Prizes for Newcomers

Puzzle Corner:
Allan J. Gottlieb

Hi. Alice and I are back East and ready for the academic year. I am teaching at the State College in North Adams, Mass. (01247), and all mail should be sent to me at the Department of Mathemaatics here.

This summer we drove fairly slowly across the country, stopping at many national parks. A year ago I had never been to one; now I've seen seven, and I heartily recommend all of them: Haleakala, Volcanos, Yosemite, Sequoia, Yellowstone, Grand Teton, and Glacier. Having lived in New York City and Boston all my life—until two years ago—I had a very distorted view of this sparsely populated country of ours. Although the constant travelling was tiring, the trip was educational and quite rewarding.

The only regrets I have about our California stay are having to leave another set of friends behind. Let me therefore dedicate this column to my colleagues at the University of California in Santa Cruz and to our neighbors Eric and Gail Heit and Bob and Diane Jackson. We miss you all.

Before starting a new year's columns, let me review the "rules" for Puzzle Corner: every month we publish five problems and several "speed problems," selected from those suggested by readers. The first selection each month will be either a bridge or a chess problem. We ask readers to send us their solutions to each problem, and three issues later we select for publication one of the answers—if any—to each problem, and we publish the names of other readers submitting correct answers. Answers received too late or additional comments of special interest are published as space permits under "Better Late Than Never." Except under unusual circumstances, no answers or discussions are published concerning "speed problems." And I cannot respond to readers' answers and queries except through the column itself.

You've Never Written Me?

As you see, readers' participation is not only welcome; it's essential to the success of "Puzzle Corner." We're grateful to the many who have contributed in the six years since "Puzzle Corner" began in *Technology Review*; but the Editors tell us that lots of people comment on the column but say they've never submitted answers or problems. So here is something special, just in time for Christmas, 1972: *Technology Review* will send a valuable prize to each reader who writes me for the first time ever in response to this installment of "Puzzle Corner" by December 15, the deadline for the column in which answers to these problems will appear (February, 1973, issue). Note on your letter that you've never before corresponded with "Puzzle Corner," and the prize will reach you just in time for Christmas.

Now on with "Puzzle Corner" 's seventh year in *Technology Review*.

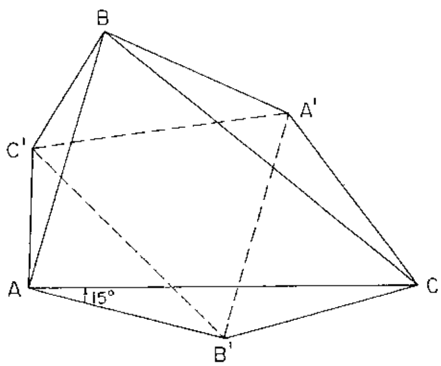
Problems

This month we begin with a chess problem from Douglas Goodman (by the way, my supply of chess problems for future issues is critically low):

O/N1 Suppose your arrogant opponent, instead of just giving you the first move, let's you set up your pieces in any positions you want, as long as you keep them on your half of the board. He then reserves to himself the privilege of the first move (his pieces begin in the normal position). What is the best arrangement in order for you to force mate as quickly as possible?

A geometry problem from Frank Rubin:

O/N2 On the sides of a triangle ABC are erected three isosceles triangles with base angles of 15° and vertices A', B', and C' external to ABC. Prove that triangle A'B'C' is equilateral.



Here is a geography problem from Karel Jan Bossart which I particularly enjoy:

O/N3 We cover the globe with a set of geodetic points in such a way that the distances from any point to three of its closest neighbors are the same. If we further stipulate that one of the points lies in Cambridge, Mass., and that another one lies due north of the first one, (1) what is the location of the second point? and (2) how many points fall in the U.S.? Assume the earth to be a perfect sphere.

A. Porter submitted the following combinatorial problem; he writes that he is "almost certain" that the answer to his final question is negative, but he is "stumped trying to formulate a general proof":

O/N4 Given a set of N elements arranged in a particular lineal order, rearrange the elements in a new lineal order to satisfy the following two conditions: (1) no element to be in its original position; and (2) no two elements which were originally consecutive to be consecutive (they may still be adjacent as long as their order is reversed). It may be seen that this can easily be done as long as N is even. As an example, consider the case $N = 4$ and the set defined as $\{1,2,3,4\}$ arranged thus:

1 2 3 4

Rearranging the set thus:

4 3 2 1

satisfies both conditions. Can this be accomplished if N is odd? If not, prove.

