Diamonds Are Forever: Reasoning about Heap Space in a Concurrent and Garbage Collected Language

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Iris'23

Formal Verification of Heap Space Bounds

Following Hofmann [1999], let $\Diamond 1$ represent one space credit.
$\rightsquigarrow$ the right to allocate one memory word.

Without GC:

- `alloc` consumes space

  $$\{\, \Diamond \mathit{size}(b) \,\}\; \ell := \mathtt{alloc}(b)\; \{\, \ell \mapsto b \,\}$$

- `free` produces space

  $$\{\, \ell \mapsto b \,\}\; \mathtt{free}(\ell)\; \{\, \Diamond \mathit{size}(b) \,\}$$

With GC: no syntactical point to recover space credits.

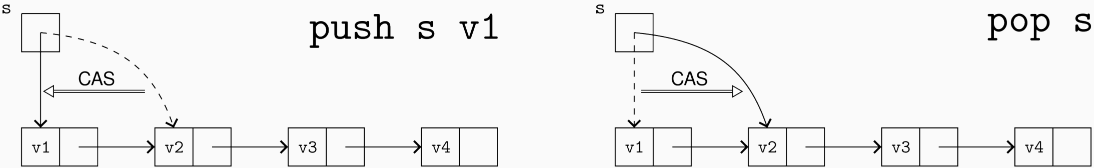
Formal Verification of Heap Space Bounds Under Garbage Collection

Space credits can be recovered as soon as a location becomes unreachable:
$\rightsquigarrow$ from the roots following heap paths.

Separation Logic for Heap Space with GC	High-Level	Concurrency
Madiot and Pottier [2022]	X	✓ (no examples)
Moine, Charguéraud and Pottier [2023]	✓	X
This work	✓	✓

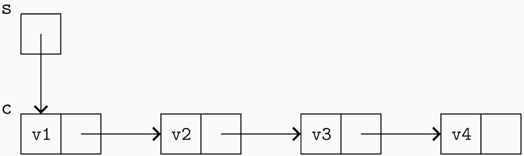
A Motivating Example: The Treiber's Stack

A lock-free linearizable stack implemented as a reference on a immutable list.

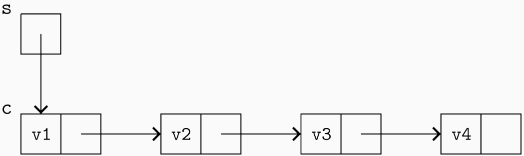


What is a space-aware specification for `push` and `pop`?

A Space-Leaking Interleaving for pop



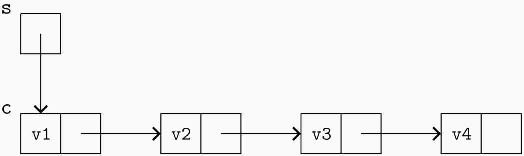
A Space-Leaking Interleaving for pop




```
pop s
```

```
pop s;
pop s;
pop s;
...
```

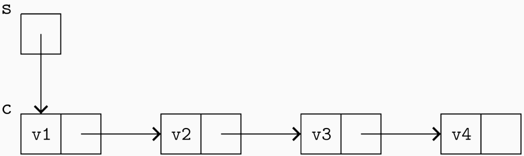
A Space-Leaking Interleaving for pop




```
let l = !s in
match l with
...
```

```
pop s;
pop s;
pop s;
...
```

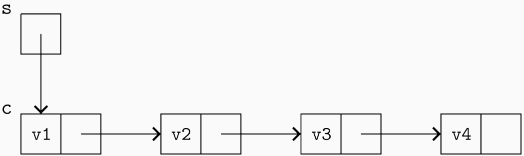
A Space-Leaking Interleaving for pop




```
let l = c in
match l with
...
```

```
pop s;
pop s;
pop s;
...
```

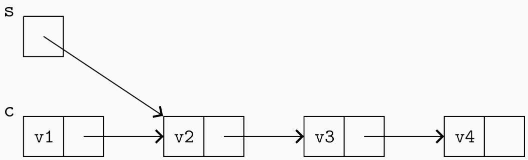
A Space-Leaking Interleaving for pop




```
let l = c in
match l with
...
```


```
pop s;
pop s;
pop s;
...
```

