Puzzle Corner

Allan J. Gottlieb, '67

Hi (again). After a summer "off" (40-hours-a-week job in industry) I'm ready to start work. I tell my colleagues at Grumman that working there is my vacation. This, of course, is greatly received. After completing his doctoral program at Brandeis, your editor will have to find permanent work. Does *Newsweek* need a puzzle column?

Last spring at the end of the term I received a master's from Brandeis. What a nice looking diploma—it wasn't even folded! When President Johnson handed me the S.B. and I noticed the crease down the center, my mind was blown. By the way, the fold loses none of its charm after a year behind glass.

One M.I.T. tradition I really appreciate is Ashdown House. If you think orchids are hard to find in a desert, try to locate an apartment in Waltham. After two days all I have to show for my efforts is a near miss and two bald tires.

In order to acquaint the "rookie" readers and refresh the memories of veterans, let me go over the ground rules:

1. The problems which appear are those you send in. The scheduling algorithm is rather complex but neatness does count heavily.
2. Answers to all but speed problems appear in the third following issue. When submitting solutions, please indicate which problem (by number) you are trying to solve. The problems within a given volume are numbered consecutively from issue to issue.
3. Our printer is Arabic, not Greek.
4. If you disagree with any solution (or editorial comment), submit your grievances in writing.
5. All correspondence should be addressed to Allan J. Gottlieb, Department of Mathematics, Brandeis University, Waltham, Mass., 02154.

Problems

This letter was mailed by James L. Manganaro, '61, Assistant Professor of Chemical Engineering at Manhattan College:

1. Here is an interesting geometrical problem which resulted from a study of solid waste disposal. [Is this a sly comment about the usual material I print?—Ed.] Given an arbitrary triangle, find (by geometrical construction) the point such that the sum of the distances to the three vertices is a minimum.

2 P. W. Parsons would like all integral solutions to
$x^2 - 8xy - 2y^2 - 6x + 1 = 0$.
(Perhaps *none* exists.)

Here's one Douglas J. Hoylman, '64, found in Knopp's *Infinite Series:*

3 Consider the series

$$\sum_{n=1}^{\infty} \sin(n!\pi x).$$

a Show that it converges when x is rational. (Easy.)
b Show that it converges when $x = e$. (Tough.)
c Find a value of x for which the series diverges. (Not too bad once you've done b.)

This problem from Thomas M. Cover, '60, Associate Professor at Stanford University, should make us all rich. He writes: "During the history of gambling there have appeared many famous gambling systems. These include martingale systems (doubling up) and paroli systems. All seem to have in common the accumulation, with high probability, of a small amount of capital at the expense of a small probability of a large loss. This original problem shows that any 'reasonable' distribution on the gambler's terminal capital may be achieved. All of the previously known gambling systems on fair binary events are encompassed in this problem. Solution of the problem follows by simple induction.

4 Consider a sequential gambling system on sequences of Heads and Tails in which the bet at each stage may depend only on the outcomes of the previous events. The gambler has an initial capital w and may never bet more than he currently has. Given capital a after k trials, the gambler may bet any amount b, $0 \leq b \leq a$, on either Heads or Tails, and the gambler's fortune at the next stage will then be $a + b$ or $a - b$ accordingly as the $k + 1$st event is correctly guessed or not. There are 2^n possible sequences of Heads and Tails of length n. Let $w_1, w_2, \ldots, w_{2^n}$ be the *corresponding* terminal fortunes (for a given betting scheme). Suppose that a sequence of n bets is to be made. Show that there exists a sequential gambling system on n trials achieving $w_1, w_2, \ldots, w_{2^n}$ if and only if $w_i \geq 0$, all i, and $(1/2)^n \Sigma w_i = w$. Thus, subject to the above rather obvious constraints, every terminal distribution on the gambler's fortune is achievable.

5 For obvious reasons I am printing this letter exactly as received from Donald B. Silverman, '60, in reference to SD 12:

86 Blodgett Avenue
Swampscott, Mass., 01907
April 29, 1968

Mr. Allan J. Gottlieb
Department of Mathematics
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DGHGJ CTPSC TLGHQ TPLIF MQFDE
PHVMC IETLP PDTQR HORHA HQRTP
NLXQH SOXJQ MPQHN GHVJC
DQMQT ILPPQ HJIBP

The cryptogram by A Non Plus was quite easy to solve; not much time wasted there. What was annoying was to decode and get the tripe that passes for eloquence. Since you are a mathematician I hope that you cannot resist a puzzle. My revenge on you is above. Not as easy a cryptogram as yours but not exceptionally difficult. All you need is the key, which is in this letter.

