Puzzle Corner

By Allan J. Gottlieb, '67

I have received letters from several very considerate readers asking about my graduate school acceptances. I should like to thank them very much. The final outcome was that after much deliberation, I decided to accept the advantages of small classes and a new location over those of a somewhat better name (i.e., I respectfully declined M.I.T.'s offer in favor of one from Brandeis).

Several people have requested that I mail to them solutions to various problems. The advantage of mass-media communications is that many can be reached at once. In order not to surrender this advantage, and because I do still have some academic involvements, I have ignored all such letters.

Problems

78—The first problem is from Lester R. Steffens, '30.

Given: 11 balls of same weight plus one heavier or lighter (12 total). Identify the odd ball and determine whether it is heavier or lighter with three weighings on a two-pan balance. This problem and the solution have been published frequently. To make it more difficult, what is the maximum number of balls from which the odd one can be identified with four weighings? with five weighings? with n weighings?

79—The following problem is from John A. Maynard, '46.

Five island natives spend all day gathering coconuts. They finish the job when it is dark and decide to leave the coconuts in a pile and come back in the morning to divide them up. At midnight one native wakes up and decides he wants his 1/5 now. He goes to the pile, divides it into five equal piles, and finds that there is one coconut left over. He gives it to a monkey. The native takes away one of the five piles, leaving the other four piles there. He pushes the four piles into one big pile as he leaves. At 1 A.M. native #2 wakes up and decides he wants his 1/5. He goes to the pile, divides it into five equal piles, finds one coconut left over, gives it to the monkey, takes away one pile, and leaves the rest. At 2 A.M. native #3 does the same thing; divides into five equal piles, one nut left over goes to the monkey, takes away one pile. At 3 A.M. native #4 does the same thing. At 4 A.M. native #5 does the same thing. After native #5 takes his coconuts away, the quantity of nuts left is evenly divisible by five.

Question: How many coconuts were gathered by the natives?

80—I have received the following letter:

The Stamp Problem: Suppose the government wants to revise its postal system and create only seven denominations of stamps. To aid in automatic processing, a maximum of only three stamps is to be permitted on an envelope (two or one are also permitted). Up to what value of postage will this cover without a break? What are the denominations of the stamps required to achieve this?

Answers: 70 cents using stamps of the following denominations: 1,4,5,15,18,27,34 cents respectively. I proved this result using a brute force technique on a digital computer. When I first heard this problem five years ago, the goal was $1.00. It seems that eight denominations using three maximum will get to 93 cents, but no higher. I believe that six denominations using four maximum will achieve the $1.00 result and I am currently working on this. What has bothered me about this problem, however, is the lack of my own mathematics in solving this general class of problem without resorting to the brute force method. Perhaps you know of some techniques which would be helpful here, or could put me in touch with someone similarly interested. I would appreciate hearing from you in this regard.

Sincerely yours,
Richard L. Heimer, '56

Can anyone help Mr. Heimer out?

81—Bridge problems seem popular. This is from Russell A. Nahigian, '57.

Here is one of my favorite non-mathematical puzzles I'm sure your readers will enjoy. It is a bridge problem where all four hands may be viewed.

North
♥ 8 5
♠ 5
♣ A K 8 6 4 3 2
♦ A K 7

West
♥ 9
♠ K 10 7
♣ Q J 10 9 7
♦ Q 10 8 3

East
♥ Q 7 6 2
♠ 8 6 4 3 2
♣ 5
♦ J 6 2

South
♥ A K J 10 4 3
♠ A Q J 9
♣ —
♦ 9 5 4

Bid seven hearts by South. Lead Queen of Clubs.

Problem: Make bid against *any* defense (after the Club lead).

