Lately several readers have been requesting that I phone them to discuss some of the problems. I am afraid that this is impossible on my schedule. As I have said before, that would destroy the essence of mass communication. Please submit letters and state your grievances in writing to me at the Mathematics Department, Brandeis University, Waltham, Mass., 02154. Since often many people notice the same mistakes or ambiguities (there always seem to be some), one correction will suffice.

I have been asked what I plan to do to avoid the draft next year. Should the present situation remain, I most likely will enlist. Perhaps next year my address will include A.P.O. Does the Army need puzzle editors?

Problems

The first problem was sent in by Mark D. Horowitz, '71, about three months ago. I must apologize for the unreasonable delay in publication, but we thought we may have printed it before and ensuing investigations accounted for most of the delay. Here it is at last:

25 Given a rectangular solid box with inside dimensions of 18"x51"x69" and incompressible golf balls 1.82" in diameter, find the maximum number of golf balls that can be packed into the box. Assume that gravity is present and that the box may be rotated to any spatial orientation, so long as it is not deformed.

The following arrived from Bojan Popovic, a mathematics student in Belgrade, Yugoslavia; I was very pleasantly surprised to see that my column has traveled so far:

26 On the table are given six matches of equal length. Make four identical triangles without breaking them.

The next two problems are from Shih-Ping Wang, S.M.'61, and they have also been somewhat delayed in their appearance:

27 Find the solution for the case of two odd balls both defective by the same absolute amount (but they may be opposite in sign).

28 Along the same vein, using a two-pan balance: given a set of n weights, each of integral weight w₁, w₂, . . . , wₙ such that any object with an integral weight from unity to w₁ + w₂ + . . . , wₙ, the sum of all weights can be determined.

This problem arrived from Winslow H. Hartford, '30, along with these comments: "Just a line to let you know how much I enjoy Puzzle Review. You are running Martin Gardner of Scientific American a good second; I've met a lot of old favorites, like the coconut problem. I don't have any football problems handy, but I suppose the type of problem which ends up 'the engineer's name is Smith' could readily be done over to end up 'the flanker back's name is Wojchieszczojysi.' Here is an old favorite of mine—a bridge problem which is a stinker."

29 With the following, South is the declarer at seven spades. West leads the 8. The problem is to make seven against any defense.

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Solutions

10 A farm horse is tethered to one corner of a barn 25 feet square, in the middle of an open field, with a rope 100 feet long. What is the area the horse can graze on?

The following solution is by Marshall Greenspan, '61: In the following drawing, the area for grazing equals twice the area of sector EOD plus twice the area of sector BAE plus twice the area of triangle ABC.

Speed Department

SD9 Joseph W. Lovell, '13, asks you to find the fallacy in his proof: given that a = b + c, then a = b. This is the proof:

\[ a = b + c \]
\[ a(a - b) = (a - b)(b + c) \]
\[ a^2 - ab = ab + ac - b^2 - bc \]
\[ a^2 - ab - ac = ab - b^2 - bc. \]
\[ a(a - b - c) = b(a - b - c), \]
\[ a = b \]

SD10 Richard P. Bishop, '59, submitted the following problem of Dr. Murray Spiegel:
The first step is to determine the angle $\angle BAC$, which equals $90^\circ - (\angle 75^\circ 25^\prime 49^\prime)$, which is $13^\circ 38^\prime$. Therefore angle $BAC$ equals $180^\circ - 135^\circ - 13^\circ 38^\prime$, or $31^\circ 22^\prime$. Next, determine angle $BAE$, which equals $90^\circ - \angle BAC$, or $58^\circ 38^\prime$. Now we can determine the areas:

- $BAC = \frac{1}{2}(25)(75\sin 31^\circ 22^\prime) = 800$ sq. ft.
- $BAE = \pi(5\sin 58^\circ 38^\prime/360^\prime) = 2880$ sq. ft.
- $EOD = \pi(10\sin 13^\circ 38^\prime/360^\prime) = 11780$ sq. ft.

The total of these areas is $15460$ sq. ft.; the total grazing area is therefore $30920$ sq. ft.

To check this result, note that $\pi(100)^2 = 31400$, which is greater than $30920$.

Also solved by Douglas K. Severn, '23, Eric Rosenthal (son of Meyer S. Rosenthal, '47), Mark H. Yu, '70, Douglas J. Hoylman, '64, Kenneth B. Blake, '13 (who adds that "my old classmate Howard S. Currier, '13, will have to dig up a tougher problem if he wants to stop me!"), Arnold B. Staubach, '19, and Mark and John D. Pfeil (sons of John S. Pfeil, Jr., '43).

