

Puzzle Corner

By Allan J. Gottlieb, '67
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After a year's experience in *Tech Engineering News*, Puzzle Corner now also appears in *Technology Review*. So I have the honor and privilege (or so I was told) to see that there are several puzzles in each issue. Being a basically lazy individual, I would prefer to be deluged with puzzle suggestions. If necessary, however, I have a supply of my own.

I will print solutions in the second issue following the one in which the problems are published, and I'll print the name(s) of those who send correct solutions (sorry, no partial credit). To snow your family and make me feel the warmth of popularity, send in your solutions . . . and your problems, too. Reach me at Box 4380, Baker House, M.I.T., or if for some reason you feel an urgent need to communicate, try Institute extension 3161 (M.I.T.'s number is 617-864-6900).

The problems vary in difficulty. Some will be so easy even I can do them. Others will be more challenging; and occasionally, a problem will appear for which I have, as yet, no solution. The latter will be noted by a diamond (♦).

Here We Go

1—The problem:

W A S M A R C H
+ T H E B E S T
C A N D I D A T E

And the product H•E•R•B•I•D equals zero.

This problem was first run in *Tech Engineering News* last spring, and I am informed that there are several trivial solutions; for example, all the characters may be zeros. To force a unique solution, I now add the condition that M, which does not equal zero, equals C•W.

2—♦ I have received the following letter:

Dear Mr. Gottlieb:

We have discovered (we think) an interesting chess problem which you and your readers may find amusing. It is moderately difficult, but there is a solution. Given the eight rear-rank pieces, place them on a board in such a way that they cover every square (i.e., any piece of the opposing color placed anywhere on the board may be taken in one move). The two bishops may not be of the same color.

Lawrence Ribbecke, '69
Mitchell Wand, '69

I am not sure whether one must pro-

tect his own pieces. The reader may attempt to solve the problem either way.

3—Simplify the following (a handbook will be helpful and is permitted):

$$\int_1^{\infty} \lim_{z \rightarrow \infty} \left(1 + \frac{1}{z}\right)^z \left(\frac{\pi}{2} - \tan^{-1}x + \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(2k+1)x^{2k+1}}\right) dx - \frac{1}{2}(e^{i\alpha} - e^{-i\alpha})^2 + \cos 2\alpha - \sum_{n=0}^{\infty} \frac{\cosh y \sqrt{1 - \tanh^2 y}}{\left(\sum_{j=0}^{\infty} \frac{\cosh \gamma \sqrt{1 - \tanh^2 \gamma}}{2^j}\right)^n}$$

4—What nonzero five-digit number has its digits reversed when multiplied by 4?

5—Prove the well-known theorem in geometry that if two angle bisectors of a triangle are equal, then the triangle is isosceles.

The Speed Department

6—If a chicken and a half lays an egg and a half in a day and a half, how many eggs do six chickens lay in six days?

7—The following is false:

$$VII = I$$

Move one line to form a true equality.

Solutions and Discussion

Here are two earlier problems, now mostly solved, from last April's *Tech Engineering News*:

22—Let (a_α) be a non-empty set of reals. Define the distance set of (a_α) to be $(b$ s.t. $b = a_\beta - a_\gamma)$. What can be said about distance sets (measure, open, closed, connected, etc.)? In particular, what are some necessary and/or sufficient conditions for a set A to be the distance set of any set? For example, A must contain 0 and A cannot be $(0,1,3)$.

Despite the fact that I offered a free subscription to *Tech Engineering News* for the best solution to this problem, I did not receive any solutions. Now the offer is extended, and the prize becomes the reader's choice of free subscriptions to *TEN* or *Technology Review*.

25—Daniel S. Drucker, '67, wants to know the last three digits of 7^{9999} . Richard Haberman, '67, solved this one by constantly reducing mod 1000; his solution is as follows:

$$\begin{aligned} 7^0 &= 1 \\ 7^1 &= 7 \\ 7^2 &= 49 \\ 7^3 &= 343 \\ 7^4 &= 2401 = 401 \end{aligned}$$

$$\begin{aligned} 7^5 &= 16807 = 807 \\ \therefore 7^4 - 7^0 &= 400 \\ 7^4 (7^4 - 7^0) &= 400 \text{ since } 7^4 = 401 \\ 7^8 &= 7^4 + 400 = 801 \\ \therefore 7^{12} &= 201 \\ \therefore 7^{16} &= 601 \\ 7^{20} &= 1 \\ \therefore \text{repeats every } 20 - \text{find mod } 20 \\ 20 \overline{) 9999} &\text{ mod } 19 \\ (\text{i.e., } 9999 &\equiv 19 \pmod{20} - \text{ed.}) \\ 7^3 &= 343 \quad 7^3(7^4 - 7^0) = 343(400) = 200 \\ \therefore 7^7 &= 543 \\ \therefore 7^{11} &= 743 \\ 7^{15} &= 943 \\ 7^{19} &= 143 \end{aligned}$$

Answer: 143

The solution given by the proposer is so completely different that I shall print it as well:

$$7^{9999} = \frac{7^{4 \cdot 2500}}{7} = \frac{(2400 + 1)^{2500}}{7}$$

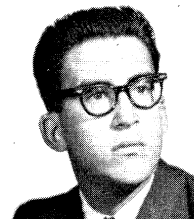
then, using the binomial expansion and sufficient gymnastics,

$$\begin{aligned} &= \frac{10^3(k+1)}{7} \text{ for suitable integer } k \\ &= \frac{10^3(k-1) + 1001}{7} = \frac{10^3(k-1)}{7} + 143 \end{aligned}$$

7 divides $(k-1)$, so $\frac{10^3(k-1)}{7}$ is a multiple of 10^3 .

\therefore the last three digits are 143.

Allan J. Gottlieb is an M.I.T. senior majoring in mathematics; his home is in Elmont, N.Y. Puzzle Corner has attained wide popularity in *Tech Engineering News*.



Review on Books

(Continued from page 9)

istic by humanistic science. Spectator knowledge is needed as well as experiential knowledge, controlled experiments as well as noninterfering receptivity, abstraction and theorizing as well as the concreteness of direct experience, reduction of experience to laws, formulas and models together with the comprehensive perception of whole experiences, mechanistic as well as humanistic science. Dr. Maslow's conception of science includes all these aspects of experience and knowledge, not as separate, im-miscible categories, but all integrated with each other.

The enlargement of the traditional scientific world to include the world of subjective experiences introduces into the scientific canon new means of ac-

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