82—The last regular puzzle for this issue is another problem from *pure mathematics*. The unit disk E^2 is defined as the set of all points (x,y) such that $x^2 + y^2 < 1$ (x and y are real numbers). Prove the Brower fixed point theorem which asserts that every continuous function f from E^2 to E^2 has a fixed point, i.e., there is an x in E^2 such that $f(x) = x$. Naturally, this theorem is proved in many books—no fair peeking.

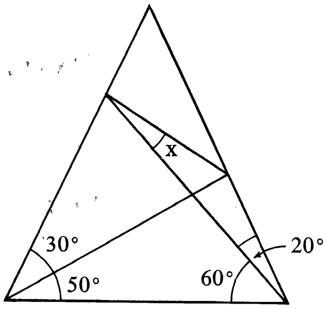
Speed Department

83—A Pythagorean triangle is a right triangle in which each of the sides is of integral length. Show that the radius of the inscribed circle in any Pythagorean triangle is of integral length.

84—Here is one from my own recent personal experience. Why do nice guys always finish last?

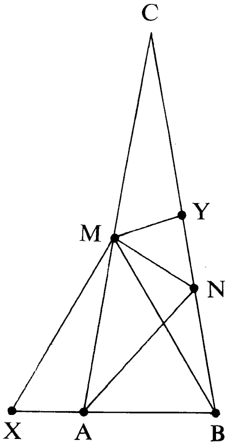
Solutions

32—Roger W. O'Dell, '68, wants you to find x:



The following is John McNear's.

Does this qualify for a year's supply of TEN? (And how!—ed.)



1) BX = BM = BY
2) <MBX = 60
3) △MBX is equilateral
4) <YBM = 20
5) <BYM = <BMY = 80
6) <BAM = 80
7) <CYM = <MAX = 100
8) <MBA = 60
9) <BMA = 40
10) <CMY = 60
11) <CBA = 80
12) <BCM = 20
13) CM = BM
14) △CYM is congruent to △MAX
15) YM = AX
16) <BAN = 50
17) <BNA = 50
18) BN = BA
19) AX = BX − BA = BY − BN = YN
20) YM = YN
21) <YNM + <YNM = <CYM
22) <YMN = 50
23) <NMB = 30

Also solved by Alan S. Ratner, Craig W. Johnson, '70, Erich S. Kranz, Mark Yu, and Arthur Mohan.

33—What is the least number of queens that can be placed on a chessboard in a manner such that any additional queen would result in three being lined up?

Mark Yu, David G. Sitter, '69, John Joseph, and Richard D. Minnick all found solutions with 12 queens. Arthur Delagrange, Peter Eirich, and Douglas Hoylman found solutions with only

(*Continued on page 9*)

Review on Books

A Matter of Life . . . .

By Joseph Mindel

—Why should I listen to you? You deal in words. If they turn out to be wrong, you can always find new ones.
—And you, what do you deal in?
—My life!
—Then we are not so different, you and I.

Language and Silence (Atheneum, New York, 1967, 426 pages, $8.00) by George Steiner, Fellow and Director of English Studies at Churchill College, Cambridge, and currently Schweitzer Visiting Professor at New York University, offers more than is indicated by the sub-title, "Essays on Language, Literature, and the Inhuman." The essays, written largely during the past five years, are concerned with the crisis of the novel, the role of the literary critic, pornography, the humanities, Marxism and literature, Homer, the Bible, Shakespeare, Thomas Mann, Kafka, Günter Grass, Trotsky, and a variety of other topics and people, listed in an excellent 31-page index.

These are the subjects; but the book's theme, which is approached from many different directions, is the role of language in our culture. "What are the relations of language to the murderous falsehoods it has been made to articulate and hallow in certain totalitarian regimes? Or to the great load of vulgarity, imprecision, and greed it is charged with in a mass-consumer democracy? How will language . . . react to the increasingly urgent, comprehensive claims of more exact speech such as mathematics and symbolic notation?"

