Puzzle Review

Allan J. Gottlieb, '67

Every month the editor and I have been having a friendly war concerning my deadline. I would like it as late as possible in order to include late responses, whereas he needs it early to purge my many mistakes. We have been effecting a compromise which causes several good solutions to be passed up and a few errors to slip by. It appears that a more reasonable alternative is to allow two issues to pass before printing solutions. I realize that some hardship may occur in having to wait an extra month for the answers but my defenses are that it *is* necessary to wait the extra month and that *American Mathematics Monthly* has a delay time of about one year.

There is a possibility that Puzzle Review may appear in another magazine outside the M.I.T. community. In order to expedite handling kindly include with your solutions the problem, number, issue, and name of the magazine in which the problem appeared.

Problems

10 The first problem this time is from Howard S. Currier, '13:

A farm horse is tethered to one corner of a barn 25 feet square, in the middle of an open field, with a rope 100 feet long. What is the area the horse can graze on?

11 Eugene W. Sard, '44, sends us the following:

Here is a problem for the Puzzle Review in the area of Diophantine analysis that had its genesis in my boyhood while listening to radio broadcasts of the old Brooklyn Dodger baseball games. About two years ago it finally crystallized in the following form:

In baseball, how is it possible for a batter to get a hit and thus raise his batting average exactly one point? A trivial solution may immediately come to mind, namely, the batter who has gone hitless for his first 999 times at bat and then gets a hit to raise his average from .000 to .001. Two related nontrivial solutions (the only ones, I believe) are enclosed.

12 Theodore M. Edison, '23, submits the following:

In the course of trying to find the factors of a number, I developed the following equation:

$$x^2 + 2xy + y = 4uv + u - v.$$

My line of reasoning led me to believe that it should always be possible to solve this equation in positive whole numbers (greater than 0) with arbitrary values assigned to either x and y, or u and v. Eventually I was able to prove that my belief was correct, but not without some trouble. Solutions are not necessarily unique. For example,

let $u = 3$ and $v = 2$: then one solution is $x = 1$, $y = 8$, and another is $x = 4$, $y = 1$ (in addition to zero solutions such as $x = 0$, $y = 25$, or $x = 5$, $y = 0$).

Perhaps this would all be obvious if I were familiar with the theory of numbers, but I thought that the problem of finding the proof mentioned might be of enough interest to merit inclusion in your puzzle department.

13 Charles D. Coltharp, '58, wonders who can handle this one:

Here's a variation on an old problem. A community has N institutions of higher learning and decides to form a football league. The committee, upon learning that one of the football players is taking a math course (his name was Ryan, or something like that), assigned to him the task of arranging the schedule.

They stipulated each team was to play every other team once, each team was to have one idle weekend during the season, and no team would play two consecutive games either at home or away. Only one team could be idle on a given weekend. Fortunately for Ryan, N was odd. What were his chances of delivering a schedule?

14 And another from Charles D. Coltharp, who wants you to prove that a nonstandard ball can be determined in n weighings from a set of $(3^n-1)/2$ plus 1 balls, one of which is marked as standard.

Speed Department

SD4 The only entry this time came from Michael R. Gabel, '65:

Let n_0 be a number, n_1 be the number of letters in the spelling (in English) of the number $n_0, \ldots, n_k$ be the number of letters in the spelling of n_{k-1}. Prove $\lim_{k\to\infty} n_k = 4$

Show this is independent of the language used.

Better Late than Never

There has been some discussion as to the validity of the solution to the seven cigarettes problem published in the *Review* last March. The problem, numbered **15** last year, is to place seven unbent cigarettes such that each one is touching the other six. The (apparently) definitive solution has now been submitted by J. W. Langhaar:

In the perspective view, right, the shortest possible cigarettes would have ends of axes of 1 and 5 coincide and outer ends of axes of 2 and 5 coincide, and similarly for other pairs. It must be shown that cigarettes have 4D large enough; for larger 4D, axes can cross. Let a cigarette have unit radius; the problem then is to determine PA.

Consider a more general case for a central cigarette of radius r possibly different from the unit radius of the other six.

The small circle, far right, is the central cigarette of radius r. The larger circle of radius (r + 1) is the circle to which the axes of the other cigarettes are tangent. If C is the origin in the usual coordinates, the location of A is $(\sqrt{3}\,r, -r - 1)$. Therefore $\overline{CA}^2 = 3r^2 + (r + 1)^2$. But $\overline{CT} = r + 1$; therefore $\overline{AT}^2 = \overline{CA}^2 - \overline{CT}^2 = 3r^2$ and $\overline{AT} = \sqrt{3}\,r$.

