Earthquake Problem

Puzzle Corner
Allan J. Gottlieb

The great event to which I referred in my last column is now accomplished, and I want to express publicly my thanks (and Alice's) to my thesis adviser and his wife for their help with the small reception on January 7; when he took me on as thesis student, I doubt that Michael Shub realized how many fringe benefits he would have to provide.

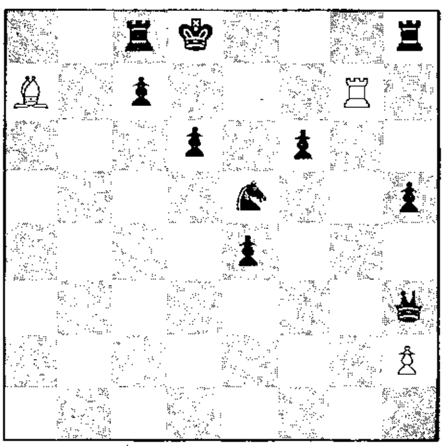
Another note of fairly local interest: John ("Boog") Rudy, a regular player in Puzzle Corner, is anxious that I call our classmates to the Class of 1967 reunion in Cambridge the first weekend in June. He used to complain that my throws from deep short would sting when he caught them; for that much of an ego boost, he deserves this plug.

Send problems and solutions to me at the Department of Mathematics, University of California, Santa Cruz, Calif., 95060.

Problems

We will start this installment with a chess problem so easy I was able to solve it. This one is from Peter J. Meschter:

66 Given the following, White to move and checkmate.



Here's a West Coast problem from R. Robinson Rowe:

67 The earthquake of February, 1971, near Los Angeles caught one Pacoima (near the epicenter) family at breakfast. Father had imbibed his orange juice, Mother had had a few swallows, and Junior had just reached for his. Then the quake quaked, and all three dashed for safety in the open patio. At the first lull, Father peeked in the window at the shambles inside, noticing one apparent inconsistency. On the breakfast table two juice tumblers had tumbled over but the third stood erect. Guess which. This suggests a problem: suppose each tumbler was cylindrical, 2" dia. by 6" high, of uniform thickness, weighing 130 g. empty and 430 g. full. What depth of juice would give it maximum stability in an earthquake and what seismic acceleration would have left it erect while tumbling two others?

Now a Phase II problem from Harry Zaremba:

68 A customer in a supermarket, observing a clerk finish stacking oranges into a pyramid with an equilateral triangular base, asked the clerk if he knew how many oranges were in the stack. Admitting he didn't, the clerk remarked that an eccentric old lady once told him if the number of oranges along an edge of the pyramid is known, any clerk worth his ability to stack oranges could find the sum. Could this be possible? thought the customer.

The following, from Ermanno Signorelli, is a sequel to the first Speed Problem this month, which should be read first:

69 There exist(s) other ratio(s) of sequential whole numbers (n, $n + 1$, $n + 2$, $n + 3$, $n + 4$; $n > 0$) which satisfy the geometry of SD1. Find the ratios and show that no other satisfying ratio(s) exist(s).

This power problem is from Harry A. Smith:

70 On TV I watched a ship skim across the water borne up by what the commentator said were jets of water and that 25,000 h.p. was being exerted. I am not a scientist, but I know that water is incompressible and therefore jets of water can be projected on the ocean surface and thus propel the ship. Now, why is it not possible to propel jets of water against fins on the outside of a turbine, and so propel the turbine to power electric dynamos and create electricity? The jets, I think, can be synchronized so that each fin is hit in turn by a high-powered jet. To begin the power process of operating the pumps to create the high-powered jets a diesel engine could be used. (I note that a garden hose projects a stream of water under only 10 to 15 lbs. pressure, and that a new fire engine purchased by the city in which I live can project water with great force using a small diesel engine.) Once the system was operating, could the jets be powered by electricity from the generator? This is a kind of perpetual motion idea. The application is to set up many small power plants for individual cities and towns. The question is, Would there be any electric power left over from the generator and turbine to transport?

Speed Department

This is the one referred to in problem 69 above, from Ermanno Signorelli:

SD1 Consider a rectangle with sides a and b, each of arbitrary length and with $a \neq b$. Inscribe five and only five triangles in the rectangle. Each triangle must have two and only two sides wholly in common with two other triangles. Identify the position of the five triangles if the ratio of their areas is 4:5:6:7:8.

