Puzzle Corner

Allan J. Gottlieb

Hi. In all fairness to Dynaco, I must mention that several readers have reported to me that, although they die at the sight of a D.E., their hi-fi equipment works fine.

The answers to problems published in the May and June issues of *Technology Review* will appear next October, in the first issue of Volume 72. Next month, the last issue of Volume 71, I'll print no new problems—only the solutions to problems published in March. So these are the last new problems to be published for four months.

Thanks to a recent surge of problems, we have a medium (six- to twelve-month) backlog; so please be patient if your proposed problem does not appear until well into next year.

Problems

36 Consider the function N(R,x) defined (for R a positive integer and x a complex) by

$N(1,x) = x^x$

$N(2,x) = x^{N(2,x)}$

. . .

$N(k,x) = x^{N(k,x)}$

Richard Lopes wonders if it can be continuously extended to R to be any real (or perhaps any complex) number.

37 Douglas J. Hoylman asks you to evaluate

$$\int_0^\infty \frac{x^3dx}{e^x - 1}.$$

38 Russell A. Nahigian can show that

$$\frac{abc,abc}{77}$$

is always an integer. Can you?

39 Marshall Greenspan claims

$$\sum_{i=1}^{n} i^3 = \left[\sum_{i=1}^{n} i\right]^2.$$

Prove, or give a counterexample.

40 The following, from Donald E. Savage, is a really cool problem; he calls it a "loyal" puzzle:

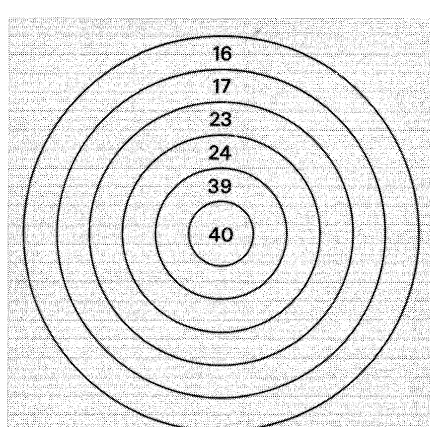
Plot, using Cartesian coordinates, the following equation:

$$\left[2y - \frac{5|x|}{1+e^{A(|x|-16)}} - \frac{\sqrt{1+\frac{10}{A}-\left(\frac{x}{16}\right)^2}}{A}\right] \cdot \left[A^2(x-30)^2 + \left(\frac{y}{20}-1\right)^2 - 1\right] \cdot \left[y - 40 + 40e^{-A(x-53)^2} + \frac{\sqrt{1-\left(\frac{X-53}{16}\right)^2}}{A}\right] = 0$$

where A is some large positive number, such as 1000.

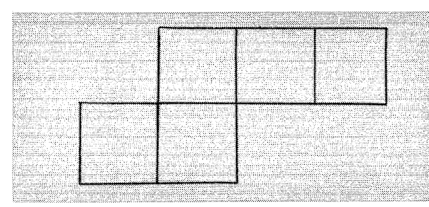
Speed Department

This is from Alec Henderson:
SD17 Using the following dart board, how many darts would you need to score an exact 100?



SD18 Mr. Hoylman wants you to punctuate the following:
That that is is that that is not is not is that it it is.

Correction: Please note that the figure for **SD12** (page 98, April) was incorrectly drawn. Hence the problem is repeated:
SD12 Mr. Nahigian wants you to move two matchsticks and change the figure below into a figure consisting of four equal squares.



Solutions

21 Each time his automobile gasoline tank is half full the owner fills it, alternating between regular fuel (92 octane) and premium fuel (100 octane). Assuming that he has been doing this for some time, what is the octane level of the fuel in the tank after he has filled it with regular fuel? (Assume that 10 gallons of 92-octane fuel plus 10 gallons of 100-octane fuel yields fuel of 96 octane.)

The following is from Donald S. Fraser: "More than one solution is possible, and mine—unlike the ones you usually receive for Puzzle Corner and may receive for this problem—are simple and straightforward.

"*Solution 1.* The problem assumes that the commuter has been mixing the given fuels in the prescribed manner for some time in his gasoline tank. However, it does not state that the tank at the point in question actually contains 10 gallons of blended fuel. The tank could be empty which he now "has filled with (his) regular fuel." A facetious answer therefore is 92 octane level.

Comment: The commuter in the problem appears confused about the market grades of motor gasoline. In a three-grade service station, "regular" is the middle quality averaging 95 (research method) octane, not 92, as stated. This latter is generally termed "sub-regular." A two-grade station normally dispenses 100 octane premium and 95 octane regular gasolines.