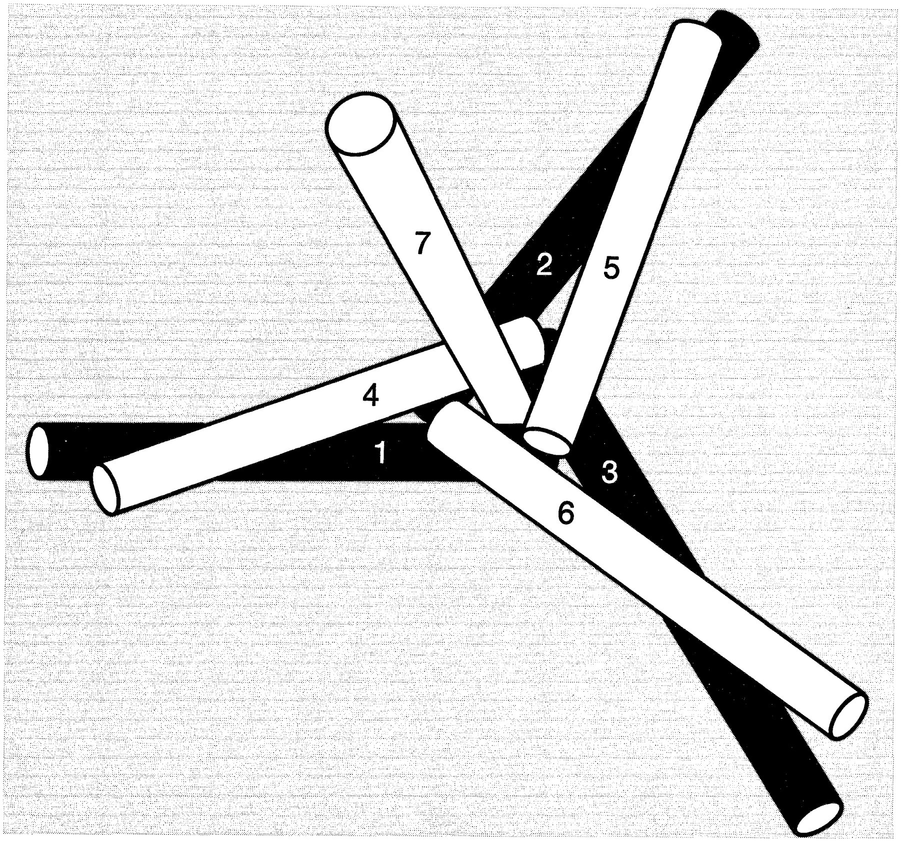
Let angle ACT $= \alpha$ and angle TCP $= \beta$. By symmetry, angle BCA $= 120°$; therefore $\alpha + \beta = 120°$

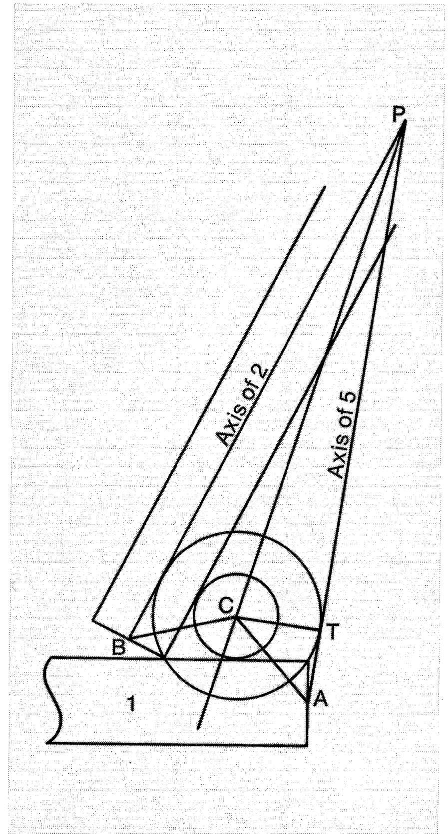
$\overline{PT} = \overline{CT} \tan\beta = (r + 1) \tan\beta$

$\overline{PA} = \overline{PT} + \overline{AT} = (r + 1) \tan\beta + \sqrt{3}\,r$

$\tan\alpha = \sqrt{3}\,r/(r + 1)$

$\tan\beta = \tan(120° - \alpha) = (\tan\alpha + \sqrt{3})/(\sqrt{3}\tan\alpha - 1)$





$$\overline{PA} = \frac{(r+1)\,[\sqrt{3}r/(r+1)+\sqrt{3}]}{\sqrt{3}[\sqrt{3}r/(r+1)]-1} + \sqrt{3}\,r$$

$= \sqrt{3}\,(r + r + 1)(r + 1)/(3r - r - 1)$
$+ \sqrt{3}r$
$= \sqrt{3}(2r^2 + 3r + 1 + 2r^2 - r)/(2r - 1)$
$= \sqrt{3}\,(4r^2 + 2r + 1)/(2r - 1)$
$= \sqrt{3}[2(r + 1) + 3/(2r - 1)]$
$=$ the minimum length/radius.

This leads to the following table of values:

r	*L/R min.*	*L/D min.*
0.5	∞	∞
1	$7\sqrt{3} = 12.12$	6.06
1.366*	$6.46\sqrt{3} = 11.20$	5.60
2	$7\sqrt{3} = 12.12$	6.06
3	$8.60\sqrt{3} = 14.90$	7.45
4	$10.43\sqrt{3} = 18.06$	9.03
6	$14.27\sqrt{3} = 24.72$	12.36
10	$22.16\sqrt{3} = 38.38$	19.19

* i.e. $(1 - \sqrt{3})/2$, which yields minimum $(L/R) = (3 + 2\sqrt{3})\,\sqrt{3}$.

For the basic problem, $r = 1$. A king-size cigarette has $4D \simeq 3.9/.33 \simeq 11.7$, which is greater than the minimum required; the construction is therefore possible.

78 George H. Borrmann, Jr., '57, has a generalization which is similar to problem 14 in this issue.

79 Many people noticed that a smaller answer is possible. Among them were: Kenneth B. Blake, '13, Theodore M. Edison, and J. Edward Philbrick, '32.

80 George H. Ropes, '33, solved the coconut problem and proposes a generalization which will appear next time. Russell A. Nahigian, '57, has a method for finding the *smallest* number of coconuts.

81 Many have noticed the trouble with last year's answer: Russell A. Nahigian, Howard S. Currier, Martin D. Landau, '47, Donald A. Trumpler, Ph.D.'58, James Theodosopoulos, '48, Edward C. Booth, '25, J. Charles Forman, '53, R. K. Broome and L. R. Mazur, John W. Sanborn, '26, Bruce P. Layton, '60, Charles D. Coltharp, David W. Ulrich, '52, and Peter E. Lobban, '66.

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