Puzzle Review

Allan J. Gottlieb, '67

By now you have probably all heard of Harvey M. Friedman, Ph.D.'67, the boy-wonder who received his doctorate in math from M.I.T. before he turned 19. I'd like to give you my rather personal account of this story. If you notice a tinge of envy, you might remember that I am a 22-year-old first-year graduate student.

During second term sophomore year at M.I.T. I was feeling quite proud of myself. Taking three upperclass math courses, I was sure no one could do better. One day, during logic class, I noticed a rather young looking handsome "little guy" in the front, who knew the material so well it seemed he should be giving the course, not attending it. Upon inquiry I ascertained that he was a freshman, a lad of merely 16, who was enrolled in all three of my math courses—plus a graduate course to boot. I was duly impressed (and my bubble was permanently collapsed). The next year, when "Puzzle Corner" first appeared in *Tech Engineering News,* I again found my math courses to be a very proper subset of Harvey's. But something strange happened. The "little guy" began to read *my* column avidly. He answered most of the problems and posed several hard ones of his own. With this common meeting ground we became friends. He seemed to enjoy having his problems printed, and I certainly benefitted from his "lectures" on some of the cooler aspects of logic. Sometime during that winter Harvey became very involved in some deep aspect of logic and "there was one week where [he] made a significant discovery nearly every day," he wrote me. Sometime during this year he was officially reclassified as a graduate student.

My closest contact with Harvey came during reading period that spring. We'd spend all day in the library worrying a little about 18.242 and a hell of a lot about 18.36 (if I only studied as much as I worried about not studying . . .), and discussing life in general. It was during this week and a half that I began to really appreciate Harvey's good-natured attitude. Last year results poured forth and the "little guy" began to correspond with a few of the leaders in American logic. As a result he flew through (perhaps "by" would be more appropriate) M.I.T. grad school and is currently an assistant professor at Stanford.

That, very sadly, brought to a close my only acquaintance with a prodigy. I'll probably never meet Harvey, nor anyone like him, again, and I don't know if the "little guy" still reads this column.

But if he does, I'd like to dedicate this current installment to him. Thank you for letting me into your life, Harvey—thank you very much.—Allan

Enough for nostalgia. Let's have some problems:

Problems

5 Will someone please tell John P. Rudy, '67, whether π^e is greater than, equal to, or less than e^π. Congratulations, Janice.

6 All you origami—geometry experts might try this one from John B. Nugent, '37:

1. Take a strip of paper with parallel edges.
2. Tie an overhand knot making sure there is no looseness where the free ends "leave" the knot.
3. Hold it up to a light source and observe all but one diagonal of a five-point star.
4. Do the five points determine a *regular* pentagon?

7 and **8** The next two problems come from Donald E. Savage, '54, who writes that though he is a "Course VI type," he enjoys mathematical puzzles, especially if they are a bit off-beat. Here, he says, are a couple of "weird" puzzles he dreamed up a few years ago:

Presumably few who have studied calculus would have difficulty solving the differential equation:

$dy/dx = y,\ y(0) = 1.$

But what is the solution if one makes a slight(?) change in the differential equation:

$dy(x)/dx = y(x - 1),\ y(0) = 1.$

Note: $y(x - 1)$ should be read "y of $(x - 1)$;" that is, find the function y such that the slope at any x equals the function at any $(x - 1)$. An alternative way of writing it might be:

$dy/dx|_x = y|_{x-1},\ y|_0 = 1.$

Using this idea of writing differential equations in which the derivative is evaluated one place and the function some other place, one can go on to invent problems *ad libitum.* May I suggest:

$dy/dx = y(x^2),\ y(0) = 1$
or
$dy/dx = y[y(x)],\ y(0) = 1.$

Note, again, that the above should be read y of x^2 and y of y of x, respectively.

Throughout the ages mathematicians have devised answers to problems that the nonmathematician might be tempted to believe had no answer. For example, when the question, "What is 3 minus 5?" was first asked, the nonmathematician presumably answered that there isn't any answer. But some mathematician apparently decided "minus 2" was a good answer. Much later when asked what is the number "x" such that $x^2 = -1$, the nonmathematician presumably said that there isn't any, but some mathematician decided "i" was a good answer. (*Somewhat of an over-simplification—Ed)* Therefore I, the non-mathematician, wish to ask this question of the mathematicians: I have heard about the first derivative of a function, I have heard about second derivatives, I have heard about third derivatives, etc., but what is a ½ derivative? Or how about a πth derivative, or even an ith derivative?

9 Here's one I just solved for my topology course under Aldridge Bousfield, '63:

Let Y be the comb space
$Y = \{(x,y)\epsilon R^2 |\ O \leq Y \leq 1 \text{ and } x = O, 1/n \text{ for } n \geq 1\}$
$\cup\ \{(x,y)\epsilon R^2 |\ Y = 0 \text{ and } 0 \leq x \leq 1\}$

Prove Y is not contractible relative to (0,1).

Speed Department

SD3 George A. W. Boehm submitted the following:

Given: an elimination tournament with 57 entries. If you arrange the bracket (with necessary byes) in the optimal way, how many matches will be played?

Better Late Than Never

Problems from last year for which solutions have recently arrived include:

23 Douglas J. Hoylman, '64, and Eric Rosenthal. (Thanks for the good wishes, and I have found someone to keep up the column—me.)

