Allan J. Gottlieb

Packages and Digits

Both Brandeis and M.I.T. are "on strike" as this is written, so this place—Brandeis—is alive with anti-war activity, a model of nonviolent demonstration. Classes are available for those students who wish to attend, but finals are optional and any student may opt for a grade based on work up to May 5. Meanwhile, the Waltham area is being canvassed by Brandeis undergraduates, and much lobbying is going on in Washington by older students and faculty members. I only hope that all these sincere efforts for peace bear some well-deserved fruits.

Enough for politics. Send speed problems!

Problems

The first selection is from Frank Rubin:

36 Show that the equations
$a^2 + b^2 = c^{16}$
and
$a^{16} + b^2 = c^2$
each have an infinite number of solutions with a, b, and c nonzero integers.

The following, by William J. Deane, should be read carefully, as his usage of "n's place" is not standard:

37 What is the smallest number (N) of n digits, which, when removing the digit from the units place and relocating it in front of the n's place, exactly doubles the number. For example, try the number 1,052. Relocating the 2 from the units place gives 2,105. This trial does not quite satisfy the conditions of the problem because 1,052 doubled is $2{,}104 \neq 2{,}105$.

A bridge problem from Paul D. Berger:

38 With bidding and lead as shown, how do you play the following hand to maximize the probability of making the contract?

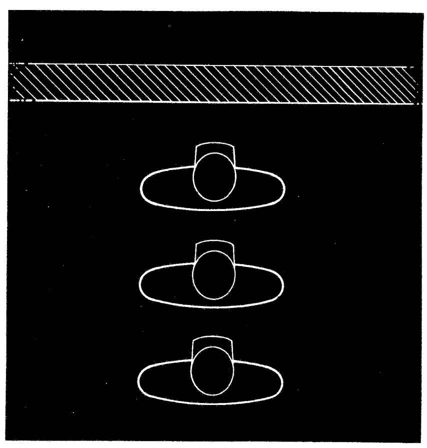
North:	South:
♠ A Q J 8 5	♠ K 10 9 6 3
♥ A K 7	♥ 5 4 3
♦ A 7 5	♦ 9 6 3
♣ K 3	♣ A 2

The bidding: West, four clubs; North, double; East, pass; South, four spades; West, North, and East, pass. West opens with the ♣Q!

Here's one David Dewan (the proposer) can't do; can you?

39 I want to send a record to a friend but don't want her to guess what's inside from the size and shape of the package. What's the smallest size *cubic* box that will hold a 12-in. record (without its jacket, of course)?

My friend John P. Rudy has a problem of current interest:



40 Three prisoners stand in a row, each facing the back of the one in front of him, the front one facing a wall perpendicular to the prisoners' line. The prisoners can neither turn around nor see their own heads. But they know that there are five hats, three red ones and two black ones; each prisoner is wearing a hat, and the remaining hats are hidden from view. If any prisoner can state the color of his own hat and provide sufficiently good reasons for his choice (law of averages excluded), he will be set free. After a suitable time, the prisoner nearest the wall announces his answer, is correct, and is set free. How?

Speed Department

The only contribution is from John E. Prussing:

SD21 Find the fallacy in the following proof that 2 = 1:

$$x^2 = x \cdot x = \underbrace{x + x + \ldots + x}_{x \text{ terms}}.$$

Differentiating:

$$2x = d/dx\,(x + x + \ldots + x) = \underbrace{1 + 1 + \ldots + 1}_{x \text{ terms}} = x$$

Thus for $x \neq 0$, $2 = 1$.

Solutions

21 Define two functions, f and g recursively by

$$f(n, a) = \begin{cases} a \\ 1 - \log f(n-1, a) \end{cases}$$

$$g(n, a) = \begin{cases} 1/a \\ g(n-1, a)/f(n, a) \end{cases}$$

in each case if $n = 0$ and $n > 0$.
Then determine whether either of the following converge:

$$\sum_{n=1}^{\infty} g(n, n)$$

$$\sum_{n=1}^{\infty} g(n, 2)$$

Only Mr. Prussing is Johnny-on-the-spot:

$\sum_{n=1}^{\infty} g(n, n)$ does not converge. This can be shown by examining the ratio $g(n, n)/g(n-1, n-1) = 1/f(n, n) = 1/[1 - \ln f(n-1, n)]$.
This ratio of successive terms in the series is unity (since $f(0, 1) = 1$, $f(n-1, n) = 1$ for any n). In fact, every term in the series is infinite, since $g(0, 0) = \infty$. The second series, $\sum_{n=1}^{\infty} g(n, 2)$, is not even defined for real-valued functions. This is due to the fact that real-valued members of the sequence $f(n, 2)$ do not exist for $n > 7$, due to the fact that $f(7, 2) < 0$.

22 Find the smallest integers m and n such that $m - n^3$, m, and $m + n^3$ are all perfect squares, and give a general solution showing infinitude.

The following is by R. Robinson Rowe, an M.I.T. alumnus ('18) who formerly

conducted a column not unlike this one for *Civil Engineering* magazine:
The squares $(a^2 - 2ab - b^2)^2$, $(a^2 + b^2)^2$, and $(a^2 + 2ab - b^2)^2$ are in arithmetic progression with differences of $4ab(a - b)(a + b)$. Multiplication of each of the three squares by a fourth square will generate an analogous set with differences also multiplied by the fourth square. We may choose this fourth square so as to make the differences the required cube n^3. First factor the difference
$4ab(a - b)(a + b) = rs^2t^3$
in which r, s, and t may be unity, prime, or composite. Then choose for the fourth square r^2s^4, making
$m = r^2s^4(a^2 + b^2)^2$ and $n = rs^2t$.
Any pair of integers a and b will generate a primitive solution; hence an infinitude may be determined. For the least solution,
$a = 2, b = 1, rs^2t^3 = 24 = 3 \times 1^2 \times 2^3$,
$r = 3, s = 1, t = 2, r^2s^4 = 9, n = 6$,
$m = 225$,
and the squares are $225 - 216 = 9$, 225, and $225 + 216 = 441$.

Also solved by Winslow H. Hartford.

23 The following pertains to a boat and crew cruising the Lesser Antilles:

We blew this one, as the contributor, Lawrence S. Kalman, points out: he writes to add a clue ("somewhat useful though possibly not essential") which we failed to publish: the puzzle was concocted some years ago. He also notes an error; number 7 across should read, "Miles logged in nine days *minus* 1 down." And he challenges you: "I hope you will give your readers another chance at solving the puzzle with this additional information." His request is hereby granted; fire away.

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