drawing, areas $A_1 = A_2 = A_3 = A_4$.)



The following trigonometry-algebra problem has been submitted by Paul Schweitzer.

DE-5 Find all solutions to

$\sin(x + y) = \sin x + \sin y.$

Speed Department

SD1 Clark Tompson wants you to devise an algorithm for extracting cube roots.

SD2 Victor Sauer wants to know if this is modern mathematics or an Italian story: Seven persons buy a car and the mathematician in the group arrives at $13 per person for a $28 car:

```
  13
7)28
   7
  21
  21
```

One of the other six of the group questions the amount, but the following addition proves the mathematician to be correct:

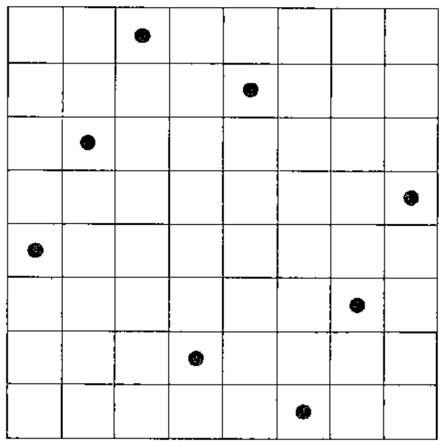
```
13
13
13
13
13
13
13
7 + 21 = 28
```

Solutions

The following are solutions to problems published in *Technology Review* for June, 1972:

JN-1 (a) On a chess board, place eight queens such that no queen is vulnerable to other queens—i.e., no queen should be in the path of other queens.
(b) Obtain one or two general patterns for placing 2n dots in a grid 2n x 2n ($n > 1$), with the same restriction as above.
(c) Extend the solution for (b) for placing n dots in a grid n x n ($n > 3$).

Arthur L. Kaplan believes his solutions are unique; the solution to (a) is in the form of this diagram:



(b, c) (1) For n equal to any of the numbers 3i + 4 in the case of the 2n x 2n grid, or for n equal to any of the numbers 6i + 8 in the case of the n x n grid, use the same pattern as that for the 8 x 8 grid in (a), starting at the left side three squares from the bottom, working toward the upper edge one square to the right and two squares up until the upper edge is reached, and then placing one queen two squares to the right and one square down, and then repeating the pattern symmetrically on the other half of the grid.
(2) For n equal to all other digits, start at the left side, one square from the bottom, working toward the upper edge one square to the right and two squares up; when the upper edge is reached, start in the next column to the right at the bot-

tom square and work up one square to the right and two squares up, until the right edge of the grid is reached.

Also solved by Robert L. Bishop, R. E. Crandall, David Merfeld, R. Robinson Rowe, Victor Sauer, Brian Witzke, Harry Zaremba, and the proposer, Benjamin Whang.

JN-2 What simple rule governs this sequence:
1,1,2,1,2,1,2,3,1,3,2,1,2,3,3,1,3,2,1,3, 2,3,4, . . .

The proposer, Mary S. Krimmel, notes that the entries are half the differences between successive odd primes.

Also solved by R. E. Crandall

JN-3 As mentioned last month, there was a serious omission in the original published version of this problem. Complete, it should read:
Show or prove that

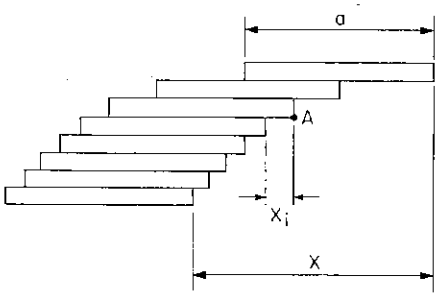
$$\left(\frac{1-x}{1+x}\right)(2x+1)\prod_{k=1}^{\infty}\left\{[1+x^{2^k}]\left[1+\left(\frac{x}{1+x}\right)^{2^k}\right]\right\}=1$$

$$-\frac{1}{2}<x<1.$$

I will give the solution along with those to the October/November problems, in the February issue of *Technology Review.*

JN-4 You are given as many playing cards as you wish to use, each of length a and width b. They are stacked one upon another in the usual way, so that each card is in contact with only the cards immediately above and below it, with the objective of achieving the maximum overhang before they fall over. What is the length of this overhang?

The following is from Harry Zaremba:



Assume the edges of all cards along their length (a) are in vertical planes. Beginning from the top, let the edge (b) of each successive lower card be placed under the center of gravity of the cards above. The center of gravity of i cards located from edge A of the ith card is given by

$$x_i = a/2i.$$

For n cards, the total overhang will be

$$x = \sum_{i=1}^{n-1} x_i = a/2 + a/4 + a/6 + \ldots + a/2(n-1), \text{ or}$$

$$x = a/2[1 + 1/2 + 1/3 + 1/4 + \ldots + 1/(n-1)].$$

The overhang could be constructed without a limit since the series in brackets is divergent for $n \to \infty$. If the diagonals of all cards are in a vertical plane, and the corner of each successive lower card is placed under the center of gravity of the cards above, then the maximum overhang for n cards is

$$x = \{(a/2)^2 [1 + 1/2 + 1/3 + \ldots + 1/(n-1)]^2 + (b/2)^2 [1 + 1/2 + 1/3 + \ldots + 1/(n-1)]^2\}^{1/2}$$

or

$$x = \frac{1}{2}[1 + 1/2 + 1/3 + \ldots + 1/(n-1)]\sqrt{a^2 - b^2}.$$

Also solved by R. E. Crandall, R. Robinson Rowe, and the proposer, Roy Sinclair.

JN-5 Find the cube root of INVENTORY and verify that it can be eaten or drunk.

The "best" solution is from David E. Anderson:
The cube root of INVENTORY is RYE:

$$\sqrt[3]{317214568} = 682$$

$$\sqrt[3]{\text{INVENTORY}} = \text{RYE}$$

The problem therefore has two solutions: rye can be eaten as in ham-on-rye or drunk as in rye whiskey.

Also solved by Winslow H. Hartford, Richard Hill, John Prussing, R. Robinson Rowe, Greg Schaffer, Harry Zaremba, and the proposer, R. E. Crandall.

Better Late Than Never

The bridge problem which appeared as number **51** in the January, 1972, issue of the *Review* comes in for criticism from Ed Nordstrom, and my lack of bridge mastery has obviously again caused problems. Mr. Nordstrom writes:
If play proceeds as outlined in the "solution" published in the *Review* for May, with four cards to play the hands would be

West:
♥ J 7 6
♣ K

South
♠ 5
♥ Q 10 3

Now what self-respecting West would throw away the ♣K if the last trump (♠5) were led? I still maintain that proper defense will set this contract.

Comments on the solution have also come from Alan La Vergne, Richard C. Lesser, and Lars Sjodahl.

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