This problem, from Harry Zaremba, perhaps should have appeared prior to the 1972 winter meeting of the American Mathematical Society; that meeting was in Las Vegas. Mr. Zaremba himself writes that it "perhaps may discourage some current or would-be gamblers from rattling the 'bones' ":

O/N5 In four tosses of a pair of dice, what are the odds against making a seven on the first throw and the point six on the second and fourth tosses without losing one's turn to roll?

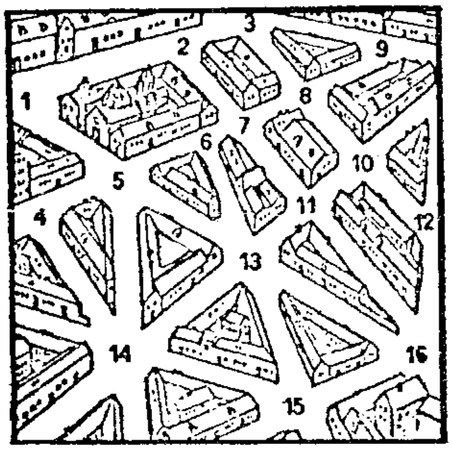
Speed Department

This well known fallacy may interest some new puzzle enthusiasts. It was submitted by John Bobbitt:

O/NSD1 Given two concentric circles of radius a and b, $a > b$. The larger circle is rolled along a straight line one complete revolution, without slipping; thus it travels $2\pi a$. Meanwhile, the smaller circle has rolled also $2\pi a$ without slipping. Thus its circumference is also $2\pi a$. What's wrong?

Russell A. Nahigan sends this problem which originally appeared in *Boys' Life*:

O/NSD2 Can you tell at what intersections (see chart) four policemen must be posted so that they can observe every street?



A Correction

For those who may still be working on problems published in June, please note: On problem **JN3**, add the requirement $-\frac{1}{2} < x < 1$.

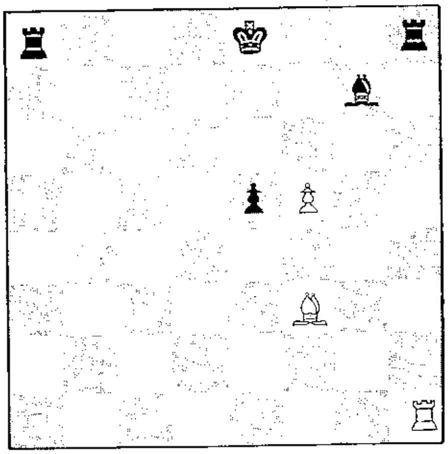
Solutions

Here are solutions for problems published in *Technology Review* for May, 1972.

M1 In the laws governing the game of chess, provision is made for the claim of a draw under a variety of circumstances. In particular, Law 12 states (in part), "The game is drawn . . . 3. Upon demand by one of the players when the same position appears three times, provided that the same player has the move after each of the three appearances of the same position on the chess board. The position is considered the same if men of the same kind and color occupy the same squares, and the possible moves of all the men are unchanged. . . ."

I have found a position reachable by legal play which can occur 21 times without either player being able to demand a draw; can any of your *Technology Review* readers do better?

Only Harry Nelson, the proposer, tackled this one. Here is his amazing solution:



The diagram shows the position after the 18th moves, and in this list of later moves we italicize each which results in a repeated position:

	White	*Black*		*White*	*Black*
18.	—	P—K4	40.	B—N4	B—R3
19.	B—K2	B—R3	41.	B—K2	B—N2
20.	B—B3	*B—N2**	42.	*B—B3*	B—R3
21.	B—K2	B—R3	43.	B—K2	B—N2
22.	B—B3	*B—N2*	44.	*B—B3*	QR—N1
23.	B—N4	B—R3	45.	B—K2	R—R1
24.	B—K2	B—N2	46.	*B—B3*	B—R3
25.	*B—B3*	B—R3	47.	B—K2	B—N2
26.	B—K2	B—N2	48.	*B—B3*	B—B1
27.	*B—B3*	KR—R2	49.	B—K2	B—R3
28.	B—K2	R—R1	50.	B—B3	*B—N2*
29.	*B—B3*	B—R3	51.	B—K2	B—R3
30.	B—K2	B—N2	52.	B—B3	*B—N2*
31.	*B—B3*	B—B1	53.	QR—N1	B—R3
32.	B—K2	B—R3	54.	R—R1	*B—N2*
33.	B—B3	*B—N2*	55.	B—K2	B—R3
34.	B—K2	B—R3	56.	B—B3	*B—N2*
35.	B—B3	*B—N2*	57.	B—N4	B—R3
36.	KR—R2	B—R3	58.	B—K2	B—N2
37.	R—R1	*B—N2*	59.	*B—B3*	B—R3
38.	B—K2	B—R3	60.	B—K2	B—N2
39.	B—B3	*B—N2*	61.	*B—B3*	

