Glued to the center of each membrane is a weight. The "cercus wheel" is now submerged in water. The weight at cup A stretches the membrane, increasing the cup's volume. The weight at cup B compresses the cup's volume. Thus, cup A is more buoyant and the "wheel" rotates in a clockwise direction forever.

A number-theory teaser from Frank Rubin:

64 Consider the infinitely nested square root

\[ \sqrt{a_0 + \sqrt{a_1 + \sqrt{a_2 + \cdots}}} \]

Prove that the nest converges when \( a_0 = n \). Does it converge when \( a_0 = n^2 - n \)? How about when \( a_0 = (n^2) - n \)?

Please recall problem 29 from last year (April, 1971): "In how many different ways can eight numbers be rearranged such that no number occupies its original position? Write out all the possibilities. Find the answer for n numbers in general." John Bobbitt asks the following (apparently hard) question about it:

65 What fraction of the arrangements possible meet the criteria of having every digit change its position? In other words, does the sequence \( a_k = k/n \) converge?

Speed Department

William Glassmire sends in the following, which he says has "some historical significance":

1. It is well known that an 8 × 8 checkerboard with diagonally opposite corners squares removed cannot be covered with 31 dominoes (each of which covers two squares of the board). Suppose that two squares are removed arbitrarily, subject only to the restriction that one is black and the other is white. Is it always possible to then cover the board with 31 dominoes?

A. Porter wants you to:
2. Find the relationship between the sides of a rectangle which guarantees that doubling the short side gives a new rectangle having the same relationship.

Here is a new perpetual motion machine to work on—from Morris Markovitz:

Proof that the nest converges when \( a_0 = n \). Does it converge when \( a_0 = n^2 - n \)? How about when \( a_0 = (n^2) - n \)?

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Here is a new perpetual motion machine to work on—from Morris Markovitz:
For $u = 1$ this becomes $x = 1147$.
So $x = 1147$ (mod 39·56) satisfies the condition; i.e., $1147 \equiv 16$ (mod 39).
Also solved by R. Robinson Rowe.

44 Find the set of angles $x$ and $y$ for which $\sin(x + y) = \sin x + \sin y$; and prove that your set is exhaustive.

I like John E. Prussing's solution; do you? He writes: Use the identity $\sin x + \sin y = 2 \sin \frac{1}{2}(x + y) \cos \frac{1}{2}(x - y)$, and define $w = \frac{1}{2}(x + y)$ and $z = \frac{1}{2}(x - y)$.
Then the equation to be solved is simply $2 \sin w \cos z = \sin 2w$.
Subtracting from the familiar identity, $2 \sin w \cos w = \sin 2w$, one obtains the equation $\sin w(\cos z - \cos w) = 0$.
The solutions to this are

(i) $\sin w = 0$ and (ii) $\cos z = \cos w$.
The general solution to (i) is $w = k\pi$, implying $x + y = 2k\pi$.
The general solution to (ii) is obtained by using the identity for the cosine of a sum to yield

$\cos \frac{1}{2}x \cos \frac{1}{2}y + \sin \frac{1}{2}x \sin \frac{1}{2}y = \cos \frac{1}{2}z \cos \frac{1}{2}y - \sin \frac{1}{2}z \sin \frac{1}{2}y$,
which yields

$2 \sin \frac{1}{2}x \sin \frac{1}{2}y = 0$.
The general solutions to this are

$x = 2k\pi$, $y$ arbitrary, and

$y = 2k\pi$, $x$ arbitrary.
Also solved by Harold Donnelly, R. Robinson Rowe, and Victor W. Sauer.

45 You are given a stack of 12 coins, which appear identical to one another, and are told that one is counterfeit and can be distinguished only by its weight, which is not the same as the genuine coins. Unfortunately, you do not know whether the counterfeit coin weighs more or less than the genuine ones. Using only a balance, how do you find the counterfeit in a minimum number of balancing operations?

Apparently there was some confusion about this. A balancing operation involves simply one balancing—not comparing one fixed group to each of several other groups. Benjamin Whang sent me the above "pictorial" solution, noting that the problem as stated does not require to determine whether the counterfeit is heavier or lighter; it only requires to find the counterfeit. He notes that the middle section of three in the third balancing is not really necessary, since it can be considered a mirror image of the left section.

This problem was popular; solutions also came from Captain F. O. Chapman, Carl L. Estes, II, Bruce Fauman, Raymond Gaillard, Carl J. Greever, Maurice A. Hoffman, Stanley A. Horowitz, Elmer C. Ingalls, W. J. Hart, Lowell Kolb, Hubert duB Lewis, Mrs. Martin S. Lindenborg, R. Robinson Rowe, Christopher Scholz, John R. Selin, W. H. Stephenson, Jr., Dr. Stephen Washburne, George J. Wynne, and the "team" of Ronald G. McKeown, Raul F. Pupo, William P. Quinn, and Thomas W. Schwagel.

Better Late Than Never

Raymond Gaillard has submitted a solution to problem 39.

Books

Leopold R. Michell
Visiting Lecturer, School of Management, Boston College

During the 1970's and 1980's, long-range planning—particularly that of the larger corporations—will be increasingly concerned with two kinds of change: changes in the business environment due to social and political causes, and the development of the multinational firm. These are the general themes of two books upon which this reviewer has been asked to comment.

The first is a synthesis from a General Electric study concerned with the development of social and political trends, and aims at the integration of these trends into business plans. The second book stems from work done at Salford University (England) and provides a good overview of the history, current state, and possible future of multinational companies, and discusses their political and social impacts. The authors of both books have supplemented their studies with information gained through interviews with educators, businessmen, and government representatives, as well as information from the available literature.

Developing trends affecting the business environment stemming from social changes were interpreted in the General Electric study as the interaction of eight significant forces:

- Increasing affluence
- Economic stabilization
- The rising tide of education
- Changing attitudes toward work and leisure
- The growing interdependence of institutions
- The emergence of the "post-industrial" society
- Increasing pluralism and individualism
- The urban/minority problem

Institutional changes—in government, the labor force, business, unions, and educational institutions—are segregated off into a chapter of their own. Also considered separately are the impacts of changing value systems—changes in attitudes towards work and leisure, emphasis on "quality of life," rejection of authoritarianism and dogmatism, emphasis on pluralism and individualism. Consequences of student revolts are discussed, with an anticipation of probable youth-related changes in values (utilizing Maslow's "hierarchy of needs" as a framework). An interesting profile of significant value-system changes from 1969 to 1980 is presented, in relation to such pairs as war/peace (in the sense of military might versus economic development), nationalism/internationalism, federal government/state and local government, public enterprise/private enterprise, materialism/"quality of life", work/leisure.

All of the above may suggest a picture of the world in which tastes and habits change in an autonomous fashion, and business can only keep informed and take its chances. But the authors conclude by suggesting a more positive role for active, rather than passive business. Business should not regard itself merely as self-contained or self-regulating, but be willing to work in national and community coalitions.

The Business Environment of the Seventies
Earl B. Donckel, William K. Reed, Ian H. Wilson

Invisible Empires
Louis Turner
Harcourt Brace Jovanovich, Inc., 1971, 228 pp., $8.95