The unusual quality of the book arises from Mr. Steiner's direct, personal involvement in its subject and theme. The questions and the attempted answers are clearly as important to him in his own life as he believes them to be in the life of society. He does not arrive at general principles that he does not apply to himself. "This is how the world appears to me," he says, in effect. "This is how I believe a man—that is, I—should try to live in it." It is left to the reader to agree or not, but if he reads at all, he cannot refuse to engage.

Mr. Steiner finds a "retreat from the word" that manifests itself in many ways. The success of mathematics in explaining the workings of the universe has tempted historians, economists, and sociologists to substitute formulas, tables, and graphs for words; the ease with which they have succumbed to temptation is notorious. Modern art reveals a similar retreat. A landscape, a still life, a portrait can be described in words, but if a painter paints what he feels, not what he sees, communication occurs outside of language. Mr. Steiner finds another diminution of language in the thinness of the Hemingway style and of much recent writing. On another level, comic strips, the mass media, and advertising replace words by pictures and music, reducing still further the already low popular literacy.

These are trends, forecasts of what might come to pass. In his controversial essay, "The Hollow Miracle," Mr. Steiner is concerned with what did happen. He analyzes with frightful, frightening illustrations the role of the German language in Hitlerism, and he concludes that it "was not innocent of the horrors of Nazism. It is not merely that a Hitler, a Goebbels, and a Himmler happened to speak German. Nazism found in the language precisely what it needed to give voice to its savagery. . . . A language in which one can write a 'Horst Wessel Lied' is ready to give hell a native tongue."

His purpose extends beyond exploring the relation of a language to history and culture. He inquires into the value of the study of the humanities. "We know now that a man can read Goethe or Rilke in the evening, that he can play Bach and Schubert, and go to his day's work at Auschwitz in the morning. . . . We do not know whether the study of the humanities, of the noblest that has been said and thought, can do very much to humanize. We do not know . . ." A teacher himself, he suggests that the failure may lie in teaching literature as a fact of the past, without considering its relevance in the present; that is, in confusing objectivity with neutrality. "Science can be neutral. . . . A neutral humanism is either a pedantic artifice or a prologue to the inhuman."

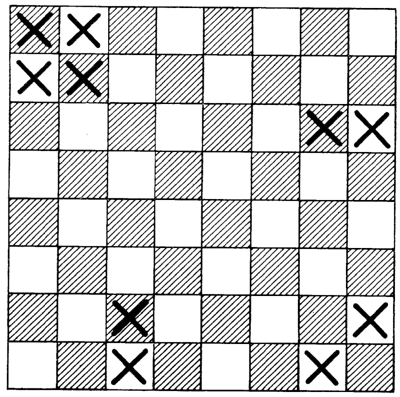
Running through the essays is the theme of silence, gliding from one meaning to another: silence as a way of understanding and communicating the ineffable; silence as the last stage in the retreat from the word ("plays in which absolutely nothing is said"); the silence beyond the extreme limits of language, creating the richness of words unspoken and music unheard ("to speak is to say less"). And finally, silence as the decision of the poet to refuse to write: "It is better for the poet to mutiliate his own tongue than to dignify the inhuman either with his gift or his uncaring. . . . Silence *is* an alternative. When the words of the city are full of savagery and lies, nothing speaks louder than the unwritten poem."

The book engages the reader wholly, so that at times there is the impulse to argue, to point out, for example, that silence is nothing until it is endowed by words or acts with form and dimension. Or that whoever does not speak, poet or plumber, is not present. Or that even though it is possible to collect evidence from newspapers, from television commercials, from modern art and literature, from our own observations in the place where we are, to file in a folder labelled "Attila Is Coming," an observer can also find materials for another folder marked "But Not Yet."

Above all, we remember that for poets, for writers like Kafka, Mann, and Mr. Steiner as well, for whom language is a matter of life, to choose silence is to relinquish a portion of life. We may be thankful that, for now, Mr. Steiner has not chosen silence.

