Puzzle Corner

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Checkmate—In Three Moves

The natural beauty of the Santa Cruz campus (Mr. Gottlieb is completing his graduate studies at the University of California—Ed.) could hardly be improved upon. One reason the campus is so lovely is the scarcity of people. There are 4,000 students here and 2,000 acres of woods and meadows occasionally intruded upon by some building. While walking back to the dormitory one evening I met a deer; and after rubbing my eyes I tried to guess the chances of this happening back at M.I.T. The only complaint I have is the proliferation of signs describing appropriate action during an earthquake. I think they’re trying to tell me something.

Many people favored chess problems, so they’ve been included. But there are so many chess and bridge columns that I cannot justify devoting 40 per cent of a Puzzle Corner to these forms of entertainment. Hence we’ll alternate chess and bridge, with chess starting out the problems this month.

Remember to send problems, solutions, and comments to me at the Department of Mathematics, University of California, Santa Cruz, Calif., 95060.

Problems

The first chess problem is from Russell A. Hahigian:
51 Given the following, show how white can mate in three moves.

Here’s a geometry problem from a regular reader, Mrs. Martin S. Lindenberg:
53 Given the parallelogram ABCD with points P and Q located anywhere on AB and CD. Segments BQ, AQ, DP, and CP are drawn; the intersection of AQ and DP is designated E, the intersection of BQ and CP is F. Line EF is drawn intersecting AD at X and intersecting CB at Y. Prove AX = CY.

A number theory challenge from Lawrence H. Smith:
54 If the sum of all the factors of a number equals that same number, that number is called perfect. For example, the first two non-trivial perfect numbers are
\[ 6 = 1 + 2 + 3 \]
\[ 28 = 1 + 2 + 4 + 7 + 14 \]
Find a general formula which computes perfect numbers.

Finally, here is a really interesting (and practical) probability problem from Frank Rubin:
55 I own N pairs of socks, all different, which I wash every washday and sort by the following method: all 2N socks are initially in the laundry bag. I withdraw one sock at a time. If its mate is not yet drawn I place it on the bed; if it matches a previously drawn sock I place both in the drawer.
1. What is the probability of the number of socks on the bed becoming 0 between the first and the (2n – 1)th draw?
2. What is the expected maximum number of socks on the bed? (that is, the maximum over the 2N-step process)?
3. What is the probability of never having more than M socks on the bed?
4. What is the expected number of times the drawer after K selections?
5. What is the expected number of times the pile on the bed will become empty?

Speed Problems

R. E. Grannell asks: Which is greater—1 or \(\sin(1 + \cos(\sqrt{2}))\)?

The following bridge problem was sent to me and signed "courtesy of The New York Times": With the hands and bidding shown, what "killing" lead can West make to ensure defeat of the contract against good defense?

West
North
East
South
1 spades
2 hearts
pass
pass
pass
3 clubs
3 hearts

Solutions

36 If you are playing South, with clubs distributed as shown:

North
x, x, x
South
Q, J, 9, x, x, x
(the location of \(\spadesuit\) A, \(\spadesuit\) K, \(\spadesuit\) 10, and \(\spadesuit\) x being unknown), how should the suit be played to minimize the loss. You are in your hand with no way of reaching dummy.

The following analysis is by Michael Kay, making the assumption that the defenders collect all the tricks coming to them: Play a low club first; this wins when the \(\spadesuit\) A or \(\spadesuit\) K is singleton (four times out of 16) and loses when the \(\spadesuit\) 10 is singleton (two times out of 16; the other ten cases are predetermined—the 4 D’s and A-K-10-9 splits always three tricks and 2-9’s only two.


37 A pipe can fill a tank in A hours, and the drain can empty it in B hours. If both are left on at the same time, it takes C hours to fill the tank. Show that there are an infinite number of integers A, B, and C which satisfy this problem; and find them.

This one must have been easy; even Joel Shwimer, my next-door neighbor, solved it: The relation that must be satisfied is \(1/A - 1/B = 1/C\), each side showing the fraction of the tank filled in one hour. Any set of integers A, B, C satisfying this relation will solve the problem. Let me consider a subset of all these solutions—nominally, those solutions where \(B = A + 1\). Now we have \(1/A - 1/(A + 1) = 1/(A(A + 1)) = 1/C\). Therefore, choose any integer A (there are an infinite number of such choices), Let \(B = A + 1\) and \(C = (A + 1)\). Both will be integers, and this will satisfy the problem.


38 It is not difficult to prove that \(x^n - x/n\) is an integer when n is a prime number. (To avoid bickering, let x and n be greater than 1.) Can someone prove, or disprove, that if n is not prime, \(2^n - 2/n\) cannot be an integer?