* E.P. capture is not available.

B—B3 at move 61. is the 21st appearance of this position. Note also that less than 50 moves have been made since its first appearance.

M2 N soldiers are lined up to be shot. The shooting proceeds as follows: The shooting begins at the left. The first man is skipped, the second man is shot, the third man is skipped, the fourth man is shot, etc., every other man being shot. If a man is not shot, he moves to the end of the line at the right. The last soldier remaining after the shooting is allowed to live. For N = 5, B is shot first, then D, A, and E, and C survives. In general for N soldiers, where should one stand in line if he wishes to live?

Professor Edwin A. Rosenberg notes that variants of this problem, and of **M3** as well, appeared in the same issue of the *Connecticut Mathematics Journal*—amazing. Paul G. N. de Vegvar found a closed-form solution, whereas Robert L. Bishop gave the most popular form of the answer; both solutions are published. First, Mr. de Vegvar's:

Construct the following table, where N is the number of soldiers to be shot, Y the position from the left where one should stand in order to survive, X the N value of the last occurrence of the value 1 in the Y sequence, and P the number of 1's in the Y sequence:

N	*Y*	*X*	*P*	*N*	*Y*	*X*	*P*
2	1	2	1	10	5	8	3
3	3	2	1	11	7	8	3
4	1	4	2	12	9	8	3
5	3	4	2	13	11	8	3
6	5	4	2	14	13	8	3
7	7	4	2	15	15	8	3
8	1	8	3	16	1	16	4
9	3	8	3	17	3	16	4

For example, let N = 10. We find that Y = 5. The last time a 1 occurred in the Y sequence was N = 8; hence X = 8. The number of 1's that have occurred in the Y sequence by N = 10 is 3; hence P = 3. Note that

$Y = 1 + 2(N - X)$, and (1)

$X = 2^P$. (2)

Also observe that

$P = [\log_2 N]$, (3)

where the brackets indicate the greatest integer less than or equal to $\log_2 N$. By substituting (3) into (2), we get

$X = 2^{[\log_2 N]}$ (4)

Substituting (4) into (1), we obtain

$Y = 1 + 2(N - 2^{[\log_2 N]})$, or

$Y = 1 + 2N - 2^{[\log_2 N] - 1}$

This problem becomes more difficult if the first man is *not* skipped.

In writing his solution, Professor Bishop notes that "although I have despaired of finding a proof, the answer seems clear enough":

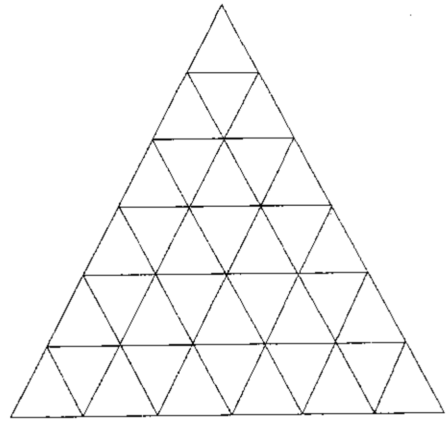
If the soldiers are labelled 1,2,...,n, write

$n = 2^m + p$

where m is the largest integer consistent with a nonnegative value of p. The last surviving soldier will then be in the position $2p + 1$. The result is more spectacular if expressed in binary form. Given the binary version of n, transfer the leftmost digit to the right end; this identifies the position of the last survivor.

