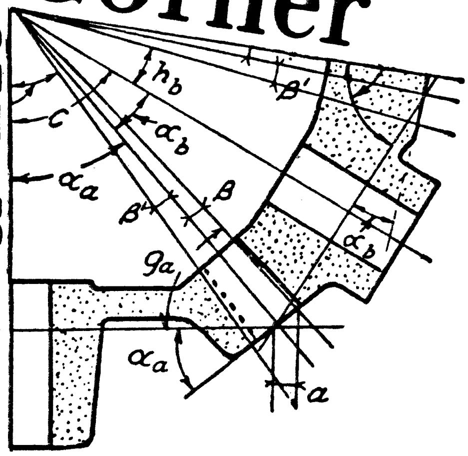
Puzzle Corner



Allan Gottlieb, '67

Hi, I sould like to reintroduce myself. My name is Allan Gottlieb, and, as your Puzzle Editor, I have the honor and privilege (or so I was told) to see that *TEN* has several puzzles each issue. Being a basically lazy individual, I would prefer to be deluged with puzzle suggestions. If necessary, however, I have a supply of my own. I will print solutions in the second issue following the one in which the problems are published. The name of any one who sends to me a correct solution will appear with the published solution (sorry, no partial credit). So to snow your date, to make the grad schools come crawling, and to make me feel that someone reads my column, send in your solutions. Send them via institute mail to Baker House 438. If you have a nickel you wish to get rid of, send the solution via US mail to me at

Kid's Place
Baker 438
362 Memorial Drive
Cambridge, Massachusetts 02139

If for some reason you feel an urgent need to speak to me, try dorm line 8-438, or institute extension 3161.

The problems will vary in difficulty. Some will be so easy even I can do them. Others will be more challenging; and occasionally, a problem will appear for which I have, as yet, no solution. The latter will be denoted by a diamond (◆).

problems

29 —

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  WASMARCH
+ THEBEST
 CANDIDATE
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The product H•E•R•B•I•D equals 0.

I have been informed that there are several trivial solutions to this problem, which was given in May 1966 as problem number 24. For example, all the characters may be zeros. To force the solution to be unique I am adding the condition that M, which does not equal zero, equals CxW.

◆30 —I have received the following letter:

Dear Mr. Gottlieb:

We have discovered (we think) an interesting chess problem which you and your readers may find amusing. It is moderately difficult, but there is a solution. Given the eight rear-rank pieces, place them on a board in such a way that they cover every square. (i.e., any piece of the opposing color placed anywhere on the board may be taken in one move.) The two bishops may not be of the same color.

Lawrence Ribbecke
Mitchell Wand
FroshHacKomm HQ
553 Lounge
Burton House

I am not sure whether one must protect his own pieces. The reader may attempt to solve the problem either way.

31 —Simplify the following:

$$\int_{1}^{\lim\limits_{z\to\infty}\left(1+\frac{1}{Z}\right)^{z}} \left(\frac{\pi}{2} - \tan^{-1}x + \frac{\sum\limits_{k=1}^{\infty}(-1)^{k+1}}{(2k+1)x^{2k+1}}\right)dx$$

$$-\frac{1}{2}(e^{i\alpha}-e^{-i\alpha})^2 + \cos 2\alpha$$

$$-\sum_{n=0}^{\infty} \frac{\cosh y\sqrt{1-\tanh^2 y}}{\left(\sum\limits_{j=0}^{\infty}\frac{\cosh\gamma\sqrt{1-\tanh^2\gamma}}{2^j}\right)^n}$$

CRC's are permitted. *SHOW ALL WORK.*

32 — What non-zero five digit number has its digits reversed when multiplied by 4?

33 — Prove the well-known theorem in geometry that if two angle bisectors of a triangle are equal, then the triangle is isosceles.

SPEED department

Do not bother submitting solutions for *Speed Department* problems. Each should take no longer than one minute.

This summer I worked for Grumman Aircraft (in future issues I'll tell you what the employees think of MIT work) and had to pass the following "qualifying examination" before I could obtain my "order of the practical" merit badge. Do you qualify?

34 — Assume you are in a boat floating in a strategic section of the bathtub with the water level at the top of the tub. You hear the enemy (mommy) coming. Just as she enters the room you take a brick, which happened to be in the boat with you, and toss it overboard. Naturally, it sinks to the bottom. Neglecting waves, what happens to the water level?