D. B. Silverman

I've checked for typo errors; found none; refuse to be responsible if there are any.—DBS

Speed Department

I received the following suggestion from Reino W. Hakala:

SDI You might ask your readers to multiply 142857 by 2, 3, 4, 5, and 6 and ask them to predict from these products what the product with 7 would be. Once they find out what the correct product is, they should explain the reason for the observed behavior.

John W. Colton, '48, asking "How about another oldie?" makes the following contribution:

SD2 Suppose we have a hopper which is 10′ x 10′ in plan view (its height doesn't matter). The bottom of this hopper is four triangular planes pitched at 45° to the horizontal. Within 30 seconds, without using paper, slide rule or computer, tell me what the solid angle is between any two adjacent bottom plates.

Solutions

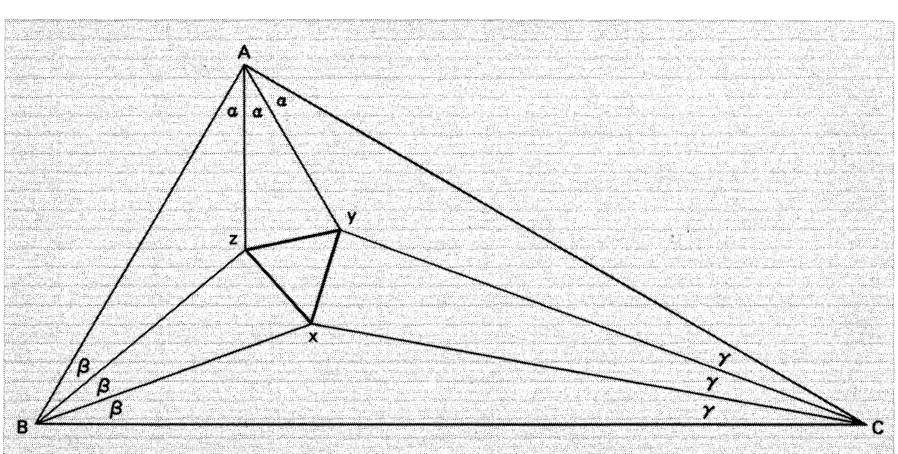
35 Given the drawing below in which it is known that triangle XYZ is equilateral and in which it can be shown that angle BZX is $\alpha + \pi/3$. Show that its sides are of length 8 R sin α sin β sin γ, where R is the perimeter of triangle ABC.

Since this is a geometrical problem, Mark H. Yu, '70, solved it; but he notes that R is given incorrectly as the perimeter of triangle ABC; it should be the circumradius. [Sorry.—Ed.]
Apply the law of sines to triangle BXC; we obtain
$\overline{BX} = [\sin\gamma/\sin(\beta+\gamma)]$
$\overline{BC} = [\sin\gamma/\sin(\pi/3-\alpha)]$ BC.
Also, $\overline{XZ} = [(\sin\beta)\,\overline{BX}]/\sin(\alpha+\pi/3) =$
$[(\sin\beta\sin\gamma)\,\overline{BC}]/[\sin(\pi/3+\alpha)$
$\sin(\pi/3)-\alpha)] = [\sin\alpha\sin\beta\sin\gamma]/$
$[\sin\alpha\,(3\cos^2\alpha/4-\sin^2\alpha/4] =$
$[8\,(\sin\alpha\sin\beta\sin\gamma)\,\overline{BC}]/2\sin 3\alpha$.
But $R = (\overline{BC})/2\sin 3\alpha$ (just check any geometry textbook). Q.E.D.

Also solved by the proposer, Eric Rosenthal, son of Meyer S. Rosenthal, '47.

36 A quadrilateral is inscribed in a circle such that one side is a diameter of the circle and the other three sides have lengths 1, 2 and 3, respectively (above, right). Find the length of the diameter to three decimal places.

