(Continued from page 9)
season, the creatures would spring to life, multiply, and return to dormancy. There is an analogy on Earth. On mountains in Antarctica, only 400 miles from the South Pole, tiny mites have been found which get all the business of life done in a month or two in tiny pools of melt-water. Is it possible that the pair of Viking craft, designed to land on Mars around July 4, 1976, could spot evidence of such creatures?

And Now to Study the Sun

When Mariner went into orbit around Mars last November, Earth and Mars were only 75 million miles apart, and so radio signals took only about seven minutes to reach the 210-foot antenna at Goldstone, Calif., from the one-meter antenna perched on top of Mariner 9.

But Earth was pulling ahead of Mars on the inside track. With every day, the distance increased, and so did the angle between the axis of the Mariner 9 antenna and the line of sight to Earth. On April 2, Mariner 9's orbit began to fall into the planet's shadow for a few more seconds on each pass. Each time the craft passed into shadow, it suffered the "thermal shock" of passing into extreme cold, and used some of electricity stored in its batteries by the power-generating solar cells. Transmission was stopped to save energy.

Once the period of shadowing is past, however, by early summer, Mariner 9 is to deliver one or two orbit's worth of scientific data to Earth each week. Meanwhile, the Earth's more rapid motion around the sun will carry it around to a point on the opposite side of the sun from Mars September 7. This point is called "superior conjunction." The two planets will be 240 million miles apart.

For a week before and after superior conjunction, radio signals from Mariner 9 will be passing within a few degrees from the sun, and through its corona. The behavior of the signals will probe the violently moving clouds of particles in the corona, and test further Einstein's predictions of the effect of gravity on light.

The Many-Royalty Problem

Puzzle Corner:
Allan J. Gottlieb

Many people sent very thoughtful letters to Alice and me wishing us well and including an occasional tip for a successful marriage. So far everything is working out fine; and we are, of course, very greatful (sic., but a feminine hand corrected it to *grateful*—Ed.) to everyone who corresponded. (Perhaps sparked by our success, Doug Friedman, an old roommate, is marrying Bonnie Koski in June.)

On the home front, Alice and I are planning an Hawaiian vacation for this summer. We are a little excited, and now "surfing" music emanates daily from the old hi-fi.

We were discussing chemistry labs today, and I mentioned that in weighing precipitates I used a balance where one manually adds small brass weights (masses?) to one pan. Alice was a little surprised to hear this, as she thought such balances went out with steam locomotives—another example of the "generation gap" a seven-year age difference can bring.

To answer a frequent question, a Harry Nelson "puzzle invention" is a puzzle (hand held, requiring thought, manipulation, but no calculations) invented by Harry Nelson.

I have received an issue of *Chess Ultimate,* a small magazine for chess positions which maximize (or minimize) various chess phenomena. Anyone interested should contact the Editor, Thur Row, at 12039 Gardengate Drive, St. Louis, Mo., 63141.

Send problems and solutions to me at the Department of Mathematics, University of California, Santa Cruz, Calif., 95060; we pick one solution to publish and note the names of others who submitted solutions. Following the suggestion of Smith D. Turner ($\int dt$), I will say if "also solved by 's' " are different solutions from that published.

Problems

Our first problem has three parts—the first an old chess problem, and the next two mathematical generalizations; it comes from Dr. Benjamin Whang:

JN-1 (a) On a chess board, place eight Queens such that no Queen is vulnerable to other Queens—i.e., no Queen should be in the path of other Queens. (For those who do not play chess: on an 8×8 grid, place eight dots, one on each row, such that no two dots have the same column or same diagonal. For example, once a dot is placed as shown below, the spaces marked "x" are taboo for other dots.)

		×			×		
×		×		×			
	×	×	×				
×	×	●	×	×	×	×	×
	×	×	×				
×		×		×			
		×			×		
		×				×	

(b) Obtain a general pattern (or two) for placing 2n dots in a grid $2n \times 2n$ $(n > 1)$, with the same restriction as above.

(c) Extend the solution for (b) for placing n dots in a grid $n \times n$ $(n > 3)$.