Solution 2. The commuter's engine knocks on 92 octane gasoline but does not knock with any of the blended fuels possible under the terms of the problem, not even with the 92+ obtained whenever 92 octane fuel is added to the tank mixture. A "tiger" is unnecessary in this situation, and following Warren K. Lewis' famous simplifying assumption hypothesis in chemical engineering principles, the solution is that the commuter should use "regular" gasoline, not the "sub-regular" mentioned in the previous paragraph. Good "orange disc" gasoline will provide a better octane cushion than the 92+ gasoline against knocking in his engine. It also should be less expensive than the described blended fuel procedure at present service station price differentials for the three grades of gasolines mentioned, and require less frequent stops for gasoline.

Also solved by Eric Callis, W. R. Campe, Edward Friedman, Arthur Hauser, Jr., John L. Joseph, Paul Karger, Stephen Owades, John E. Prussing, Frank Rubin, John Rudy, W. Allen Smith. and S. D. Turner.

22 If a pair of triangles is not co-polar, the joins of corresponding vertices form a triangle and so do the intersections of corresponding sides. The original pair of triangles has been transformed into a second pair which can be transformed into a third, and so on. How does the sequence of pairs of triangles behave?

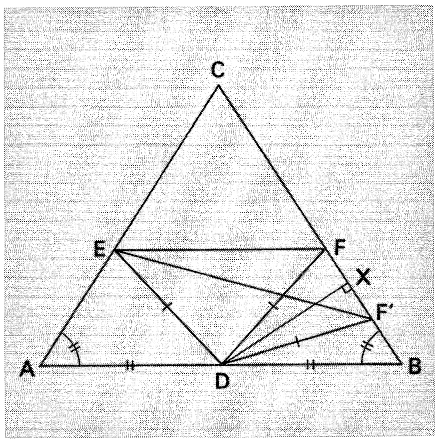
No takers so far. Keep at it.

23 Given an isosceles triangle DEF whose vertex D is the midpoint of the base of an isosceles triangle ABD. Prove that FE is parallel to AB.

Many people noticed that this problem cannot be solved. The best explanation is from Danny L. Taylor, who begins by noting a typographical error in the original publication of the problem (and repeated above): "If D is the midpoint of the base of an isosceles triangle ABD, then this is a very degenerate situation and therefore FE and AB are not parallel but colinear. Therefore, I assume the reference is to isosceles triangle ABC and submit the following:

"Consider isosceles triangle ABC with D the midpoint of the base AB. Construct DX perpendicular to BC. Set point F′ (not X) on XB such that $XF' \leq XC$. Set point F on XC such that $FX = XF'$. Construct isosceles triangle DEF with vertex at D.

"Now DX is a segment of the perpendicular bisector of FF′ and therefore $DF = DF'$ by definition of perpendicular bisector. By definition of isosceles, $DE = DF$ and therefore $DE = DF'$ by transitivity. Therefore DEF′ is also an isosceles triangle whose vertex is the midpoint of AB. Since both triangles DEF and DEF′ satisfy the conditions of the problem, an affirmative proof would require both EF and EF′ to be parallel with AB. This necessitates ignoring Euclid's parallel postulate, which takes us out of the realm of plane geometry and disproves the hypothesis."



Also solved by R. A. Bender, Richard Grant, Henri M. Gueron, Mrs. Martin S. Lindenberg, John McNeir, Peter Ross, Albert Yuncht, Jr., Messrs. Hauser, Joseph, Karger, Owades, Rubin, Rudy, and Turner, and the proposer, William J. Wagner.

24 For the case $n = 10$, $x = ½$, analytically evaluate the infinite series:

$$\sum_{k=1}^{\infty} [a_k k(k + 1) + x^k [k - (-1)^k]/k]$$

where

$a_k = ½\ [1 + \text{signum}\ (n - k + \epsilon)]$,
$0 < \epsilon < 1$.

This was solved only by Frank Rubin, who included a lucid description of "signum," and the proposer, John E. Prussing. Mr. Rubin's response is reprinted here:

"This one is sort of a hodge-podge of three series thrown together:

$$S = \sum_{k=1}^{\infty} a_k\, k(k + 1) + \sum_{k=1}^{\infty} (½)^k - \sum_{k=1}^{\infty} (-½)^k\, 1/k.$$

To evaluate the first series, note that

$a_k = 0$ if $k > 10$; and $a_k = 1$ if $k \leq 10$.

So $\sum_{k=1}^{10} k(k + 1) = 440.$

The second series of course has value 1. The third series is recognized as logarithmic, in this case — log 3/2.
Then $S = 441 + \log 3/2$.

"Signum, incidentally, is not a new but a very old notation for the sign of a number:
$\text{Sgn}\ (x) = \text{signum}\ (x) = -1$ if $x < 0$,
$= 0$ if $x = 0$, and $= 1$ if $x > 0$.
It is commonly used in conjunction with trigonometric functions in various quadrants and with square roots. A more common signum function refers to the sign of a permutation and is used in the expansion formula for a determinant

$$|(a\ i\ j)| = \sum_{\pi} \text{signum}(\pi) a_{1\pi(1)} \ldots a_{n\pi(n)}$$

sum taken over all permutations, π, of the numbers 1 to n."

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