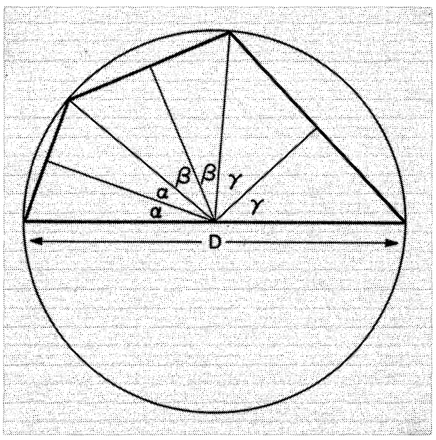
The following from Richard G. Lipes, '64, uses enough calculus even for the proposer, Doug Hoylman:



Calculus in the form of Newton's method is applied:

$\sin\alpha = 1/D$, $\sin\beta = 2/D$, and
$\sin\gamma = 3/D$, so $\alpha+\beta+\gamma = \pi/2 =$
$\sin^{-1}(1/D) + \sin^{-1}(2/D) +$
$\sin^{-1}(3/D)$.
Solving this for D gives a cubic equation
$u^3 - 28u^2 + 196u - 144 = 0$,
where $u = D^2$.
One can find the roots of this equation with Newton's method; the solution is
$D = 4.114$.

Also solved by William Dunbar, Colonel Foster L. Furphy, S.M.'48, Michael A. Goldberg, John W. McNear, '59, R. Robinson Rowe, '18, Frank Rubin, '62, Paul J. Schweitzer, '61, Lars H. Sjodahl, '35, Glenn W. Zeiders, '59, Messrs. Rosenthal and Yu, and the proposer.



37 Arrange six line segments of equal length in the plane to form eight equilateral triangles.

Mark Yu says, "Hey, man, I'm a Zionist, too." (The answer, of course, is the Star of David.)

Also solved by Robert G. Gottlieb, '60 (no known relation), John L. Joseph, '40, Sanford M. Libman, '65, Roddy R. Rogers, S.M.'57, Messrs. Lipes, Rowe, Rubin, Sjodahl, and Zeiders, and the proposer, Mr. Hoylman.

38 Given four colored cubes described as follows:

	Front	Right	Back	Left	Top	Bottom
Cube 1	Green	Blue	Red	White	Red	Red
Cube 2	White	Green	Green	Red	White	Blue
Cube 3	Blue	Red	White	Blue	Green	Green
Cube 4	Red	White	Blue	Green	Red	White

The problem is to pile the blocks one above the other so that each face of the pile shows all four colors. (Cubes are helpful but not necessary to solve the problem.)

I blew it this time. I listed the cubes in the order in which they belong. Several people noticed this, including Richard A. Bator, '65, Peter M. Kendall, '67, Karl R. Kennison, '08, Ted Leahy, Robert G. Millar, '40, John E. Prussing, '62, Messrs. Rogers, Sjodahl, and Yu, and the proposer, Russell A. Nahigian, '57.

But one virtue of the problem is that I received a reprint from the Rand Corporation containing several variants which will appear and the following letter from Milton Kamins, '48, which accompanied it:

Having enjoyed your material in *Technology Review* on more than one occasion, I am pleased to be able to pass on what I believe is an unusually neat solution to your problem 38. My own connection with (and interest in) the problem began a year ago when a friend received one of the first sets of these blocks seen locally, but without instructions. After what I think were a few months of silent frustration, he showed them to me. When I calculated the number of possible combinations, I quickly gave up the trial-and-error approach, exploited the one obvious (to me) asymmetry to eliminate more than 90 per cent of the combinations, and instituted a computer search of the remaining 10,000 or so, thus finding the solution. However, when my eight-year-old son tried the blocks he found the right combination in just about five minutes, unassisted. Tested again one day later, he took 10 minutes. At that point, I suggested that he quit while he was ahead.

39 Find n such that $n^4 + n^3$ is a 10-digit number in which each digit is used only once.

Everyone seemed to use some trial-and-error scheme to arrive at the fact that $(264)^4 + (264)^3 = 4{,}875{,}932{,}160$. Solved by Messrs. Leahy, Libman, Prussing, Sjodahl, and the proposer, John Reed, '43.

Better Late Than Never

A considerable accumulation of solutions and correspondence about previous problems in "Puzzle Corner" will be published in this department in the December issue.

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