This number-theoretic problem is from Mrs. H. E. Schabacker:

JN-2 What simple rule governs this sequence?

1,1,2,1,2,1,2,3,1,3,2,1,2,3,
3,1,3,2,1,3,2,3,4, . . .

Here's a complicated-looking algebraic formula from Greg Schaffer:

JN-3 Show or prove that

$$\left(\frac{1-x}{1+x}\right)(2x+1)\prod_{k=1}^{\infty}\left\{[1+x^{2^k}]\left[1+\left(\frac{x}{1+x}\right)^{2^k}\right]\right\}=1$$

A cute, somewhat well known problem from Roy Sinclair:

JN-4 You are given as many playing cards as you wish to use, each of length a and width b. They are stacked one upon another in the usual way, so that each card is in contact with only the cards immediately above and below it, with the objective of achieving the maximum overhang before they fall over. What is the length of this overhang?

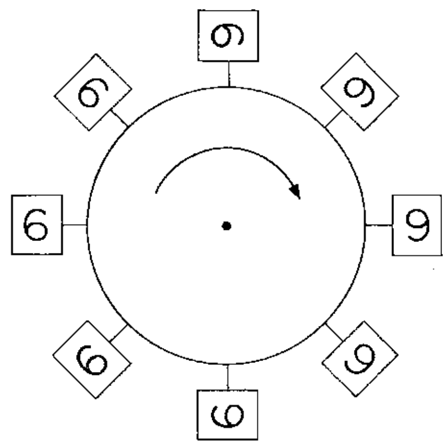
One of those "plug-in-numbers-for-letters" puzzles from R. E. Crandall:

JN-5 Find the cube root of INVENTORY and verify it can be eaten or drunk.

Speed Department

This is hardly a problem, but I cannot resist printing it—from Kenneth Morgan: he writes:

SD-1 "I have had a perpetual motion machine (below) working satisfactorily for years. Somehow I have a feeling that the Morris Markovitz machine and mine have similarities. (Six-pound weights automatically become nine-pound weights on the right side. There is no limit to the power that can be developed by this machine; for instance, try using 6,666-lb. weights)."



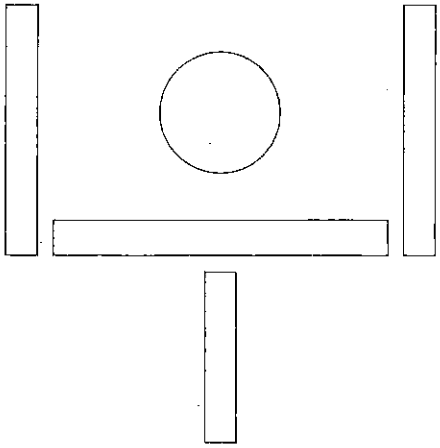
A pencil-pushing problem from Les Servi:

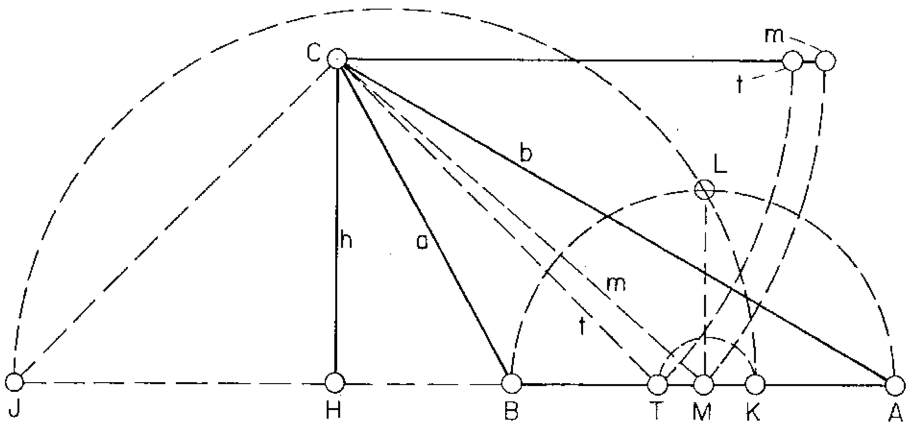
SD-2 With four pencils and one penny in the formation shown, move just two pencils to make the penny no longer in the cup. (See diagram on p. 71 top.)