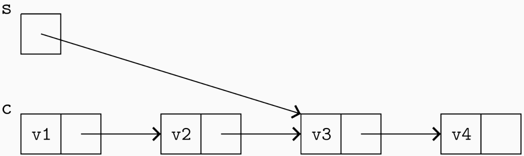
A Space-Leaking Interleaving for pop




```
let l = c in
match l with
...
```


```
pop s;
pop s;
pop s;
...
```

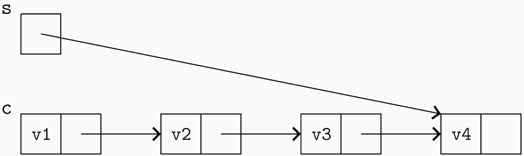
A Space-Leaking Interleaving for pop




```
let l = c in
match l with
...
```


```
pop s;
pop s;
pop s;
...
```

A Space-Leaking Interleaving for pop




```
let l = c in
match l with
...
```


```
pop s;
pop s;
pop s;
...
```

- The sleeping thread maintains reachable a morally dead structure.
- pop cannot produce space credits!

Contributions

We present the first program logic to reason about

heap space usage for a high-level and concurrent language with GC.

Key contributions:

- A new pointed-by-thread assertion, tracking in which thread a location is a root.
- Examples:
  - Lock-free data structures: Treiber's stack, Michael and Scott's queue
  - Closures: Concurrent counter
- Theory and examples are fully mechanized in Iris.

Prior Work: Pointed-by-Heap Assertions to Track Heap Predecessors

From Kassios and Kritikos [2013], Moine et al. [2023]:

$$\ell \mapsfrom_q L$$

(signed) multiset

$\in [0, 1]$

- $\ell \mapsfrom_1 L$ asserts that L is an over-approximation of the reachable predecessors of ℓ.
- $\ell \mapsfrom_1 \emptyset$ asserts that ℓ is unreachable from the heap.

$$\ell \mapsfrom_1 \{+\ell_1; +\ell_2\} \quad \mathrel{-\!\!*} \quad \ell \mapsfrom_{\frac{1}{2}} \{+\ell_1\} * \ell \mapsfrom_{\frac{1}{2}} \{+\ell_2\}$$

$$\ell \mapsfrom_{\frac{1}{2}} \{+\ell_1\} * \ell \mapsfrom_0 \{-\ell_1\} \quad \mathrel{-\!\!*} \quad \ell \mapsfrom_{\frac{1}{2}} \emptyset$$

Main invariant: if $\ell \mapsfrom_0 L$ then L must only contain negative elements.

What Are the Roots?

- We consider an interleaving semantics.
- The GC is interleaved with the per-thread reduction.

The Free Variable Rule (FVR), adapted from Felleisen and Hieb [1992]

In a substitution-based semantics, the roots of a threadpool consist of the union of the locations occurring in each thread.

Separation Logic Triples & Pointed-By-Thread Assertions

We use a ghost thread identifier π to identify a thread. $\{\,\Phi\,\}\,\pi\colon t\,\{\,\Psi\,\}$

The pointed-by-thread assertion:

$$\ell \Leftarrow\!\!|_p\ \Pi$$

set of thread identifiers

$\in (0, 1]$

- $\ell \Leftarrow\!\!|_1\ \Pi$ asserts that Π is an over-approximation of the threads in which ℓ is a root.
- $\ell \Leftarrow\!\!|_1\ \emptyset$ asserts that ℓ is not a root.

$$\ell \Leftarrow\!\!|_{(p_1+p_2)}\ (\Pi_1 \cup \Pi_2) \quad \equiv \quad \ell \Leftarrow\!\!|_{p_1}\ \Pi_1 \ * \ \ell \Leftarrow\!\!|_{p_2}\ \Pi_2$$

Our Logical Deallocation Rule

$$\ell \mapsto_p \vec{w} \;*\; \ell \hookleftarrow_1 \emptyset \;*\; \ell \Lleftarrow_1 \emptyset \quad\Rrightarrow\quad \ell \mapsto_p \vec{w} \;*\; \Diamond size(\vec{w}) \;*\; \dagger\, \ell$$

