

The answer to the problem is that the fraction of time you should try new restaurants to maximize overall enjoyment is the square root of the death rate. The solution follows.

The problem is one of rates. Let us assume that the favorites die of at an average interval of N_d (measured in meals eaten out). The sampling of new restaurants must be sufficient to produce, on average, one new favorite over the same interval. If we measure the enjoyment (E) on a scale of 0 to 1, the enjoyment of a new restaurant will be assumed to be a random value from 0 to 1. It will also be assumed that the pool of new restaurants is unlimited. Let us now assume the favorites have a value of E_0 or higher. Since the distribution is linear, the average enjoyment value of the favorites will be

$$E_f = \frac{(1 + E_0)}{2}$$

and the average value of the rejects in the pool (values of less than E_0)

$$E_p = \frac{E_0}{2}$$

Now on average, for every $1/(1-E_0)$ samples, there will be one sample above E_0 , and this number of sampling visits must be taken in the interval N_d . Now for enjoyment purposes, one of the sampling visits was enjoyable, so the number of inferior members of the pool visited will be

$$N_a = \frac{1}{1 - E_0} - 1 = \frac{E_0}{1 - E_0}$$

and the average enjoyment will be:

$$E_{av} = \frac{(E_p \cdot N_a + E_f \cdot (N_d - N_a))}{N_d}$$

$$E_{av} = E_p \cdot \frac{N_a}{N_d} + E_f \cdot \left(1 - \frac{N_a}{N_d}\right)$$

$$E_{av} = \frac{E_0}{2} \cdot \frac{N_a}{N_d} + \frac{(1 + E_0)}{2} \cdot \left(1 - \frac{N_a}{N_d}\right)$$

$$E_{av} = \frac{E_0}{2} \cdot \frac{N_a}{N_d} + \frac{(1 + E_0)}{2} - \frac{N_a}{2N_d} - E_0 \cdot \frac{N_a}{2N_d}$$

$$E_{av} = \frac{(1 + E_0)}{2} - \frac{N_a}{2N_d}$$

$$E_{av} = \frac{(1 + E_0)}{2} - \frac{E_0}{2N_d \cdot (1 - E_0)}$$

$$E_{av} = \frac{1}{2} \cdot \left(1 + E_0 - \frac{E_0}{N_d \cdot (1 - E_0)}\right)$$

The average value is thus a function of E_0 and N_d . Assuming the death rate is constant, the value of E_0 that will maximize average enjoyment can be found by differentiating, and setting the derivative equal to zero

$$2 \cdot E_{av} = 1 + E_0 - \frac{E_0}{N_d \cdot (1 - E_0)}$$

$$2 \frac{dE_{av}}{dE_0} = 1 - \left(\frac{N_d \cdot (1 - E_0) - E_0 \cdot (-N_d)}{(N_d \cdot (1 - E_0))^2} \right)$$

$$2 \frac{dE_{av}}{dE_0} = 1 - \frac{1}{N_d \cdot (1 - E_0)^2} = 0$$

$$(1 - E_0)^2 = \frac{1}{N_d}$$

$$E_0 = 1 - \frac{1}{\sqrt{N_d}}$$

Now the problem asked for the fraction of the time you should try new restaurants, and this fraction is:

$$Frac. = \frac{\frac{1}{(1-E_0)}}{N_d} = \frac{1}{N_d \cdot (1 - E_0)} = \frac{1}{\sqrt{N_d}}$$

Now N_d is the interval between deaths, or the reciprocal of the the death rate, thus

$$Frac. = \sqrt{DeathRate}$$