11 How is it possible for a batter to get a hit and thus raise his batting average exactly one point? (A trivial solution may immediately come to mind, namely, the batter who has given hitless for his first $999$ times at bat and then gets a hit to raise his average from $.001$ to $.001$. Nontrivial solutions are desired.)

Richard Haberman, '67, sent in this solution:

Let $x = \text{number of hits}$ and $y = \text{number of times at bat}$. The problem is to solve for $x$ and $y$, positive integers, $x \leq y$, such that $(x + 1)/(y + 1) = x/y = .001$. Consequently, $x = .001 y(999 - y)$; for $x$ to be an integer, $y(999 - y) = 999n$, where $n$ is a positive integer. Therefore $y^2 - 999y + 999n = 0$.

The quadratic formula implies $y = [999 \pm \sqrt{(999)^2 - 4(999)999}] / 2$. For $y$ to be an integer, $(999)^2 - 4(999)999$ must be a perfect square $= z^2$; but $(999)^2 = 998001$ and hence $\text{mod}(999,2) = 1$.

Furthermore $z \leq 999$. A quick look through C.R.O. shows the only numbers $(0, 999)$ which end 001 are 1, 249, 251, 499, 501, 749, 751, and 999. We know that $n = \frac{1}{2} \left( (999)^2 - z^2 \right)/1000$ and we easily see that $z = 249$ is the only solution except the trivial one ($z = 999$):

$z^2 - (999)^2 + z^2 = 1000n$,

$\begin{align*}
1 & 1 & 998 & \text{not integer} \\
249 & 62001 & 936 & 234 \\
251 & 62001 & 935 & \text{not integer} \\
499 & 249001 & 749 & \text{not integer} \\
501 & 251001 & 747 & \text{not integer} \\
749 & 561001 & 437 & \text{not integer} \\
751 & 564001 & 434 & \text{not integer} \\
999 & 998001 & 0 & 0
\end{align*}$

Hence $y = (999 + 249)/2 = 624$ or 347, and $x = 234$. The two possibilities are then:

- $234/624 = \frac{1}{2} = .375$ and $235/625 = 47.125 = .376$
- $234/375 = 78/125 = .624$ and $235/375 = \frac{1}{2} = .625$

(Note the symmetry.) Both are great hitters!

The "big three" solved this as well—namely, Messrs. Yu, Rosenthal and Hoylman—and so did Mr. Greenespan and Arthur W. Anderson, '63, who included a small epistle for his proof.

12 It should always be possible to solve this equation

$4uv + u - v$ in positive whole numbers (greater than 0) with arbitrary values assigned to either $x$ and $y$ or $u$ and $v$.

Arnold B. Staubach, '19, writes as follows: "The solution may consist of the application of the New Math ideas of arrays and one-to-one correspondence, which reveal the hidden structure of the expressions on each side of the equation, together with proof by induction, as shown in the following diagrams and equations."

The following super-elegant solution is by James E. Rubenstein, '63 (my money says he's a math major):

Let $N = 2M + 1$ be the number of teams. We shall construct an $N \times N$ matrix representing the schedule. The $ith$ row shall represent the schedule for the $ith$ week, to wit: $a_{ij}$ designates the team that is idle on the $ith$ week. The remaining $2M$ elements are considered as $M$ ordered pairs, the first of a pair being the home team, who are scheduled with the second of the pair, the away team. The requirements are:

1. Each row shall contain the integers $1, \ldots, N$ exclusively; i.e., each week, each team either plays or is idle.
2. The first column contains the integers $1, \ldots, N$ exclusively; i.e., each team is idle exactly once.
3. If a team (integer) appears in an odd column one week, it must appear in an even column the next week, and vice versa; i.e., a team alternates home and away. Column 1 is ignored for this consideration.

The following matrix satisfies these requirements:

$a_{ij} = \begin{cases} 
1 & \text{if } j = i \\
1 & \text{if } i = j \\
0 & \text{otherwise}
\end{cases}$

Addition is taken modulo $N$, and we take $N \text{ mod } N = N$, not $N \text{ mod } N = \phi$.

Requirements 1. and 2. are obviously satisfied. As for 3. the appearance of the integer $k$ in the $ith$ row implies $i + k = k$ for some $j$. In the next row, $i$ has increased by 1, so that $j$ must decrease by 1 for $k$ to remain constant. An increase of 1 means a change in parity. Transitions into and out of the first and last columns are seen to satisfy this parity shift. The following is an example for $N = 5$:

$\begin{align*}
1 & 2 & 3 & 4 & 5 \\
2 & 4 & 5 & 1 & 3 \\
3 & 5 & 1 & 2 & 4 \\
4 & 1 & 2 & 3 & 5 \\
5 & 2 & 3 & 4 & 1
\end{align*}$

Obviously, any mapping that is 1-1 from the set $\{1, 2, \ldots, N\}$ to itself will preserve the schedule. Therefore there are $N!$ solutions, each one reflecting a different permutation of the order of teams idle.