- Free does not consume the points-to. Useful for:
  - Persistent objects
  - Closing invariants

Our Logical Deallocation Rule

ℓ is a valid block

ℓ is not a root

$$\ell \mapsto_p \vec{w} \;*\; \ell \mapsfrom_1 \emptyset \;*\; \ell \Mapsfrom_1 \emptyset \quad \Rrightarrow \quad \ell \mapsto_p \vec{w} \;*\; \Diamond size(\vec{w}) \;*\; \dagger\,\ell$$

ℓ is unreachable from the heap

- Free does not consume the points-to. Useful for:
  - Persistent objects
  - Closing invariants

Our Logical Deallocation Rule

$$\ell \mapsto_p \vec{w} \ * \ \ell \longleftarrow\!\!\!\!\!\mid_1 \emptyset \ * \ \ell \Longmapsfrom_1 \emptyset \quad \overset{\text{only applicable in a precondition}}{\Rrightarrow} \quad \ell \mapsto_p \vec{w} \ * \ \Diamond \mathit{size}(\vec{w}) \ * \ \dagger \ell$$

- Free does not consume the points-to. Useful for:
    - Persistent objects
    - Closing invariants

Our Logical Deallocation Rule

$$\ell \mapsto_p \vec{w} \;\ast\; \ell \mapsfrom_1 \emptyset \;\ast\; \ell \Mapsfrom_1 \emptyset \quad \Rrightarrow \quad \underbrace{\ell \mapsto_p \vec{w}}_{\ell \text{ is still a valid block}} \;\ast\; \overbrace{\Diamond \mathit{size}(\vec{w})}^{\text{space credits}} \;\ast\; \underbrace{\dagger\,\ell}_{\ell \text{ is unreachable}}$$

- Free does not consume the points-to. Useful for:
  - Persistent objects
  - Closing invariants

Life of the Pointed-by-Thread Assertion, Part 1

The rule for allocation.

$$\{\, \Diamond n \,\}\; \pi \colon (\mathsf{alloc}\ n)\; \{\, \lambda\, \ell.\ \ \ell \mapsto [\,\overbrace{(), \ldots, ()}^{n\text{ times}}\,] \;*\; \ell \hookleftarrow \emptyset \;*\; \boxed{\ell \Lleftarrow \{\pi\}} \,\}$$

After a load, a thread points-to the loaded value.

$$\frac{0 \le i < |\vec{w}| \qquad \vec{w}(i) = v}{\{\, \ell \mapsto_r \vec{w} \;*\; \boxed{v \Lleftarrow_p \emptyset} \,\}\; \pi \colon \ell[i]\; \{\, \lambda\, v'.\, \ulcorner v' = v \urcorner \;*\; \ell \mapsto_r \vec{w} \;*\; \boxed{v \Lleftarrow_p \{\pi\}} \,\}}$$

Life of the Pointed-by-Thread Assertion, Part 2

The $\ell \Lleftarrow_p \{\pi\}$ assertion can be cleaned when ℓ does not appear in the focused part of π.

$$\frac{\ell \notin \mathit{locs}(t) \qquad \{\, \ell \Lleftarrow_p \emptyset \, * \, \Phi \,\}\, \pi \colon t \,\{\, \Psi \,\}}{\{\, \ell \Lleftarrow_p \{\pi\} \, * \, \Phi \,\}\, \pi \colon t \,\{\, \Psi \,\}}$$

$\ell \Lleftarrow_p \{\pi\}$ is force-framed when ℓ becomes a root of the evaluation context in π.

$$\frac{\mathit{locs}(t_2) = \{\ell\} \qquad \{\, \Phi \,\}\, \pi \colon t_1 \,\{\, \Psi' \,\} \qquad \forall v.\, \{\, \boxed{\ell \Lleftarrow_p \{\pi\}} * \Psi' \; v \,\}\, \pi \colon [v/x]t_2 \,\{\, \Psi \,\}}{\{\, \boxed{\ell \Lleftarrow_p \{\pi\}} * \Phi \,\}\, \pi \colon (\mathsf{let}\, x = t_1 \,\mathsf{in}\, t_2) \,\{\, \Psi \,\}}$$

Forking!

