Puzzle Corner

By Allan J. Gottlieb, '67

Unfortunately, there is no issue of *Tech Engineering News* corresponding to the July TR. Therefore, in order to avoid confusion next year, I shall not print any new problems this month.

Several readers have asked what will happen to Puzzle Corner next year when I go to Brandeis. By popular request (i.e., from the editor and my mother), the column shall remain in my hands for at least one more year. I should again like to thank everyone for their concern.

John B. Joseph, '51, claims that the answer given for the "seven cigarettes" problem (#15) is wrong—the cigarette just cannot be made to touch as illustrated (Technology Review for March). It still looks right to me, but I would welcome hearing anyone else's opinion.

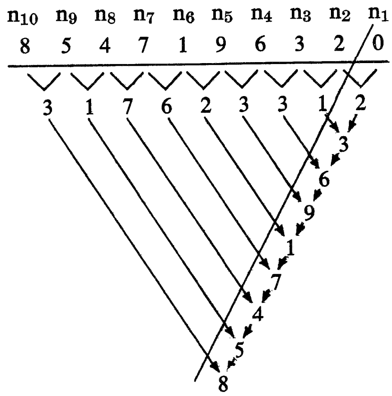
Solutions

68—Insert the remaining digits 7, 8, and 9 into the proper place in the following sequence:
—, 5, 4, —, 1, —, 6, 3, 2, 0.

The intended sequence was one determined by alphabetically ordering the words one, two, etc. This solution was found by George H. Ropes, '33. Eric Rosenthal, and Douglas Hoylman, '64. Benson P. Ho, '70, found a rather different correspondence. Its complexity is rivaled only by the difficulty the printer will have in setting the type:

From Mr. Ho:

My solution to problem 68 is: 8, 5, 4, 7, 1, 9, 6, 3, 2, 0. I think it's easier to draw than to explain it:



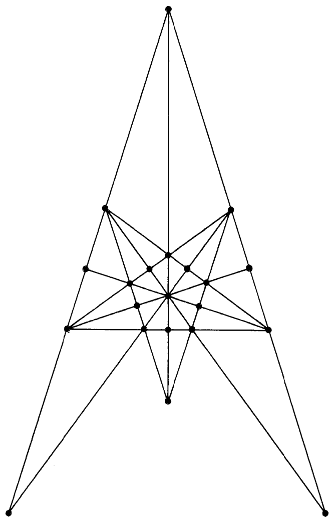
Key:

∨ —subtraction: $n_i - n_{i-1}$; if it is less than 0, then add 10 to it.

↘↙ —addition: add the two directed numbers; if their sum is greater than 10, subtract 10 from it.

69—Arrange 21 points so that they form 12 lines each having precisely 5 points.

The following artistic endeavor is from Ann Giffels, musician, the daughter of John E. Giffels, '14:



Also solved by Messers. Ropes and Joseph.

70—Find a relation among the betti numbers of the homology of a compact orientable non-bounded n-dimensional manifold. Do the same for the corresponding torsion subgroups. A hint: compute many "simple examples." Proofs were *not* required.

In case anyone is interested, $\beta_i = \beta_{n-i}$ and $T_i = T_{n-i-1}$.

71—Thomas F. Hickerson, '09, would like to know the minimum integral values of A, B, and C such that $A(A + 8) = B(B + 28) = C(C + 34)$.

Many readers solved this by brute force. One elegant solution was sent in by Neil K. Cohen, '69:

Let $B(B + 28) = C(C + 34)$
(Note that $C < B$, since x^2 is an increasing function and everything is integral.)
$B^2 - C^2 = 34C - 28B$
$B^2 - C^2 = (6C) + 28\,(C - B)$
$(B + C)\,(B - C) = (3)\,[(C - B) + (B + C)] + 28\,(C - B)$
Let $x = B - C$, $y = B + C$. Then
$xy = 3(y - x) - 28x$
$x(y + 31) = 3y$.
Since everything is positive and
$y + 31 > y$, $x < 3$.
If $x = 0$, $y = 0$, $B = C = 0$, illegal.
If $x = 1$, $2y = 31$, illegal.
Therefore $x = 2$, $y = 62$ and $B = 32$ and $C = 30$.
$A^2 + 8A = (32)(60)$.

$$A = \frac{-8 + \sqrt{64 + (4)(32)(60)}}{2} = 40.$$

This must be the unique as well as minimal solution since 2 is the only possible value of x.

Also solved by Messers. Roper, Joseph, and Rosenthal, Thomas B. Jabine, '48, Mark H. Yu, '70, and Mrs. Marion Giffels (Do you have a daughter Ann?).

72—Arthur Mohan, '08, sent in the following problem:

In how many ways may "m" different

blue books and "n" different red books be arranged on a shelf so that no two of the red books are together? What inequality must "m" and "n" satisfy? This is supposed to be solved without the use of any theory of groups or sets.

According to Mr. Mohan, a text he has used gives the answer as $m!(m+1)!/(m-n+1)!$ and $m>n-2$. Well, everybody agrees with $m>n-2$, but four other people sent in solutions and none of them agrees with the text or any of the others! Rather than throw my two cents' worth into that rapidly increasing pot, I shall leave the issue as it stands. As a final comment, my roommate Steven M. Slutsky, '68, obtained the same answer as Mr. Mohan in about half a minute but then noticed he had confused the red and blue books. As of press time no further pearls of wisdom have poured forth.

73—Show that if $2^p - 1$ is a prime then $2^{p-1}(2^p - 1)$ is perfect.

Doug Hoylman gave a neat solution: Since $2^p - 1$ is prime, $2^{p-1}(2^p - 1)$ is already decomposed into prime factors. Hence the divisors of this number other than itself are $1, 2, 4, \ldots 2^{p-1}, 2^p - 1, 2(2^p - 1), \ldots, 2^{p-2}(2^p - 1)$ and the sum of these is

$$\sum_{n=0}^{p-1} 2^n + \sum_{n=0}^{p-2} 2^n(2^p - 1)$$

$$= 2^p - 1 + (2^{p-1} - 1)(2^p - 1)$$
$$= 2^{p-1}(2^p - 1)$$

Hence the number is perfect. (This suggests a related problem: If $2^p - 1$ (or, in general, $m^p - 1$) is prime, show that p is prime.)

Also solved by Mr. Rosenthal.

Better Late Than Never

Eric Rosenthal has sent in solutions for #63 and #68, and Capt. Roger A. Whitman, '61, solved #63.