Puzzle Corner

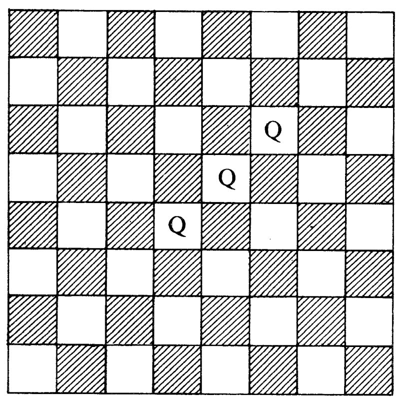
(*Continued from page 7*)
10 queens. The following is Art's.



And then there is the following:

Dear Mr. Gottlieb,

I believe that a glance at this diagram will show that it meets the conditions of your problem 33 as stated. Apparently accurate technical English is more difficult than it looks.



Sincerely,
W. Richard Ristow

(*Continued on page 68*)

between M.I.T. and the African Studies Program of Boston University, the School of Dental Medicine at Tufts University and the Woods Hole Oceanographic Institution.

There was enthusiastic response to the Wellesley-M.I.T. announcement on both campuses and observers at each school seemed to assume that the flow of students would be all in his direction. Presidents Adams and Johnson refused to speculate, emphasizing the plan's purpose of "extending the opportunities open to students in each school while maintaining the integrity of each home base."

The two presidents guessed that the plan when it first goes into effect may involve 60 to 80 students from each school. The program will be subject to continuing faculty review and to final evaluation after five years of experience.

President Johnson was at pains to assure M.I.T.'s 369 women students that he expected the plan to strengthen the position of full-time women students at M.I.T. "The success of M.I.T.'s coeds," he said, "led us to think that this new step would be appropriate."

"The achievement of increasing numbers of girls at M.I.T.," he said, "has convinced us that there need be no reluctance in admitting those who are adequately prepared and sufficiently motivated. We expect to increase the number of women students admitted at M.I.T., and we expect that many of them will have the rewarding experience of taking one or more courses at Wellesley."

Puzzle Corner

(Continued from page 9)

I suppose you're right, but how can my *glance* meet the conditions of Problem 33. Apparently conversational English is more difficult than it looks.

34—You sit south and must make 7 spades. West's opening lead is the king of clubs. How do you play the hand given the following distribution?

North (dummy)
♠ A K Q J
♥ A Q 8
♦ Q J 10 9 8 7
♣ —

West
♠ 5 4 3 2
♥ K J 10 9 7 6
♦ —
♣ K Q J

East
Immaterial

South (declarer)
♠ 10 9 8 7 6
♥ —
♦ A K
♣ A 10 9 8 7 6

Here is Richard Minnick's solution.

Dear Mr. Gottlieb:

I enjoy your column in Technology Review each month. However, I find most of the problems too taxing for my lesser mathematics (S.M./S.B. Industrial Management, '66)—or, rather I should say that I do not have enough time available in my busy schedule! Nevertheless, I was amused by Problem 34 enough to (I think) see it through:

Given the disclosure of West's hand and therefore East's, one does not have to protect against the rare chance that East may be void in Clubs on the first trick. West opens with the King of Clubs which is taken by South's Ace of Clubs, while dummy discards a diamond. A Spade is then led to dummy, drawing West's first trump. The dummy cashes the Ace of Hearts, with South discarding a Diamond. Dummy then leads a Heart which is trumped by South. South again leads a Spade toward dummy. The third Heart is returned and ruffed by South. South then leads his last Spade to dummy. North returns the last Spade, declarer discards the second Diamond and West plays his last Spade. At this point North has five Diamonds, South has five Clubs and West is left with two Clubs and three Hearts, thus making dummy's five Diamonds good.

Keep up the good work. I'd like to see some more Bridge problems.

♦63—What is the remainder when 5^{100} is divided by 101?

The following is from James S. Kaltenbronn, '60.