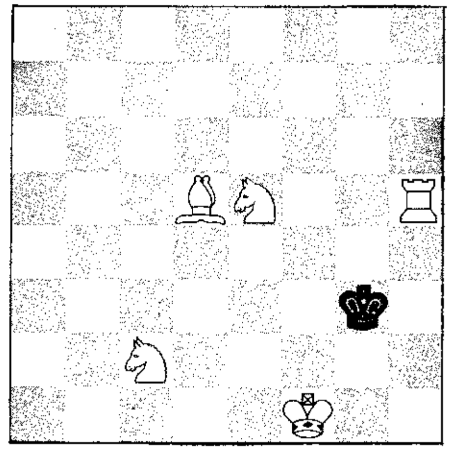
Greg Gagarin submits the following:

SD2 A man in an outboard motorboat is travelling upstream on a river crossed by two bridges exactly one mile apart. As the boat passes the upstream bridge, the man's hat falls overboard. This is unnoticed for 10 minutes—at which time the man, feeling the hot sun upon his head, turns the boat around (assume no time lost). Without changing the power setting, he catches up to his floating hat under the downstream bridge. What is the speed of the current in the river?

Solutions

Here are solutions to the problems published in *Technology Review* for December:

51 Given the following, show how white can mate in three moves.



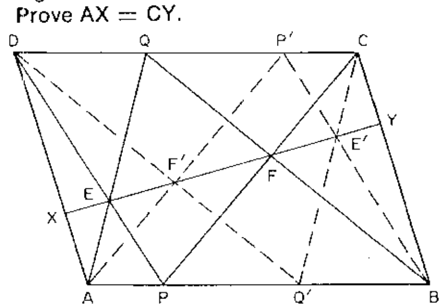
The solution comes from Richard Bennett:

1. B to R1; K to B5
2. K to N2; K to K5
3. K to N3

52 A railway-car is travelling in a straight line at a constant velocity. A helium-filled balloon is tied to the floor of the car with a piece of string. The car is then decelerated at a constant rate. Relative to the car, what is the initial direction of movement of the balloon? What is the steady-state direction during deceleration (i.e., at what angle is the string to the floor during deceleration)?

Everyone says the balloon moves forward, but I disagree. I feel that the air in the car will go forward creating a high-pressure area forcing the balloon backwards. Opposing views have been expressed by Alan LaVergne, Jeffrey Miller, R. Robinson Rowe, Robert Shooshan, and Harry Zaremba.

53 Given the parallelogram ABCD with points P and Q located anywhere on AB and CD. Segments BQ, AQ, DP, and CP are drawn; the intersection of AQ and DP is designated E, the intersection of BQ and CP is F. Line EF is drawn intersecting AD at X and intersecting DB at Y. Prove AX = CY.

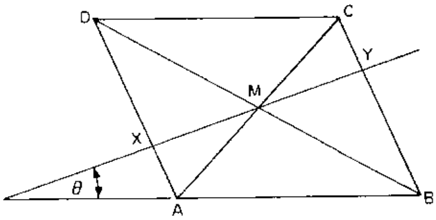


George Clahane submitted both geometric and analytic solutions. For his geometric solution, he adds the dashed lines in the drawing above:

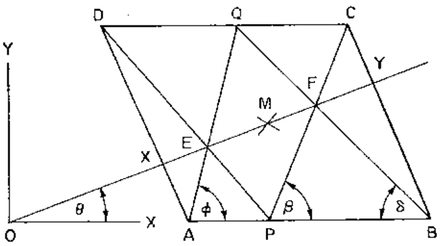
BP′ parallel to DP
AP′ parallel to PC

DQ′ parallel to BQ
CQ′ parallel to AQ.
Triangle CBP′ = triangle ADP;
therefore CP′ = AP.
Triangle CP′E′ = triangle APE;
therefore CE′ = PE.
Triangle CYE′ = triangle AXE;
therefore AX = CY. Q.E.D.

Mr. Clahane's analytical solution begins with this drawing.



If M is the intersection of the diagonals of the parallelogram, triangles AXM and CYM are equal. Therefore AX = CY. Now lay out a parallelogram of randomly selected dimensions, direction of XY and OX—OY axes.



Drawn to scale, it can be shown that:
For line OXY, $\tan\theta = 3.5/9$
For line AQ, $\tan\phi = 7/2$
Line OXY: $y = 3.5/9 \cdot x$ (1)
Line AQ: $y = (x - 6) \cdot 7/2$ (2)
Equate (1) and (2) and find
$x_E = 6.75$,
$y_E = 2.625$
$x_P = 6.75 + 2.625/4.375 \cdot 3.75 = 9.00$
For line QB, $\tan\delta = 7/7 = 1$,
$y = (15 - x)$ (3)
For line PC, $\tan\beta = 7/3$,
$y = (x - 9) \cdot 7/3$ (4)
Equate (1) and (3) and find $x_F = 10.8$, as determined by the intersection of QB and OXY.
Equate (1) and (4) and find $x_F = 10.8$ as determined by the intersection of PC and OXT.
This seems to prove that points E and F lie on a line through the intersection of the diagonals of the parallelogram. Therefore AX = CY.