Solutions

The following are solutions to problems published in *Technology Review* for February:

61 Set up chess pieces as though to start a game. White *must,* in proper sequence, make the following four moves: 1. P—KB3; 2. K—B2; 3. K—N3; 4. K—R4. What are the four legal moves to be made





by Black after which White is checkmated?

The following solution is from Abraham Schwartz:

The problem has one general solution with two key moves:

1. P—KB3; Black moves P—K3 *or* P—K4.
2. K—B2; Black moves Q—B3.
3. K—N3; Black moves Q × P +.
4. K—R4; Black moves B—K2. Mate.

The solution is easy from the chess point of view but psychologically hard since one doesn't look for Q × P +; it is somewhat similar to a helpmate problem.

Also solved by Allen Andersson, Robert B. Anthonyson, Bill Cain, Claude W. House, Richard Jenney, John L. Joseph, Marc Judson, Chip Melvin, Russell A. Nahigian, E. A. Nordstrom, Hunter Platt, and Mike Rolle.

62 Given line segments of length h_c, t_c, and m_c, construct a triangle such that the altitude has length h_c, the angle bisector has length t_c, and the median has length m_c when these three lines emanate from the same angle.

The following is from R. Robinson Rowe: Given h, t, and m concurrent at C, construct triangle ABC. (See diagram at top right.)

With CH = h, draw the perpendicular at H as the base of the triangle. Draw arcs with radii t and m to the base at T and M. Draw the bisector CT and the median CM. Draw CJ perpendicular to CT to the base at J. On the base, lay off MK = MT. On diameter JK, draw arc JLK. Draw ML perpendicular to the base, intersecting semicircle JK at L. With M as the center and ML as the radius, draw semicircle ALB intersecting the base at A and B. Draw AB, BC, and CA; ABC is the required triangle. *Proof*: Let a = BC, b = AC, c = AB, d = BT, e = AT, f = MT, g = HM, h = CH, and i = HT. Then

$a^2 = h^2 + (g - \frac{1}{2}c)^2$,

$b^2 = h^2 + (g + \frac{1}{2}c)^2$,

$bd = ae$, $c = e + d$, and

$2f = e - d = 2(g - i)$.

Eliminating unknowns a, b, d, and e, we get

$(\frac{1}{2}c)^2 = f(g + h^2/i)$.

By construction $JH = h^2/i$,

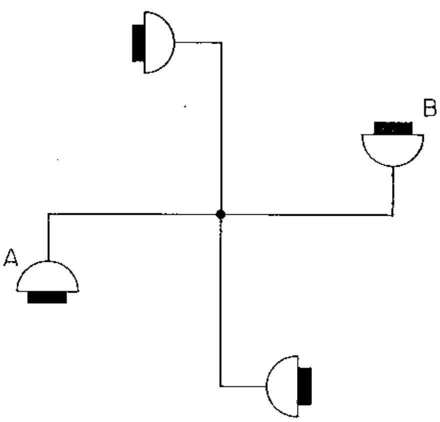
$JM = g + h^2/i$,

$MK = f$, and $\overline{ML}^2 = JM \cdot MK$.

So $ML = \frac{1}{2}c = MA = MB$.

Also solved by Richard Jenney, Raymond Gaillard, John L. Joseph, Mary Lindenberg, P. Markstein, and Mike Rolle; all solutions are, at least slightly, unique.