Dear Mr. Gottlieb:

Occasionally we chemists can work out some of your problems. Enclosed are solutions to problems 63 and 64 posed in the April, 1967, issue of Technology Review. These solutions were arrived at with the help of Duane Morrow.

Problem 63: Since 101 is a prime, $5^{100}/101$ is simply an example of Fermat's theorem, which states that for any prime p and any integer a not divisible by p, $a^{p-1} \equiv 1 \bmod p$; that is, the remainder is 1.

This theorem is easily proven: any integer a not divisible by p is congruent mod p to one of the integers 1,2,3, . . ., p—1. If p is a prime, it is readily seen that the series of integers a, 2a, 3a, . . ., (p—1)a are congruent mod p, in some order, to the series of integers 1, 2, 3, . . ., p—1. Therefore, by elementary algebra of congruences, $(a)(2a)(3a) \ldots [(p-1)a] \equiv (1)(2)(3) \ldots (p-1) \pmod p$, or $a^{p-1} (p-1)! \equiv (p-1)! \bmod p$.

Since p is a prime, (p—1)! and p must be relatively prime, so that $a^{p-1} (p-1)!/(p-1)! \equiv [(p-1)!/(p-1)!] \bmod p$ or $a^{p-1} \equiv 1 \bmod p$.

Also solved by Douglas J. Hoylman, '64, Robert L. Knighten, '62, John L. Joseph, '40, Henry S. Lieberman, '61, and Mark H. Yu, '70.

64—A courier is in the rear rank of a column one mile long. He leaves his position to deliver a message to his commanding officer in the front rank, and then returns to his original position, arriving there precisely at the same moment that the column has reached one mile. How far did the courier walk? (It must be assumed that the courier walked at a constant rate of speed, and the column at a constant though different rate of speed.)

Mr. Lieberman sent in the following:

As for Mr. Moore's problem, it's intriguing but readily solved with a bit of elementary algebra. Wherefore, let $c =$ the speed of the courier and $m =$ the speed of the column (in miles per hour).

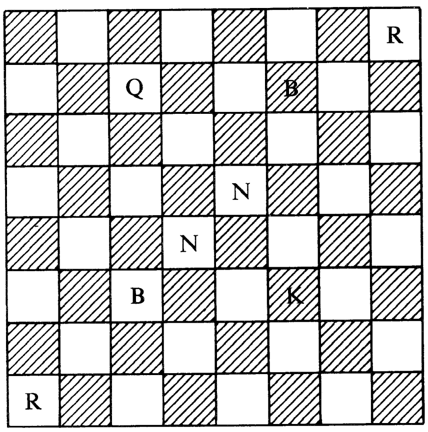
1. The time it takes for the column to move ahead one mile is simply $1/m$ hours, and in this same amount of time the courier has walked $c(1/m) = c/m$ miles. It is therefore the ratio of the speeds that we must discover.
2. The courier takes $1/(c-m)$ hours to reach the front of the column (clearly $c > m$) and $1/(c+m)$ hours to get back to the rear of the column. Thus the total time of his walk is $1/(c-m) + 1/(c+m)$ hours.
3. According to the condition of the problem, the total amount of time that the courier spends walking is equal to the amount of time it takes the column to advance one mile, i.e. $1/(c-m) + 1/(c+m) = 1/m$. Therefore, $(c/m)^2 - 2(c/m) - 1 = 0$. Hence, $c/m = 1 + \sqrt{2}$ miles.

Please never let the Puzzle Corner lose any of its spirit. It's a wonderful diversion. Moreover, do not lose this letter!

I'll do my best, Henry. Also solved by Messers Joseph, Hoylman, Yu, Kaltenbronn, Kenneth B. Blake, '13, the wife of Martin S. Lindenberg, '39 (thanks for the kind word, Mary), Guy H. Trindell, '66, D. P. Gaillard, '11, and H. Kelsea Moore, '32.