A fork updates pointed-by-thread assertions.

$$\frac{\begin{array}{c} \mathit{locs}(t) = \{\ell\} \\ (\forall \pi'.\quad \{\, \ell \Lleftarrow_p \{\pi'\} \ast \Phi \,\}\, \pi' : t \,\{\, \lambda\_.\, \ulcorner \mathsf{True} \urcorner \,\}) \end{array}}{\{\, \ell \Lleftarrow_p \{\pi\} \ast \Phi \,\}\, \pi : \mathsf{fork}\; t \,\{\, \lambda\_.\, \ulcorner \mathsf{True} \urcorner \,\}}$$

The Soundness Theorem

Our semantics

- is parameterized by a maximal heap size S
- interleaves reduction steps and GC steps

An allocation is stuck if, after a full GC, there is not enough free space.

Soundness Theorem

If $\{\ \Diamond S\ \}\ \pi\colon t\ \{\ \Psi\ \}$ holds, then t cannot reach a stuck configuration.

Reformulation: the live heap space of any execution of t cannot exceed S.

Treiber's Push – The Standard Specification Using Atomic Triples

Private precondition $\longrightarrow$ Public precondition $\longleftarrow$ Public postcondition $\longleftarrow$ Private postcondition $\longrightarrow$

$$\frac{\{\ \text{stack-inv}\ s\ \}}{\dfrac{\begin{array}{c}\langle\ \forall L.\ \text{content}\ s\ L\ \rangle \\ \pi : \text{push}\ s\ v \\ \langle\ \text{content}\ s\ (v :: L)\ \rangle\end{array}}{\{\ \lambda\_.\ \ulcorner\text{True}\urcorner\ \}}}$$

- The public precondition is atomically updated into the public postcondition.
- The user can open invariants around a public precondition and postcondition.

Space-Aware Treiber's Push

- Stack of unboxed values $\rightsquigarrow$ focus on the space consumption of the structure.
- A fractional access token for the stack ($r \in (0, 1]$):

$$\mathsf{stack}\ s\ r$$

If $r = 1$ then the stack is not being concurrently used.

$$\frac{\dfrac{\{\ \mathsf{stack}\ s\ r\ *\ \Diamond 2\ \}}{\langle\ \forall L.\ \mathsf{content}\ s\ L\ \rangle}}{\dfrac{\pi\colon \mathsf{push}\ s\ v}{\dfrac{\langle\ \mathsf{content}\ s\ (v :: L)\ \rangle}{\{\ \lambda\_.\ \mathsf{stack}\ s\ r\ \}}}}$$

Space-Aware Treiber's Pop

- pop cannot give back space.
- It returns virtual credits $\blacklozenge_s\, 2$.

$$\frac{\{\ \text{stack}\ s\ r\ \}}{\dfrac{\langle\ \forall\, v\, L.\ \text{content}\ s\ (v :: L)\ \rangle \quad \pi : \text{pop}\ s \quad \langle\ \text{content}\ s\ L\ \rangle}{\{\ \lambda\, v'.\ \ulcorner v' = v \urcorner\ *\ \text{stack}\ s\ r\ *\ \blacklozenge_s\, 2\ \}}}$$

If the stack is not being concurrently used,
virtual credits can be converted to physical credits.

$$\text{stack}\ s\ 1\ *\ \blacklozenge_s\, 2 \quad \Rrightarrow \quad \text{stack}\ s\ 1\ *\ \lozenge 2$$

Allocation and Deallocation of a Stack

Allocation of a stack:

$$\{\,\Diamond 1\,\}\,\pi : \mathsf{create}\,()\,\{\,\lambda s.\ \mathsf{stack}\ s\ 1\ *\ \mathsf{content}\ s\ []\ *\ s \hookleftarrow \emptyset\ *\ s \Lleftarrow \{\pi\}\,\}$$

