Puzzle Corner

Allan J. Gottlieb

Hi. Several readers have asked why their proposed problems have not appeared. The answer is quite simple: I have a rather large backlog of questions, some submitted nearly a year ago, and it will take time until I am able to print any sent in this year. But I am certainly not complaining; rather, I consider it an honor that you take the time to send so many. (I am, however, running low on bridge problems—enough said?)

I now understand how L.B.J. must feel being a lame duck. My ankle is sprained, and I hobble around the math building with a limp. But since math types don't actually "do" anything (so says my girl friend), I have managed to survive.

My biggest success in math has been in the book department. Among the graduate students at Brandeis it is considered a sign of great wisdom to own an impressive array of math books (even if you can't understand a single page). One of my friends at the office owns both volumes of Zariski, thus making him twice as smart as I with my humble volume I. The fact that he knows infinitely more commutative algebra than I do is not as significant as the fact that he owns the two volumes. Well, I still don't know any commutative algebra but I just bought volume II and feel very wise indeed. Now do you see why my girl friend says math types "do" nothing?

Problems

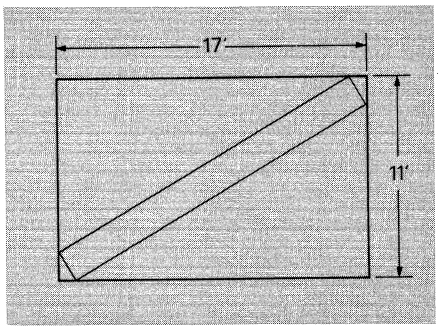
The answers to this month's problems will appear in the April issue.

John E. Prussing (M.I.T.'62) has a problem similar to one we have seen before:

11 Find the range of positive values of x such that, given x, the only positive value of y which satisfies $y^x = x^y$ is the trivial solution $y = x$. For those values of x for which nontrivial solutions for y exist (e.g., $x = 2$, $y = 4$), how many solutions are there for a given x? If a value of x is selected at random from the open interval (o, e), what is the probability that a nontrivial solution for y lies in the same interval?

The following comes from H. G. Holderness of Texas A & M:

12 Given a two-foot-wide carpet laid so that all four corners touch the four walls of a room 17 by 11 feet (see the drawing below), find the length of the carpet.



Steven Scott, the son of John H. Scott, Jr. (M.I.T.'43), wants you to . . .

13 . . . set up all 32 chessmen on the board so that each one has at least six legal moves— except pawns, of course. Bishops belonging to each player must be on different colors. One king may be in check but not both. Pawns may be placed in the first rank but not in the eighth rank.

John W. Langhaar of Kennett Square, Pa., writes, "A friend has recently been passing along copies of the Puzzle Corner from *Technology Review* since I am involved in a somewhat similar column for *The Bent* of Tau Beta Pi. We swipe problems from any available source and acknowledge the source as best we can. I find your column stimulating, but I must admit to not attempting some of the problems because I belong to the pre-Hilbert-space-set of college students. A problem which caused some controversy and which has many interesting features not immediately apparent is the following:

14 Let $n_1 = x^x$, $n_2 = n_1^x$, . . . $n_i = (n_{i-1})^x = x^{x^{\cdot^{\cdot^{\cdot^x}}}}$ and let $N = \lim n_i$ as i grows large beyond bound. Do real values of x exist for all positive N? If so, what is the relationship? If not, what are the limitations?"

The final problem comes with a nice letter from Carole A. Clarke, Secretary of the M.I.T. Class of 1921: "Since I have always been interested in mathematical recreations, I have for years urged Editors of *Technology Review* to run a regular puzzle section and expand the popular sets of puzzles which have appeared occasionally over the years. I cannot claim any credit for having worked on the present Editor to this end, but yours has been an excellent feature of the *Review*—although I must admit that I find most of the items too tough to handle. I have been awaiting an appropriate new problem to come along for your collection. The following, sprung on me by my son, Alfred L. Clarke (University of Pennsylvania Wharton School '54), has stumped quite a few people and I don't believe you have used one like it:

15 This is a logical set of numbers, mathematically derived: 10, 11, 12, 13, 14, 15, 16, 17, 20, 22, 24, 31, 100, —, 10,000. What is the missing number?"

Speed Department

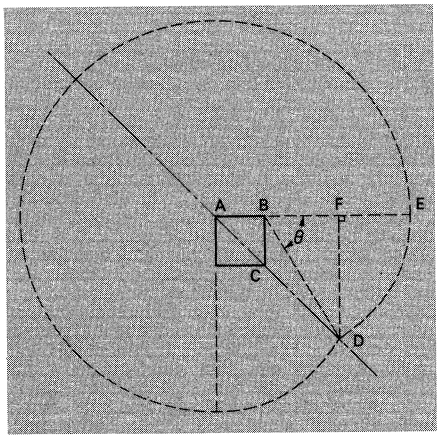
SD5 If a chicken and a half lays an egg and a half, how many eggs do six chickens lay in six days? (Your first answer is probably wrong.)

SD6 Here is an old *Voo Doo* puzzle: How many M.I.T. janitors does it take to change a light bulb? (Answer: five—one to hold the bulb and four to turn the ladder.)

Better Late Than Never

32 The published answer is incorrect. The problem: A cow is tethered to the corner of a square barn in a level field. The length of the tether equals the perimeter of the barn. The cow can graze over just one acre. What is the size of the barn, "give or take" a small decimal fraction? George Piotrowski, '64, presents the following:

The grazing area of the cow must be symmetrical about the diagonal of the barn which passes through the corner to which the cow is tethered. The grazing X corner is delineated in the sketch on the next page, and the solution of the problem proceeds as follows:
We know that
$AE = 4a$
$BD = BE = 3a$
$BC = AB = a$



Let $x = BF$, then
$AF = DF = x + a$
For triangle BFD, by the Pythagorean Theorem,
$(x + a)^2 + x^2 = (3a)^2$
Solving for x we find that
$BF = (\sqrt{17} - 1)a/2$
We can also find θ to be
$\theta = \cos^{-1}(\sqrt{17} - 1)/6$
The total area is given by
$3/4\ [\pi(4a)^2] + 2(\theta/2\pi)\ [\pi(3a)^2] + 2(BC \cdot BF/2) = 1$ acre.
Solving for a gives the length of a side of the barn to be 29.98 feet.

Mr. Piotrowski was not alone; there were also corrections from John L. Freeman, '58, Stephen G. Hovemeyer, '70, Brian R. Kashiwagi, '64, Henry C. King, W. B. Ladd, Thomas D. Landale, S.M.'54, Mark Lively, '69, Benjamin Madero, '06, William T. Moody, '31, Donald G. Parrish Jr., '51, Frank Rubin, '62, John T. Rule, '21, John G. Ryan, S.M.'60, Herbert G. Shaw, '13, James W. Shearer, '45, Robert D. Shooshan, '48, Waite H. Stephenson, Jr., '45, William H. Wannamaker, Jr., '30, Jerry Wolf, David A. Wright, and Mark H. Yu '70.

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