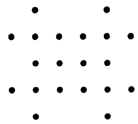
65—Arrange eight chess pieces so that a maximum number of squares, including those on which the pieces are situated, is guarded. You are allowed to place the two bishops on squares of the same color.

Robert Sinnott submitted the only solution.



66—Peter L. Eirich, '69, wants to see 24 dots arranged to form 24 line segments of four dots each.

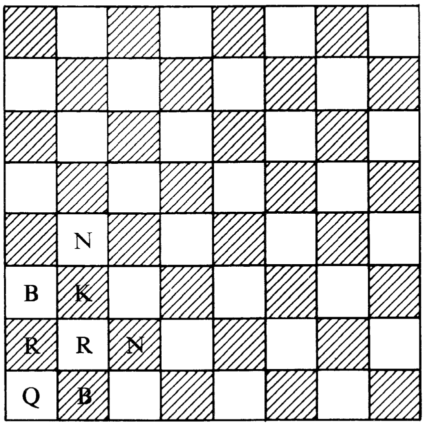
Mr. Hoylman found a solution with 26 segments.



Also solved by Mr. Joseph.

67—Arrange the eight pieces so that a minimum number of squares is guarded. (The count must necessarily include absence or presence of guard of the squares on which the pieces themselves are situated.)

Mark Yu managed to cover only 17 squares.



Also solved my Messers Hoylman and Sinnott.

Better Late Than Never Department

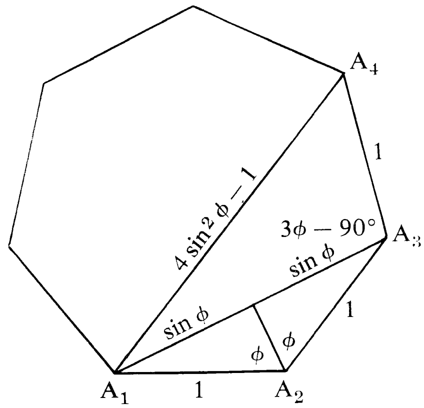
12—Arthur D. Delagrange, '61, and W. Whitticar.
22—Eric Rosenthal ('73?).
24—Eric Rosenthal.
25—Frank G. Smith, '11, J. Spotto McDowd, '16, Arthur F. Mohan, '08, and Eric Rosenthal.
26—Daniel S. Diamond, '65.
27—Eric Rosenthal.
28—Mark Yu and Eric Rosenthal.
30—Jon Livingston, '68, John A. Maynard, '46, Martha Morrison (The William School), Lester Steffens, '30 (six shapes), Robert Goodstein, '46, Yukikazu Iwasa, '61, and Eric Rosenthal.
32—William R. Osgood, '19, and Eric Rosenthal.
33—Martha Morrison, Jon Livingston, and George H. Wiswall, '19.
34—Joseph Kozol, '54, Ralph Seferian, '44, John A. Maynard, Yukikazu Iwasa, Robert Goodstein, Henry B. Cochran, '61, Homer S. Davis, '24, Capt. Allan J. MacLaren, '60, Leon M. Kaatz, '64, and A Honolulu Housewife (a very cool letter; I'm sorry I haven't the room to print it.)

Problems were submitted by John A. Maynard, Russell A. Nahigian, and Richard L. Heimes, '56.

Better Late Than Never (Part II)

Eric Rosenthal has bailed me out by sending me a copy of his February solutions. As you may remember, the original versions have disappeared. Thanks, Eric.

22a—Don B. Zagier, '70, would like to know, given a regular n-gon $A_1A_2 \ldots A_n$ having the property that $1/A_1A_2 = 1/A_1A_3 + 1/A_1A_4$, what is n?