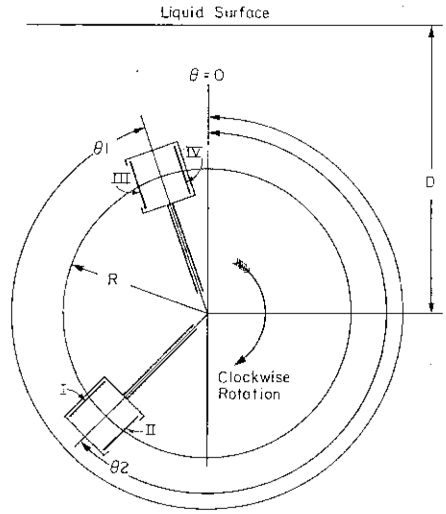
63 The "ferris wheel" (below) is constructed under atmospheric pressure. Metal cups are attached to each arm, and a pliable membrane seals the top of each cup. Glued to the center of each membrane is a weight. The "ferris wheel" is now submerged in water. The weight at cup A stretches the membrane, increasing that cup's volume. The weight at cup B compresses that cup's volume. Thus, cup A is more buoyant and the "wheel" rotates in a clockwise direction forever.



A magnificent solution has come from T. Davidson who writes that he built such a machine some 60 years ago and "can assure you with no success. The obvious reason for failure is that energy to raise liquid from the lower expanding chamber to the upper contracting chamber must come from somewhere." Here is his mathematical "proof" (those quotation marks are his):

Some liberties have been taken with the design:

1. The cups have been replaced by cylinders with "frictionless" pistons, as it is easier to visualize how their expansion and contraction occurs.
2. The surface area of the liquid is assumed to be large enough that the depth D is not changed appreciably by the expansion and contraction of cylinders which do not occur simultaneously.
3. The residual air volume is assumed to be large enough that piston movement is not affected appreciably by changes of air pressure as the cylinder volume expands or contracts.
4. The pistons are assumed to be held at both inner and outer positions by "frictionless" latches, for mechanical release at the angular position at which the balance of piston weight and liquid pressure will just achieve full stroke.



ρ = liquid density, positive downward
w = piston weight, positive downward
s = piston stroke, from closed end;
piston area = 1
E = energy, positive clockwise

The weight of cylinder and support can be neglected.

Rotation clockwise from θ1 to θ2:

$E = w[R(\cos\theta 1 - \cos\theta 2) - s/2(\sin\theta 1 - \theta 2)]$ (1)

Unbalanced piston force from I to II:

At $I = w\sin\theta 2 - \rho(D - R\cos\theta 2 + s/2\sin\theta 2)$

At $II = w\sin\theta 2 - \rho(D - R\cos\theta 2 - s/2\sin\theta 2)$ (A)

Net average $= \rho s/2 \sin\theta 2$

$E = \rho s^2/2 \sin\theta 2.$ (2)

Rotation clockwise from θ2 to θ1:

$E = w[R(\cos\theta 2 - \cos\theta 1) + s/2(\sin\theta 2 - \sin\theta 1) - \rho s R(\cos\theta 2 - \cos\theta 1)]$ (3)

Unbalanced piston force from III to IV:

At $III = w\sin\theta 1 - \rho(D - R\cos\theta 1 - s/2\sin\theta 1)$

At $IV = w\sin\theta 1 - \rho(D - R\cos\theta 1 + s/2\sin\theta 1)$ (B)

Net average $= \rho s/2 \sin\theta 1$ (4)

$E = \rho s^2/2 \sin\theta 1.$

Total energy in one revolution (eq. 1 + 2 + 3 + 4):

$E = w\,s(\sin\theta 2 - \sin\theta 1) + \rho\,s\,R(\cos\theta 2 - \cos\theta 1) + \rho s^2/2(\sin\theta 2 + \sin\theta 1)$ (5)

If $\theta 1$ and $\theta 2$ are the angles at which liquid pressure just balances piston weight at positions IV and II, expressions A and B equal zero, or:

From A: $w \sin\theta 2$
$= \rho(D - R\cos\theta 2 - s/2 \sin\theta 2)$ (6)

From B: $w \sin\theta 1$
$= \rho(D - R\cos\theta 1 + s/2 \sin\theta 1)$ (7)

Combining (6) and (7) and multiplying by (5):

$w\,s(\sin\theta 2 - \sin\theta 1) =$
$-\rho\, s\, R(\cos\theta 2 - \cos\theta 1)$
$- \rho\, s^2/2\, (\sin\theta 2 + \sin\theta 1)$ (8)

When (8) is substituted in (5), total energy E is zero.