Logical deallocation of a stack:

$$\mathsf{stack}\ s\ 1\ *\ \mathsf{content}\ s\ L\ *\ s \hookleftarrow \emptyset\ *\ s \Lleftarrow \emptyset \quad \Rrightarrow$$

$$\mathsf{stack}\ s\ 1\ *\ \Diamond(1 + 2 \times |L|)$$

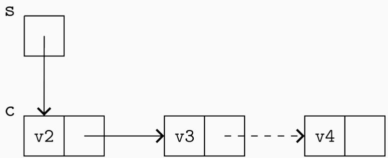
Stack With Boxed Values

$$\begin{array}{c} \{\ \mathsf{stack}\ s\ r \ \ast\ v \mapsfrom_q \emptyset \ \ast\ v \Mapsfrom_p \{\pi\} \ \ast\ \Diamond 2\ \} \\ \hline \langle\ \forall L.\ \mathsf{content}\ s\ L\ \rangle \\ \pi : \mathsf{push}\ s\ v \\ \langle\ \mathsf{content}\ s\ ((v, q, p) :: L)\ \rangle \\ \hline \{\ \lambda\_.\ \mathsf{stack}\ s\ r\ \} \end{array}$$

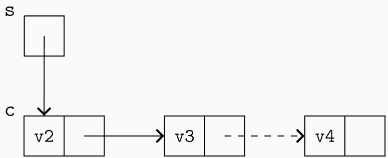
The specification of pop is also richer:

- The pointed-by-heap assertion cannot be returned without an additional write.
- Can the pointed-by-thread assertion be returned?
  $\rightsquigarrow$ yes, but we need to change the semantics.

Stack With Boxed Values – Another Dangerous Interleaving

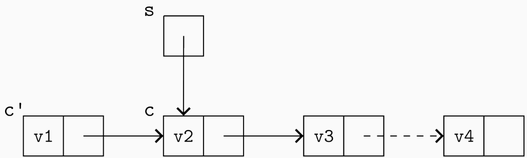


Stack With Boxed Values – Another Dangerous Interleaving



`push s v1`	`pop s;` `...`

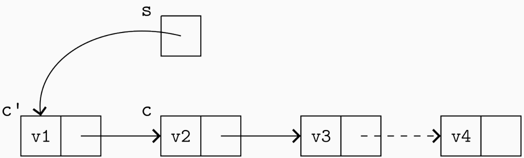
Stack With Boxed Values – Another Dangerous Interleaving




```
if CAS s c c'
then ()
else push s v1 -- retry
```

```
pop s;
...
```

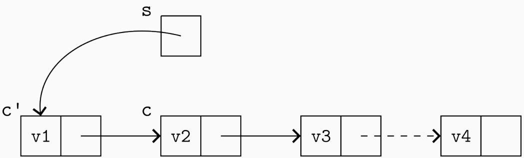
Stack With Boxed Values – Another Dangerous Interleaving




```
if true
then ()
else push s v1 -- retry
```

```
pop s;
...
```

Stack With Boxed Values – Another Dangerous Interleaving

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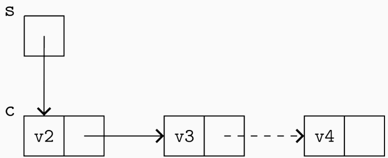
Stack With Boxed Values – Another Dangerous Interleaving




```
if true
then ()
else push s v1 -- retry
```

```
pop s;
...
```

The sleeping thread apparently maintains reachable the pushed value.

Safe Points to the Rescue

The scenario of the previous slide does not occur in OCaml 5.

- The GC is stop-the-world.
- Threads run independently and, at safe points, lookup if a GC is pending.
- The GC runs only when every thread reached a safe point.
- In particular, no safe point before `if true then _ else _`.
- This expression always releases the roots of the `else` branch.
- Interleaving of small steps is too fine. We need a coarser interleaving.

For now: an atomic ifCAS primitive.

Not Shown Here

More meta-theory:

- Encoding of the logic for sequential programs
- Simplified mode where no deallocation is required
- Closures

More examples (with the indirect help of some of you!):

- Michael & Scott's queue, based on the proof of Vindum and Birkedal [2021]
- Async-Finish library (proof involving later credits [Spies et al., 2022])

$\rightsquigarrow$ Some automation thanks to Diaframe [Mulder et al., 2022]

Conclusion & Future Work

We present the first program logic to reason about

heap space usage for a high-level and concurrent language with GC.

Please talk to me if you know data structures with a cool space usage under GC!