25 Frank G. Smith, '11, submitted a simpler proof.

68 Edmund Blau, Norman D. Davis, '64, and L. William Sardes, Jr., have pointed out errors in Mr. Ho's solution.

72 A partial solution was sent in by Robert Mullen, eighth grade East Prairrie Eleamenty (sic) School, who includes an illuminating exsampal (sic). Keep up the good work, Bobby; in a few years I hope you will be sending in some problems, too.

79 Mr. Rosenthal and George Schnitzler, '21.

81 Steven R. Gordon, '70, and Pauliine Orwin (wife of Milton O. Orwin, '23) found a flaw in the published solution. The following letter from Jeffrey S. Passel, '69, President of the M.I.T. Bridge Club, clears up the mystery entirely:

I have enjoyed your column ever since I've been reading *Tech Engineering News*. I usually cannot work the math problems, but I enjoy trying. The ones I enjoy most are the bridge problems, and I would like to see more of them. One thing, however: the solution you printed to last April's bridge hand (**81**) is incorrect. The hand is:

	♠ 5 ♥ 8 5 ♦ A K 7 ♣ A K 8 6 4 3 2	
♠ K 10 7 ♥ 9 ♦ Q 10 8 3 ♣ Q J 10 9 7		♠ 8 6 4 3 2 ♥ Q 7 6 5 ♦ J 6 2 ♣ 5
	♠ A Q J 9 ♥ A K J 10 4 3 ♦ 9 5 4 ♣ —	

The solution as printed is:

North to win first club, South throwing diamond. Trump finesse. Spade ruffing finesse, Club ruff (East is assumed to discard a spade). However, if East throws a diamond, he will beat the hand, as follows: Diamond to the ace. Club ruff (East throwing his last diamond). South must now lose a trick (via a ruff when he tries to get on the board with a diamond or just losing a trump trick).

The proper solution is more complex and involves two complex lines of play, as follows: North wins ♣ A, throwing diamond. North leads ♣ K. East has three choices:

1. East trumps, South over-ruffs. South leads ♠ A and ♠ Q, ruffing out West's king. Then a trump finesse. Trumps are pulled and south wins the remainder.
2. East throws a spade. The hand becomes a trump coup. South must ruff the ♣ K! ♠ A and ♠ Q ruffing if covered, leading another if not covered. Trump finesse. Cash remaining high spades. Diamond to ace. Club from board (if East ruffs, overruff, pull trump, and claim—East should throw diamond) ruff. Diamond to board, leading to this position:

	♠ — ♥ — ♦ 7 ♣ 8	
Irrelevant		♠ — ♥ Q 7 ♦ — ♣ —
	♠ — ♥ K J ♦ — ♣ —	

Lead a club for trump finesse.
3. The most complex line. East throws a diamond, the hand becomes a progressive squeeze against West. South throws a spade. Trump finesse. Diamond to the board. Second trump finesse. South plays his remaining trumps, throwing clubs from the board. Before the lead of the last trump, this is the position:

	♠ 5 ♥ — ♦ A 7 ♣ 8 6	
♠ K 10 ♥ — ♦ Q 10 ♣ J		♠ 8 6 4 3 ♥ — ♦ J ♣ —
	♠ A Q J ♥ 4 ♦ 9 ♣ —	

West must guard spades, high club, and diamonds. On the lead of the last heart, West is squeezed in three suits. If he throws a spade, South wins the last four tricks with ♠ A, ♠ Q, ♠ J, and ♦ A. If West throws a diamond, South throws a club from the board and cashes two diamonds on the lead of the last diamond, and West is squeezed again in clubs and spades.

	♠ 5 ♥ — ♦ — ♣ 8	
♠ K 10 ♥ — ♦ — ♣ J		♠ 8 6 ♥ — ♦ — ♣ —
	♠ A Q ♥ — ♦ — ♣ —	

West still must discard; if a spade, south wins ♠ A and ♠ Q; if a club, South wins club and ♠ A. If West throws a club, North throws a diamond and wins two club tricks and aces in spades and diamonds. The hand is far more complex than was analyzed in the last issue of *Tech Engineering News.*

As for Mr. Ciaramaglia's hand, this is almost a book example of a double-squeeze.

	♠ A 7 6 5 ♥ A 7 6 ♦ J 9 5 ♣ A K 10	
♠ K J 8 4 3 ♥ K J 8 5 4 ♦ 7 2 ♣ 5		♠ Q 9 2 ♥ Q 9 6 2 ♦ — ♣ Q J 9 6 3 2
	♠ 10 ♥ 10 ♦ A K Q 10 8 6 4 3 ♣ 8 7 4	

The play is as follows:

1. Win ♣A
2. ♠A
3. Spade ruff with ♦ A.
4. Diamond to the ♦ J.
5. Spade ruff with ♦ K.
6. High diamond (pulling last trump)
7. ♣ K

South now runs his trump. West must hold hearts and the high spade. East must hold hearts and the high club. This is the end position:

	♠ 7 ♥ A 7 ♦ — ♣ —	
♠ K ♥ K J ♦ — ♣ —		♠ — ♥ Q 9 ♦ — ♣ Q
	♠ — ♥ 10 ♦ Q ♣ 8	

On the lead of the last diamond, if West throws a spade, South can just win ♥A and ♠7. Therefore West must throw ♥J. There is no more need for the ♠7, so it is discarded from the dummy. It is now East's turn to be squeezed. If he throws the ♣ Q, South wins the ♣8 and ♥A. So East must throw the ♥ 9. South leads the ♥10 and wins the last trick with the ♥A and ♥7. The end position is an example of a simultaneous double squeeze. Thanks for the column and keep up the good work.

I should like to thank John E. Giffels, '14, for his kind words.

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