35 — After escaping from the enemy you now encounter an iceberg floating menacingly towards you. The enemy, after leaving the battle grounds, really turns on the heat. Since the home oil burner is more than adequate, the bathroom temperature rises sharply and the iceberg melts. Again, what happens to the water level?

solutions

These problems appeared in the April 1966 issue. The problems are reprinted to aid the reader.

22 — Let (a_α) be a non-empty set of reals. Define the distance set of (a_α) to be (b such that $b = a_\beta - a_\gamma$). What can be said about distance sets (measure. open. closed. connected. etc.)? In particular, what are some necessary and/or sufficient conditions for a set A to be the distance set of any set. For example, A must contain 0 and A cannot be (0,1,3). The best solution to the above problem will win a free subscription to TEN (*wow — ed.*).

Despite the fact that I offered a free subscription to *TEN* for the best solution to this problem, I did not receive any solutions. I intend to leave this issue in a state of limbo until I receive some solution. The offer of a free subscription still stands.

23 — Here's one practically everybody at MIT is working on already. Is it true that the boundary of any open subset of the plane must contain a subset which is homeomorphic to the open interval (0,1)?

The "little guy" (that is, Harvey Friedman, '67, for any new readers who might not know) tells me that an extensive library search has turned up a counter-example. He adds, however, that just describing the set is a major achievement. All that I was able to get out of him is that it has something to do with the concept of being infinitely squiggly (*whatever that means —ed.*).

24 —See problem 29 above.

25 — Daniel Drucker, '67, wants to know the last three digits of 7^{9999}.

Richard Haberman, '67, of Sigma Alpha Mu solved this one by constantly reducing mod 1000 (*Long live the Sammies. May they have continued success after they lose to Baker A in football —ed.*). His solution is as follows:

$$7^0 = 1$$
$$7^1 = 7$$
$$7^2 - 49$$
$$7^3 = 343$$
$$7^4 = 2401 = 401$$
$$7^5 = 16807 = 807$$
$$\therefore 7^4 - 7^0 = 400$$
$$7^4 (7^4 - 7^0) = 400 \text{ since } 7^4 = 401$$
$$7^8 = 7^4 + 400 = 801$$
$$\therefore 7^{12} = 201$$
$$\therefore 7^{16} = 601$$
$$7^{20} = 1$$
$$\therefore \text{ repeats every 20 — find mod 20}$$

$$20\overline{)9999} \text{ mod } 19$$
(*i.e., 9999 ≡ 19 (mod 20) — ed.*)

$$7^3 = 343$$
$$7^3(7^4 - 7^0) = 343(400) = 200$$
$$\therefore 7^7 = 543$$

$\therefore 7^{11} = 743$
$7^{15} = 943$
$7^{19} = 143$

Answer: 143

The solution given by the proposer is so completely different that I shall print it as well:

$$7^{9999} = \frac{7^{4 \cdot 2500}}{7} = \frac{(2400+1)^{2500}}{7}$$

then, using the binomial expansion and suitable gymnastics,

$$= \frac{10^3(k+1)}{7} \text{ for suitable integer } k$$

$$= \frac{10^3(k-1)+1001}{7} = \frac{10^3(k-1)}{7} + 143$$

7 divides $(k-1)$, so $\frac{10^3(k-1)}{7}$ is a multiple of 10^3.

$\therefore$ the last three digits are 143.

26 — Prove that 6 is the only square-free perfect number. *(Thank you, MAA).*

I had a little trouble understanding the MAA's solution so here is a slightly clarified form:

If N is squarefree and perfect, then $N = p_1 p_2 \cdots p_n$ where each p_k is a prime, $n > 1$ (no prime is perfect), $1 < p_1 < p_2 < \ldots < p_n$,

$$\sigma(N) = \prod_{i=1}^{n} \frac{p_i^2 - 1}{p_i - 1}$$

$$= (p_1+1)(p_2+1)\ldots(p_n+1),$$

and $\sigma(N) = 2N$. For odd squarefree N, $p_i + 1$ is even for all i so $4/\sigma(N)$. But $\sigma(N) = 2N$ so $2/N$ which is impossible. Hence N is even and p_1 is 2. Thus $2^n/\sigma(N)$. But 4 does not divide N (N is squarefree) so $n = 2$. Since $4p_2 = 2p_1p_2 = 2N = (p_1+1)(p_2+1) = 3(p_2+1)$, $p_2 = 3$, so $N = 6$. ∎