Let each angle of the polygon be 2ϕ. Draw A_2P perpendicular to A_1A_3. If each of the sides of the polygon is the unit length, A_1P and PA_3 will both have length $\sin\phi$, giving A_1A_3 length $2\sin\phi$. A_2P bisects angle $A_1A_2A_3$ because A_2P is the altitude of isosceles triangle $A_1A_2A_3$. Since angles $A_1 + A_2 + A_3 = 180°$ and angle $A_1 =$ angle A_3, then angles $A_2 + 2A_3 = 180°$, or angle $A_3 = \frac{1}{2}(180° - A_2) = \frac{1}{2}(180° - 2\phi) = 90° - \phi$. From this angle $A_1A_3A_4 = 2\phi - (90° - \phi) = 3\phi - 90°$.

By the law of cosines in triangle $A_1A_3A_4$,

$A_1A_4^2 = (2\sin\phi)^2 + 1^2 - 2(2\sin\phi)(1)\cos(3\phi - 90°)$
$= 4\sin^2\phi + 1 - 4\sin\phi\cos(3\phi - 90°)$
$= 4\sin^2\phi + 1 - 4\sin\phi\sin 3\phi$

Now, $\sin 3\phi = 3\sin\phi - 4\sin^3\phi$, so

$A_1A_4^2 = 4\sin^2\phi + 1 - 4\sin(3\sin\phi - 4\sin^3\phi)$
$= 16\sin^4\phi - 8\sin^2\phi + 1$
$= \sqrt{16\sin^4\phi - 8\sin^2\phi + 1}$
$= 4\sin^2\phi - 1$

Since $1/A_1A_2 = 1/A_1A_3 + 1/A_1A_4$,

$1/1 = 1/2\sin\phi + 1/(4\sin^2\phi - 1)$,

$$\frac{4\sin^2\phi - 1 + 2\sin\phi}{2\sin\phi(4\sin^2\phi - 1)} = 1$$

$2\sin\phi(4\sin^2\phi - 1) = 4\sin^2\phi - 1 + 2\sin\phi$

$8\sin^3\phi - 2\sin\phi = 4\sin^2\phi - 1 + 2\sin\phi$

$8\sin^3\phi - 4\sin^2\phi - 4\sin\phi + 1 = 0$

Solving the above for $\sin\phi$ (only for the interval $0 < \sin\phi < 1$, because ϕ is a nonzero positive acute angle), these approximate values are found:

$\sin\phi \approx 0.2$
$\sin\phi \approx 0.901$

Because ϕ is acute, these values give, respectively:

$\phi \approx 12°$
$\phi \approx 64°$

If $\phi = 60°$, $2\phi = 128°$, since $128^{(4/7)°}$ is the angle of a regular heptagon, this is the only possible solution, for there is no regular polygon with an angle of 24°.

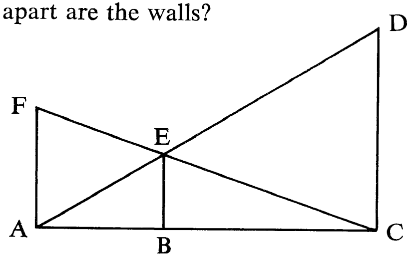
Puzzle Corner

To check,
$\frac{1}{2} \sin 64^{(2/7)°} + 1/(\sin^2 64^{(2/7)°} - 1)$ should equal 1.
$\approx \frac{1}{2} \csc 64^{(2/7)°} + 1/(4 \sin^2 64^{(2/7)°} - 1)$
$\approx \frac{1}{2}(1.1059) + 1/[4(.81171) - 1]$
$\approx .5530 + 1/(3.24684 - 1)$
$\approx .5530 + 1/2.24684$
$\approx .5530 + .4451$
$\approx .998 \approx 1.$

24—Find the missing element in the following sequence (whose rightward continuation is undefined): 10, 11, 12, 13, 14, 15, 16, 17, 20, 22, 24, __, 100, 121, 10000.

The missing number is 31. The series consists of the representation of 16 $(1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1)$ in bases 16_{10} to 2_{10}.