Also solved by John E. Prussing, '62, Leo P. Buckley, Jr., '52, and Messrs. Yu and Hoylman.

14 Prove that a nonstandard ball can be determined in $n$ weighings from a set of $(3^n - 1)/2$ plus 1 balls, one of which is marked as standard.
Here is the solution of the proposer, Charles D. Coitharp, '58:
Let Q₀ be the proposition that a non-
standard ball can be determined in n
weighings from a set of

\[(3^n - 1)/2 \text{ pl } + (3^n - 1)/2 \text{ ph or}
(3^n - 1)/2 \text{ ph } + (3^n - 1)/2 \text{ pl},
\]
where pl = possibly light ball or balls, and
ph = possibly heavy ball or balls.
Q₀ is true, since given one pl and two ph,
one ph can be weighed against the
other and an imbalance will indicate
which is heavy. A balance will indicate
that the pl is light. Because of the sym-
metry in the problem, pl and ph can be re-
versed, so the case of two pl and one
ph does not need to be considered
separately. In what follows, appeals to
symmetry will be implied rather than ex-
licit. P₂ is true since two unknown balls
can be weighed against an unknown
ball and the marked ball, and a balance
will indicate that the unweighed ball
is nonstandard. It can be weighed
against the marked ball to determine
if it is heavy or light. An imbalance
reduces the problem to Q₁. Now suppose
Pₙ and Qₙ₋₁ to be true, and examine
Pₙ₊₁ and Qₙ. Set \((3^{n+1} - 1)/2\) balls aside
and \(3^n\) balls remain, since

\[(3^n + 1)/2 = (3^n + 1)/2 = (3^{n+1} -
3^n)/2 = 3^n (3^n - 1)/2 = 3^n/2.\]

Weigh \(3^n - 1)/2 \text{ pl plus the marked ball}
against \(3^n - 1)/2 \text{ pl balls}. If a balance
occurs, the problem reduces to Pₙ, which
is true. If an imbalance occurs, we have,
without loss of generality, \((3^n - 1)/2 \text{ pl and}
(3^n + 1)/2 \text{ ph}, which is Qₙ. Set aside
\((3^{n+1} - 1)/2 \text{ pl and (3^n - 1)/2 \text{ ph,}
and there remains \(3^n - 1\) pl plus \(3^n - 1\) ph.
Weigh \((3^n - 1)/2 \text{ pl plus (3^n - 1)/2 ph}
against \((3^n - 1)/2 \text{ ph plus (3^n - 1)/2 ph}
and no matter what happens the problem
reduces to Qₙ₋₁, which is true. Therefore
Pₙ and Qₙ₋₁ imply
Pₙ₊₁ and Qₙ. Couple this with the
truth of P₂ and Q₁, and we have
by induction that Pₙ is true for all n.

Also solved by Mr. Yu.

**SD4** Let \(n₀\) be a number, \(n₁\) be the
number of letters in the spelling (in
English) of the number \(n₀ \ldots \text{ and } nₙ\) be the
number of letters in the spelling of \(nₙ₋₁\).
Prove \(\lim_{n \to \infty} \left(\frac{n₀}{n₁}\right) = 4\),
and show that this is independent of
the language used.

In general I do not print solutions to
speed problems, but for this one I will
make an exception. Mr. Hoyman's
is the most interesting:
It is easily seen that for \(n > 4\), the
number of letters in the name of \(n\) is
less than \(n\). (If you don’t believe me, write
them all out.) Hence the sequence
\(n₀, n₁, \ldots\) is strictly decreasing until one
of the \(nₙ\) is less than 5. Then we have
one \(\to\) three \(\to\) five \(\to\) four.

So eventually 4 must appear in the
sequence. But if \(n₄\) is 4, so is \(n₅ + 1\), and
all terms after that. So the sequence
converges to 4. I can’t figure out what he
means by, "Show this is independent of
the language used." Indeed, in German

**Better Late Than Never**

Solutions to the following problems have
come from those indicated:

79 Richard P. Bishop, ’59, and Eric
Hovemeyer.
3 John F. Simmons.
5 R. Robinson Row, ’18, and
Mr. Hovemeyer.
7 Charles S. Sutton, ’35.
8 Jeffrey D. Dodson, ’67, and Mr. Sutton.

Allan J. Gottlieb, ’67, is a graduate stu-
dent in mathematics at Brandeis
University. "Puzzle Review" is written
for Technology Review and Tech
Engineering News, the M.I.T. undergradu-
ate professional magazine.

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