Independently of heap space, our logic allows reasoning about unreachability.
$\rightsquigarrow$ Can we apply our ideas to other areas?

Thank you for your attention!

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Cycles

We handle cycles following the approach of Madiot and Pottier [2022].

$$\ulcorner \mathrm{True} \urcorner \twoheadrightarrow\!\!\!\ast\ \emptyset \,\text{☁}^{0}\, P$$

$$\begin{array}{r} D \,\text{☁}^{n}\, P \\ \ell \mapsto_p \vec{v} \ast \ell \leftarrow\!\!\dashv L \ast \ell \Leftarrow\!\!\dashv \emptyset \end{array} \;\; \twoheadrightarrow\!\!\!\ast\ (\{\ell\} \cup D) \,\text{☁}^{n+\mathit{size}(\vec{v})}\, P \quad \text{if } L \subseteq P$$

$$D \,\text{☁}^{n}\, D \Rrightarrow \Diamond n \ast \Big(\mathop{\Large\ast}_{\ell \in D} \dagger\ell \Big) \qquad \text{if } D \cap \mathit{locs}(t) = \emptyset$$

Proof Insight: Liveness-Based Cancellable Invariants

Recall standard cancellable invariants: $\mathsf{CInv}^{\gamma}\,\Phi \triangleq \boxed{\Phi \lor \fbox{1}^{\gamma}}$

We can define liveness-based invariants: $\mathsf{LInv}_{\ell}\,\Phi \triangleq \boxed{\Phi \lor \dagger\ell}$

- The user can access the invariant as long as the location is logically allocated.
- Deallocation cancels the invariant.
- Useful to avoid another fractional token.

The semantics ensures no dangling pointers.

$$\frac{\ell \in \mathit{locs}(t)}{\{\,\dagger\ell\,\}\;\pi\colon t\;\{\,\Psi\,\}}$$

Triples with Souvenir

Pointed-by-thread assertions are easy to manage in practice.

Introducing triples with souvenir $[\,R\,]\{\,\Phi\,\}\,\pi\colon t\,\{\,\Psi\,\}$

"Give a pointed-by-thread assertion once and that's it"

$$\frac{locs(t_2) = \{\ell\} \qquad [\,R \cup \{\ell\}\,]\{\,\Phi\,\}\,\pi\colon t_1\,\{\,\Psi'\,\} \qquad \forall v.\,[\,R\,]\{\,\ell \Lleftarrow_p \{\pi\} * \Psi'\ v\,\}\,\pi\colon [v/x]t_2\,\{\,\Psi\,\}}{[\,R\,]\{\,\ell \Lleftarrow_p \{\pi\} * \Phi\,\}\,\pi\colon \mathrm{let}\,x = t_1\,\mathrm{in}\,t_2\,\{\,\Psi\,\}}$$

$$\frac{locs(t_2) = \{\ell\} \qquad \ell \in R \qquad [\,R\,]\{\,\Phi\,\}\,\pi\colon t_1\,\{\,\Psi'\,\} \qquad \forall v.\,[\,R\,]\{\,\Psi'\ v\,\}\,\pi\colon [v/x]t_2\,\{\,\Psi\,\}}{[\,R\,]\{\,\Phi\,\}\,\pi\colon \mathrm{let}\,x = t_1\,\mathrm{in}\,t_2\,\{\,\Psi\,\}}$$

The NoFree Mode

If the user pledges not to clean, framing pointed-by-thread assertions is not needed.

$$\frac{\textsc{LetNoClean}\quad [\,\text{NoClean}\,]\{\,\Phi\,\}\,\pi\colon t_1\,\{\,\Psi'\,\} \qquad \forall v.\ [\,R\,]\{\,\Psi'\,v\,\}\,\pi\colon [v/x]t_2\,\{\,\Psi\,\}}{[\,R\,]\{\,\Phi\,\}\,\pi\colon \text{let}\,x = t_1\ \text{in}\ t_2\,\{\,\Psi\,\}}$$

Matthias Felleisen and Robert Hieb. The revised report on the syntactic theories of sequential control and state. Theoretical Computer Science, 103(2):235–271, 1992. URL https://www2.ccs.neu.edu/racket/pubs/tcs92-fh.pdf.

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