Also solved by 19 other readers: Bob Baird, Gerald Blum, Paul Fates, Bill Gilbert, Tom Glennon, Winslow H. Hartford, Dick Jenney, Judith Q. Longyear, Ed Nordstrom, David Tomm, R. Robinson Rowe, Don Savage, Henry Seltzer, Lars Sjodahl, Harry Zaremba, Jeff Shadrick and Thomas Woody, and Benjamin Whang and Shou-ling Wang.

M3 The main lobby floor of a new bank building was finished with a mosaic surfacing in which the central feature is a large, contrasting colored triangle patterned from small, equilateral triangles as shown in the illustration above. The floor contractor claimed that it required 10,000 individual units to form the large triangle, and he stated that he could retire comfortably if he had as many dollars as there were triangles of all sizes within



the triangular pattern. How many mosaic units M were along each side of the large triangle, and how many triangles of all sizes, including the large one, are to be found within its bounds?

Many people submitted incorrect answers to this one. L. R. Steffens, however, got it right (one does not wonder, however, that he writes that he moved from the table (below) to the formula "by virtue of some 1922-vintage algebra, a lot of muddling—with repeated comments by my daughter, "You're sick!"—and a hunch that the binomial theorem was involved"):

If the number of small triangles with their bases on a side of the multi-triangle is M, the total number of small triangles is M^2. In the problem given, $M^2 = 10{,}000$, $M = 100$. The table below pertains; in the last term at the right, the number of triangles with apex down, if M is odd, $N = (M - 1)/2$; if M is even, $N = M/2$. Whence, total triangles:

If M is odd: $(2M^3 + 5M^2 + 2M - 1)/8$, and

If M is even: $(2M^3 + 5M^2 + 2M)/8$.

For example, if M = 1, total triangles are $(2 + 5 + 2 - 1)/8 = 1$; if M = 100, total triangles are $(2 \cdot 100^3 + 5 \cdot 100^2 + 2 \cdot 100)/8 = 256{,}275$.

Also solved by Bob Baird, Gerald Blum, Marty McGowan, R. Robinson Rowe, and the proposer, Harry Zaremba.

M4 Prove that if 51 integers are chosen from the 100 integers 1, 2, 3, . . . 100, then among the 51 are two a and b such that a evenly divides b.

Bob Wolf, a colleague of mine at M.I.T. whom I met at Stanford last year, sent in the following elegant solution:

We prove more generally that if n is a positive integer and n + 1 numbers are

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M	*Total triangles apex up*	*Increment M = i + 1 vs. M = i*	*Total triangles apex down*	*Increment M = i + 1 vs. M = i*	*Total triangles*
		1		0·1	
1	1		0		1
		1 + 2		1·1	
2	4		1		5
		1 + 2 + 3		1·2	
3	10		3		13
		1 + 2 + 3 + 4		2·2	
4	20		7		27
		1 + 2 + . . . + M			
M	$\sum_{i=1}^{i=M} i\,(M + 1 - i)$		$\sum_{i=1}^{i=N} (2M + 1)\,i - 4i^2$		

again what an earlier Harvard-Yale-Columbia medical library automation project and a number of others have shown—that collaboration in the development of library automation systems doesn't work:

"It was decided that the most valuable contribution that these three institutions could make would be to develop individual systems. . . . Cooperation when it costs nothing is easy, but when it is expensive, as this is, even among men with the best will in the world, cooperation requires of all of its participants the wisdom of Solomon, the patience of Job, and the prophetic powers of blind Tiresias thrown into the bargain."

From one point of view, the collaboration worked very well. By nice grantsmanship each of the three universities received roughly $100 thousand a year for five years and has been able to develop its own system, which would certainly not have been the case if each had come in with a separate proposal.

A Sober Summary

The quality of writing in both these books is high; so is the quality of the thinking. Both books (particularly F. & V.) show a healthy scepticism: "Unless an automation budget can be completely integrated or absorbed into the regular library budget the effort is potentially in serious danger." Again, "It is possible that individual libraries are getting into an awkward position of assuming new responsibilities while at the same time they are reducing their ability to guarantee the quality of their own performance."

Both show much concern with costs in dollars, costs to the consumer, and costs to the library staff. For example, in F. & V.: "If you don't involve the users in the design, you simply cannot have a successful system. . . . For most users of the system I propose that at least the marginal cost associated with his use of the system be charged directly to the user. . . . The investment required to convert basic library operations to a heavily on-line machine system is in the range of $1 or $2 million."