54 If the sum of all the factors of a number equals that same number, that number is called perfect. For example, the first two non-trivial perfect numbers are
$6 = 1 + 2 + 3$
$28 = 1 + 2 + 4 + 7 + 14$
Find a general formula which computes perfect numbers.

Lawrence H. Smith responds, reporting that one such formula is
$(2^N - 1)\,(2^N - 1)$, which is a perfect number if $(2^N - 1)$ is prime. He lists two groups of factors:

$$\left.\begin{array}{l} 1 \\ 2 \\ 2^2 \\ \cdot \\ \cdot \\ \cdot \\ 2^{N-1} \end{array}\right\} \text{Group A}$$

$$\left.\begin{array}{l} (2^N - 1)\,1 \\ (2^N - 1)\,2 \\ \cdot \\ \cdot \\ \cdot \\ (2^N - 1)\,(2^{N-2}) \end{array}\right\} \text{Group B}$$

Group A sums to $(2^N - 1)$;
Group B sums to $(2^N - 1)\,(2^{N-1} - 1)$;
and the total is
$2^N - 1 + (2^N - 1)\,(2^{N-1}) - (2^N - 1)$
$= (2^N - 1)\,(2^{N-1})$.
Q.E.D.
The next two smallest perfect numbers are 496 = (16) (31) and 8128 = (64) (127), according to Mr. Smith. "I do not know," he writes, "if my formula generates every perfect number, but there are no others less than 8192. By the way, I believe that $(2N - 1)$ is prime if N is prime, but this has never been proved nor disproved."

Also solved by Roger Milkman.

55 The author and the Editors have conspired to confuse themselves—and in the process readers' responses to problem 55 in the December issue of the *Review* have been destroyed. The Editors have apologized to me, and I in turn do so to the readers.

Hope and Despair

Book Review:
Vincent A. Fulmer
Vice President and Secretary, M.I.T.

The New Depression in Higher Education
by Earl F. Cheit
McGraw Hill Book Co.,
New York, 1971, 170 & xxii pp.

On the theory that our society is conditioned to act most easily on the quantities we can measure, there is a clear need for a simple yardstick to prompt greater national as well as institutional concern for the cultural tragedy of our time—the financial dismantling of American higher education.

Earl F. Cheit's book, *The New Depression in Higher Education*, reports to the Carnegie Commission of Higher Education and the Ford Foundation on financial conditions at 41 public and private colleges and universities in the United States. We are all indebted to him for the suggestion that it is possible to characterize the financial dilemmas of the colleges and universities more systematically than we have in the past. Mr. Cheit skillfully cuts through the complexities of educational finance to tell us whether alma mater is on the verge of heading for financial difficulty, in financial difficulty and headed for trouble, or really in the depths of trouble itself.

He found a shocking 71 per cent "headed for trouble" financially—or already there—by the spring of 1970. Since then college finances have even worsened.

The College Depression Scale

1—Reducing further additions to reserves
2—Cancelling further additions to reserves
3—Borrowing from reserves
4—Spending reserves with no intention of replacing them
5—Borrowing from unrestricted gift income for budget balancing instead of capital additions
6—Spending unrestricted gift income for budget balancing with no intention of replacing it to capital
7—Borrowing from income accumulations in designated endowments for use in general purposes
8—Spending income accumulations in designated endowments for general purposes
9—Borrowing from the realized gains of the portfolio
10—Spending the realized gains of the portfolio with no intention of replacing them
11—Borrowing from *unrealized* gains of the portfolio
12—Spending *unrealized* gains
13—Borrowing internally from the general endowment of the college
14—Spending the general endowment with no intention of replacing it
15—Borrowing from designated endowment, with legal sanction
16—Spending the designated endowment, with legal sanction
17—Borrowing from banks to meet a chronic budget deficit, or to repay past debts
18—Merging as an alternative to bankruptcy
19—Closing the college partly, then completely
20—Selling the assets to satisfy creditors

Vincent A. Fulmer, who reviews Earl F. Cheit's study of the financial plight of colleges and universities, proposes this Depression Scale to interpret the financial status of an institution in terms of its financial practices. The Scale is intended to portray more visibly than most expositions of financial problems how a college can sink inexorably through the early stages of a 20-step process which ends in bankruptcy.

We have seen—and are due to see more—sporadic instances of college and university closings and bankruptcies. Less obvious but very real is a growing number of cancelled programs, schools, and departments—dramatic events for higher education. Mr. Cheit would classify institutions forced to take these steps as "in trouble." But what really is "trouble"?

It is a curious paradox of our time that most profit-making enterprise is regarded as unfit for survival if two or three consecutive years of losses are experienced—whereas a college might have a deficit for several years running and still be regarded as a shining jewel. That is because colleges are not supposed to make profit in our society. Yet the bald fact is that no college, public or private, can long sustain a deficit. Indeed, if a college does not realize