25—Two ladders 30 and 40 feet long are set between two vertical walls, the tops of the ladders on opposite walls. They cross at 10 feet above the horizontal between the walls. How far apart are the walls?



Given AD = 40, CF = 30, EB = 10, FA perpendicular to AC, EB perpendicular to AC, and DC perpendicular to AC. Find AC (to be called d).

By the Pythagorean theorem,
$CD = \sqrt{40^2 - d^2} = \sqrt{1600 - d^2}$, and
$AF = \sqrt{30^2 - d^2} = \sqrt{900 - d^2}$.
$1/AF + 1/CD = 1/BE$ (1)

Proof:
$AF/AC = BE/BC$, by similar triangles (a)
$BC = (AC - BE)/AF$ (b)
$CD/AC = BE/AB = BE/(AC - BC)$, by similar triangles (c)
$CD/AC = BE/[AC - (AC-BE)/AF]$, substituting (2) (d)
$CD/1 = BE/(1 - BE/AF)$ (e)
$BE = CD - (CD - BE)/AF$ (f)
$AF \cdot BE = AF \cdot CD - CD \cdot BE$ (g)
$AF \cdot BE + CD \cdot BE = AF \cdot CD$ (h)
$(AF + CD)\ BE = AF \cdot CD$ (i)
$BE = (AF \cdot CD)/(AF + CD) = 1/(1/AF + 1/CD)$ (j)
$1/BE = 1/AF + 1/CD$ (k)

From (1) is obtained
$1/\sqrt{1600 - d^2} + 1/\sqrt{900 - d^2} = 1/10$
$1/\sqrt{100\,(16 - d^2/100)} + 1/\sqrt{100\,(9 - d^2/100)} = 1/10$

Let $y = d^2/100$: (2)
$1/\sqrt{100(16 - y)} + 1/\sqrt{100(9 - y)} = 1/10$
$1/10\sqrt{16 - y} + 1/10\sqrt{9 - y} = 1/10$
$1/\sqrt{16 - y} + 1\ \sqrt{9 - y} = 1$
$1/\sqrt{16 - y} = 1 - 1/\sqrt{9 - y} = (\sqrt{9 - y} - 1)/\sqrt{9 - y}$ (3)

Squaring both sides:
$1/(16 - y) = (10 - y - 2\sqrt{9 - y})/(9 - y)$
$(16 - y)\ (10 - y - 2\sqrt{9 - y}) = 9 - y$
$y^2 + 160 - 26y - (32 - 2y)\sqrt{9 - y} = 9 - y$
$y^2 - 25y + 151 = \sqrt{9 - y}\ (32 - 2y)$ (4)

Again squaring,
$y^4 + 625y^2 + 22801 - 50y^3 + 302y^2 - 7550y = (9 - y)\ (4y^2 - 128y + 1024)$
$y^4 - 50y^3 + 927y^2 - 7550y + 22801 = -4y^3 + 164y^2 - 2176y + 9216$
$y^4 - 46y^3 + 763y^2 - 5374y + 13585 = 0$ (5)

All useful values of d are in the interval $0<d<30$ (distances are positive; $AC<CF$ because the hypotenuse of a right triangle is greater than a leg):
$0<d<30$
$0<d^2<900$
$0<d^2/100<9$
$0<y<9$

Horner's method gives $y = 6.775+$. $y = 6.775684$ is obtained from a linear approximation:
$d^2/100 \approx 6.775684$
$d^2 \approx 677.5684$
$d \approx 26.03.$

Substituting in the original equation, $d = 26.03$ gives BE a length of 10 to 5 significant figures. The distance between the walls is approximately 26.03 feet.

Congratulations on an excellent column. Since Technology Review is sent through the mail, you may be required to place a warning that "these problems may be habit-forming" and that "repeated use may be detrimental to your work."

I still need solutions to 21 and 23. Any takers?