The following is from George E. Andrews of the M.I.T. Mathematics Department, who says the assertion of the problem is false. His proof:

When \(n = 341 = 11\times31\), we note that \(2^{341} = 1024 \equiv 1 \pmod{31}\) and \(2^{166} = 1023 \equiv 1 \pmod{31}\) or \(2^{170} = 1 \pmod{31}\). Hence \(2^{342} = (2^{170})^{2} \equiv 2 \pmod{341}\). If \(2^n \equiv 2 \pmod{n}\), n is known as a pseudo-prime. Sierpinski has pointed out that all of the Fermat numbers \(2^n\) are pseudo-
primes. This is seen as follows:

\[\Phi_n = (2^{n-1} - 1) \div (2^{n-2} - 1)\]

The first time the formula is used is for \(n = 3\), and we get:

\[\Phi_3 = 2^{3-1} - 1 = 3\]

Thus, it is known that the \(\Phi_n\) is the composite for \(n = 5\), \(7\), \(8\), \(9\), \(10\), \(11\), \(12\), \(15\), \(16\), \(18\), \(23\), \(36\), \(39\), \(55\), \(63\), \(73\).

Also solved by Neil Cohen, Harold Donnelly, John W. Langhaar, Ted Leashy, R. Robinson Rowe, Frank Rubin, and Harry Zaremba.

38 Take an arithmetic progression of \(a\), \(d\), and form it into an \((m) \times (n)\) matrix by making the first \(n\) terms the first row, the next \(n\) terms the second row, and so on. What is the rank of this matrix?

Harry Zaremba submitted the following:

```
\[
\begin{array}{cccc}
  a & (a + d) & (a + 2d) & \cdots \\
  (a + nd) & (a + (n + 1)d) & (a + (2n + 1)d) & \cdots \\
  \vdots & \vdots & \vdots & \ddots \\
  (a + (m - 1)d) & (a + (m - 1)n + 1)d & (a + (m - 2)n + (n - 2)d) & \cdots \\
  (a + (m - 2)nd) & (a + (m - 2)n + (n - 2)d) & (a + (3n - 3)d) & \cdots \\
  \vdots & \vdots & \vdots & \ddots \\
  [a + (n - 1)d] & [a + (n - 1)d] & [a + (2n - 2)d] & \cdots \\
  \end{array}
\]
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If the elements of any column \(i \geq k\) are subtracted from column \(k\), the elements of the \(m\)th column will all be zero and will be a multiple of the common difference \(d\). Thus, if column 2 is subtracted from column \((n - 1)\), all elements of column \((n - 1)\) will equal \((n - 3)d\). In any square matrix \(A\) of third order or greater, the elements in two columns \(k\) and \(j\) (where \(k \geq j\)) will be multiples of \(d\) when the elements of column \(i\) are subtracted from them. When the resulting elements of column \(k\) are multiplied by a suitable factor and subtracted from column \(j\), all elements of column \(j\) will be zero, and the value of the determinant of \(A\) of the square matrix will be zero. Now, since the rank of a matrix is the order of its largest nonvanishing minor, the rank of the matrix above is two—because at least one of its minors of second order is nonvanishing. For example, the value of minor,

\[
\begin{vmatrix}
  a & (a + d) \\
  (a + nd) & (a + (n + 1)d)
\end{vmatrix}
\]

is

\[= nd^2\]

Also solved by Gerald Blum, Harold Donnelly, John E. prussing, Carl J. Rosenberg, R. Robinson Rowe, Frank Rubin, and Paul G. N. de Veygar.

40 Nine men were captured by a strange tribe. All were seated in a straight line. The tribe always served a certain potion to its captives, with every seventh cup containing a deadly poison, and they always served from left to right, continuing from the last victim. They continued serving in this way until all but one prisoner died. They never killed the last man. If you knew this, which position—starting from the left—would you choose?

The last solution is from Paul G. N. de Veygar:

We write the nine men as follows:

1 2 3 4 5 6 7 8 9

The first time the potion is served number 7 dies, so we are left with:

1 2 3 4 5 6 8 9

The second time it is served number 5 dies, so we are left with:

1 2 3 4 6 8 9

The third time number 4 dies, leaving:

1 2 3 6 8 9

The fourth time the victim is number 6, leaving:

1 2 3 8 9

The fifth time, number 9 is gone:

1 2 3 8

The sixth time it is number 3:

1 2 3 8

The seventh time number 8 dies, leaving:

1 2 3

The eighth time number 1 dies, and number 2 is the survivor. Hence the answer overtakes the \(\Delta\) with the \(\Delta\) and returns a heart to East's \(\Delta\) and East returns a club, South ruffs high and is again home free. In conclusion, I see no way to go down at four spades as long as the \(\Delta\) is on side giving South two entries to the board.

Other responses to 26 have come from Leonad V. Avaroff, Phillip Bell, Harry L. Bechon, Connie Chase, Peter Friedland, David Gross, Leonard Lewis, Peter Lobban, and Frank Westcott.

The solution to problem 30 seems to have ruffled Frank Rubin's feathers a bit. He has written a six-page argument to establish that Mr. Hiberg's solution to his own problem is "a full of holes and a Swiss cheese." Unfortunately police does not permit publication here, but interested readers may obtain a copy by writing to the Editors of the Review.

Other solutions have arrived:

22 Harold Blum
29 Roy G. Sinclair
31 Bowman Cutter

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