Also solved by Jacob A. Bernstein, M. Markovitz, and Harry Zaremba; but no other solution was as detailed as Mr. Davidson's.

64 Consider the infinitely nested square root

$$\sqrt{a_1 + \sqrt{a_2 + \sqrt{a_3 + \ldots}}}$$

Prove that the nest converges when $a_n = n$. Does it converge when $a_n = n^2$? $n!$? How about when $a_n = (n^2!)^{n2}$?

Here is a solution from Mike Rolle, who thanks me for recommending that he buy a Volvo 122S. He did, and after almost 100,000 miles it's still running well (with original brake pads). My 122S, which tempted Rolle to buy his, was wrecked after 45,000 miles (brake pads changed). The general solution to the problem for non-negative a_n is as follows: The square root converges if and only if for some x,

$a_n \leq x^{(2^n)}$ for $n = 1, 2, \ldots$

The sufficiency of this condition is seen by observing that the series

$$\sqrt{x^2 + \sqrt{x^4 + \sqrt{x^8 + \sqrt{x^{16} + \ldots}}}}$$

converges to the value $x(1 + \sqrt{5})/2$. The necessity of the condition is seen by assuming the series converges to some value y. Observe that

$$\sqrt{a_n + \ldots} \leq \left[\sqrt{a_{n-1} \pm \sqrt{a_n - \ldots}}\right]^2$$

Therefore it folows that

$$a_n < \left[\sqrt{a_n - \sqrt{\ldots}}\right]^2 \leq y^{(2^n)}$$

by induction.

Since $k! \leq k^k$ for any k, we estimate that

$(n^2!)^{n^2} \leq n^{2n^4}$

The 2^n-th root of this is $n^{(n^4/2^{n-1})}$.

It should be clear that this expression goes to 1 as $n \to \infty$, and therefore has a maximum value x. Therefore,

$(n^2!)^{n^2} \leq x^{(2^n)}$

and the square root converges in this case. Certainly this is the worst case of these mentioned in the problem, and so all the other sequences for a_n also converge.

Also solved by R. Robinson Rowe and Stephen Scheinberg, whose solutions look different.

65 In how many different ways can n numbers be rearranged such that no number occupies its original position; and what fraction of the arrangements possible meet the additional criterion of having every digit change its position? In other words, does the sequence $a_n = k_n/n!$ converge?

The following is from Harry Zaremba:

The first half of this problem (originally problem 29 of April, 1971) was treated as the following equivalent probability problem: A set of n numbers are in a certain arrangement from left to right. If arbitrary selections are made of all n numbers in a second identical set and also arranged in sequence from left to right, what is the probability that no number in the second set matches a number in the first arrangement? The probability that *at least one number* would occupy the same position it had in the original arrangement was given by:

$P = 1 - 1/2! + 1/3! - 1/4! + \ldots$
$+ -1^{(n+1)}/n!$ (n terms)

The probability that *no number* occupies the same position it had in the original arrangement is the complement probability of P:

$P' = 1 - P = k_n/n!$
$= 1 - (1 - 1/2! + 1/3!$
$- 1/4! + \ldots + -1^{(n+1)}/n!)$

or,

$k_n/n! = 1/2! - 1/3! + 1/4! - \ldots$
$- -1^{(n+1)}/n!$ (n − 1 terms)

where k_n is the number of different ways n numbers can be arranged where no number matches its original position and n! is the total number of different ways (permutations) n numbers can be arranged. When $n \to \infty$, the ratio $k_n/n!$ converges to e^{-1}.

Also solved—in the same way—by Judith Q. Longyear, Mike Rolle, and R. Robinson Rowe.

Better Late Than Never

45 Solutions have come from Norman Brenner and Everett A. Potter.

Hervé Thiriez has supplied solutions to problems **53** and **55**, and Stephen Bryant to problem **54**.