And the real kicker: "We have also learned that to date there have been no demonstrable savings in library operations through automation; that the hardware and the software don't live up to the manufacturer's claims or to our own expectations. We have learned that on-line systems represent a whole new ball game. We know that library automation involves more than just a library staff—it immediately gets us into the political arena with the people that control the purse strings of the university and with those who are in charge of that all important resource, the computer."

These books are outdated by the exponential growth of computer applications to libraries. Indeed, computers for libraries are at last approaching maturity. In a few universities they are doing financial and bibliographical record-keeping and producing machine-readable catalogs, indexes, and lists. In the spring of 1972 we see the Ohio College Library Center running an on-line real-time computer system which has been operational for almost a year, serving some 80 institutions with bibliographic information and catalog cards.

For a clear picture of the current state of the art, see the authoritative *Libraries and Information Technology—A National System Challenge*, a report of the National Academy of Sciences (Washington, D.C., 1972, xi + 84 pp., $3.25).

Puzzle Corner

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chosen from $\{1, 2, \ldots, 2n\}$ then there are numbers a, b among the $n + 1 \ni a/b$. (Throughout, when we write a/b we tacitly assume $a \neq b$.) Proof by induction on n. That $n = 1$ is obvious because 1/2. The induction step: say true for $n = k$, we want to show it also true for $k + 1$. So let X be a set of $(k + 1) + 1 = k + 2$ integers from among $\{1, 2, \ldots, 2k + 2\}$. Note that if X contains $\geq K + 1$ elements which are $\leq 2k$, then, by the induction hypothesis, we are done. If not, X must contain exactly k elements which are $\leq 2k$ and both of the integers $2k + 1$ and $2k + 2$. If $k + 1 \in X$, we're done, since $2k + 2 | k + 1$. If $k + 1 \notin X$, proceed as follows: Let $Y = \{m \in X | m \leq 2k\} \cup \{k + 1\}$. Then Y is a set of $k + 1$ distinct integers from $\{1, 2, \ldots, 2k\}$. Therefore, by the induction hypothesis, we can find $a,b \in Y \ni a/b$. Choose such an a and b. If neither $a,b = k + 1$, then $a,b \in X$, so we're done. If either a or $b = k + 1$, note the smaller of a and b cannot be $k + 1$, because any number $c(k + 1)$, where $c > 1$, would be $> 2k$, so cannot be in Y. If the larger number is $k + 1$, then the smaller is a factor of $k + 1$, so it is also a factor of $2k + 2$. Therefore, $2k + 2 \mid$ the smaller of a,b. And we've seen that a,b and $2k + 2$ are all in X. In all cases, we have shown $\exists\, a,b \in X \ni a/b$. Q.E.D.

Also solved by Bob Baird, Tom Glennon, Winslow H. Hartford, Richard Jenney, Judith Q. Longyear, and R. Robinson Rowe.

M5 Find the rational numbers A, B, and C such that

(1) $A^2 - B^2 = 6$, $B^2 - C^2 = 6$, other than the trivial solution $A = 7/2$, $B = 5/2$, and $C = 1/2$.

(2) $A^2 - B^2 = 29$, $B^2 - C^2 = 29$.

R. Robinson Rowe submitted the following general solution and a detailed solution for which there is simply not space; readers who are interested may receive a copy of the latter from the Editors of the *Review* at Room E19-430, M.I.T., Cambridge, Mass., 02139. Mr. Rowe's result: The general problem is to solve $A^2 - B^2 = B^2 - C^2 = K$ with A, B, C rational and K integral. The two particular problems are for $K = 6$ and for $K = 29$.

For $K = 6$: $A = 1249/140$; $B = 1201/140$; $C = 1151/140$.

For $K = 29$: $A = 48039601/180180$; $B = 48029801/180180$; $C = 48019999/180180$.

The proposer, Winslow H. Hartford, also solved this problem in its entirety, and partial solutions came from Gerald Blum and Rich Schroeppel.

Better Late Than Never

Solutions to several problems published in Volume 74 of *Technology Review* have been received as follows:

61 Edwin Eigel, Jr.
62 Fereidoun Farassat
67, 68 John Prussing

Letters

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article in Technology Review *was written we have developed the sensors and logic to a considerable degree. What we are now trying to set up is a multideck vibrating screen with vacuum-plus-blowing over and under it to remove paper and plastic sheets, and a band to remove single items not passing through the screens.*